## Abstract Argumentation – All Problems Solved?

#### Stefan Woltran

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Argumentation is the study of processes <u>"concerned with how assertions</u> are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held".

[Bench-Capon and Dunne: Argumentation in Al. Artif. Intell., 171:619-641, 2007]

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- "The idea of argumentational reasoning is that a statement is believable if it can be argued successfully against <u>attacking</u> arguments."



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- "The idea of argumentational reasoning is that a statement is believable if it can be argued successfully against <u>attacking</u> arguments."
- "[...] a formal, <u>abstract</u> but simple theory of argumentation is developed to capture the notion of <u>acceptability</u> of arguments."

#### Argumentation Frameworks

... thus abstract away from everything but attacks (calculus of opposition)



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How to obtain such frameworks? ... identify conflicting information

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## How to obtain such frameworks? ... identify conflicting information (it is everywhere!)

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Domain	Argument	Attack	Aim
People	person	"dislike"	coalition formation
DSupport	statement	"conflict"	conflict resolution
BBS	message	reply	identify opinion leaders
KB	$(\Phi, \alpha)$	$ eg lpha \in \mathit{Cn}(\Phi')$	inconsistency handling
LP	derivation	viol. assumption	comparison LP semantics
DL	support chain	viol. justification	nonmonotonic logics

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- Background on Argumentation Frameworks
- State of the Art and Open Questions
  - Complexity
  - 2 Expressibility (Signatures and Numbers)
  - Translations
  - O Explicit-Conflict Conjecture
  - Oynamics (Strong Equivalence and Enforcement)
  - O Labellings and some further issues
- Conclusion

#### Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathcal{A}$  is a finite set of arguments and
- $R \subseteq A \times A$  is the attack relation representing conflicts.

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## Example



## $F = (\{a, b, c, d, e, f\}, \\ \{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d), (c, f), (d, f), (f, f), (f, e)\})$

#### Conflict-free Sets

Given an AF F = (A, R), a set  $E \subseteq A$  is conflict-free in F, if, for each  $a, b \in E$ ,  $(a, b) \notin R$ .

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 $cf(F) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{a, b\}, \{a, e\}, \{a, e\}, \{b, c\}, \{b, e\}, \{d, e\}, \{c, e\}, \{c$ 

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Given an AF F = (A, R), a set  $E \subseteq A$  is a stable extension in F, if

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# Example $adm(F) = \{ \{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{a, b\}, \{a, d\}, \{a, e\}, \{b, c\}, \{d, e\}, \{c, e\},$

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#### Complete Sets

Given an AF F = (A, R), a set  $E \subseteq A$  is complete in F, if

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# Example $comp(F) = \{\{a, d, e\}, \{b, c, e\}, \}$ $\{a, b\}, \{a, d\}, \{b, c\}, \{d, e\}, \{c, e\}, \{c,$ $\{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

### Preferred Extensions

Given an AF F = (A, R), a set  $E \subseteq A$  is a preferred extension in F, if

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 $\Rightarrow$  Maximal admissible sets (w.r.t. set-inclusion).

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#### **Further Semantics**

For (A, R) and  $E \subseteq A$ ,  $E_R^+ = E \cup \{b \mid (a, b) \in R\}$  denotes the range.

- semi-stable (sem): admissible sets with subset-maximal range
- stage: conflict-free sets with subset-maximal range

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#### Unique-status semantics:

- grounded (grd): subset-minimal complete set
- ideal: subset-maximal adm set contained in each pref extension
- eager: subset-maximal adm set contained in each sem extension

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Other semantics we touch in this talk:

- naive (subset-maximal conflict-free sets)
- cf2
- resolution-based grounded (resgr)

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Meanwhile, an invasion of semantics! Bug or feature?





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# Issue 1: Complexity

σ	$Cred_{\sigma}$	$\mathit{Skept}_{\sigma}$	$Ver_{\sigma}$	$NE_{\sigma}$
cf	in L	trivial	in L	in L
naive	in L	in L	in L	in L
grd	P-c	P-c	P-c	in L
stb	NP-c	coNP-c	in L	NP-c
adm	NP-c	trivial	trivial	NP-c
сотр	NP-c	P-c	in L	NP-c
resgr	NP-c	coNP-c	P-c	in P
cf2	NP-c	coNP-c	in P	in L
ideal	in $\Theta_2^P$	in $\Theta_2^P$	in $\Theta_2^P$	in $\Theta_2^P$
pref	NP-c	$\Pi_2^P$ -c	coNP-c	NP-c
sem	$\Sigma_2^P$ -c	П <sub>2</sub> <sup>P</sup> -с	coNP-c	NP-c
stage	$\Sigma_2^P$ -c	$\Pi_2^P$ -c	coNP-c	in L
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### Some observations for ideal semantics

It is known that  $Cred_{ideal}$  is coNP-hard,  $Ver_{ideal}$  is D<sup>P</sup>-hard, and  $NE_{ideal}$  is NP-hard. Moreover,

- If  $Cred_{ideal}$  is NP-hard, then  $Cred_{ideal}$  is  $\Theta_2^P$ -complete.
- If Cred<sub>ideal</sub> is in coNP, then NE<sub>ideal</sub> is NP-complete.
- If *Cred*<sub>ideal</sub> is in coNP, then *Ver*<sub>ideal</sub> is D<sup>P</sup>-complete.

#### Some further open questions

- P-hardness for Ver<sub>cf2</sub>
- problems defined on subclasses of graphs and parameterized complexity

### Definition

The signature of a semantics  $\sigma$  is defined as

$$\Sigma_{\sigma} = \{\sigma(F) \mid F \text{ is an AF } \}.$$

Thus signatures capture all what a semantics can express.

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Some Notation

Call a set of sets of arguments  $\ensuremath{\mathcal{S}}$  extension-set. Moreover,

• 
$$Args_{\mathcal{S}} = \bigcup_{S \in \mathcal{S}} S$$

•  $Pairs_{\mathcal{S}} = \{\{a, b\} \mid \exists E \in \mathcal{S} \text{ with } \{a, b\} \subseteq E\}$ 

#### Example

Given 
$$S = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$$
:  
 $Args_S = \{a, b, c, d, e\},$   
 $Pairs_S = \{\{a, b\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, e\}, \{c, e\}, \{d, e\}\}$ 

#### Definition

#### An extension-set $\mathcal{S}$ is called

- tight if S is incomparable and for all E ∈ S and all a ∈ Args<sub>S</sub> \ E there exists e ∈ E such that {a, e} ∉ Pairs<sub>S</sub>
- pref-closed if for each A, B ∈ S with A ≠ B, there exist a, b ∈ (A ∪ B) such that {a, b} ∉ Pairs<sub>S</sub>

#### Proposition

• 
$$\Sigma_{stb} = \{ S \mid S \text{ is tight } \}$$

•  $\Sigma_{pref} = \{ \mathcal{S} \neq \emptyset \mid \mathcal{S} \text{ is pref-closed } \}$ 

Further exact characterizations avaliable for *cf*, *adm*, *sem*, *stage*, and *naive*.

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Question:

How to adapt the AF to replace  $\{a, b\}$  by  $\{a, b, d\}$  in pref(F)?

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#### Question:

How to adapt the AF to replace  $\{a, b\}$  by  $\{a, b, d\}$  in *pref*(*F*)? Impossible!  $\{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$  is not pref-closed. (An extension-set S is pref-closed if for each  $A, B \in S$  with  $A \neq B$ , there exist  $a, b \in (A \cup B)$  such that  $\{a, b\} \notin Pairs_S$ )

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Exact characterization for complete, *resgr*, and *cf2* semantics open.

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### Proposition

For  $\sigma \in \{stb, sem, pref, stage, naive\}$  the maximal number of  $\sigma$ -extensions that can be obtained by an AF with *n* arguments is

$$\sigma^{max}(n) = \begin{cases} 1 & \text{if } n \le 1, \\ 3^{s} & \text{if } n \ge 2 \text{ and } n = 3s, \\ 4 \cdot 3^{s-1} & \text{if } n \ge 2 \text{ and } n = 3s+1, \\ 2 \cdot 3^{s} & \text{if } n \ge 2 \text{ and } n = 3s+2. \end{cases}$$

For  $\sigma \in \{adm, cf\}$ , clearly  $\sigma^{max}(n) = 2^n$ .

### Proposition

For  $\sigma \in \{stb, sem, pref, stage, naive\}$  the maximal number of  $\sigma$ -extensions that can be obtained by an AF with *n* arguments is

$$\sigma^{max}(n) = \begin{cases} 1 & \text{if } n \le 1, \\ 3^s & \text{if } n \ge 2 \text{ and } n = 3s, \\ 4 \cdot 3^{s-1} & \text{if } n \ge 2 \text{ and } n = 3s+1, \\ 2 \cdot 3^s & \text{if } n \ge 2 \text{ and } n = 3s+2. \end{cases}$$

For  $\sigma \in \{adm, cf\}$ , clearly  $\sigma^{max}(n) = 2^n$ .

Question: What about  $\sigma^{max}(n)$  for the remaining (non unique-status) semantics, in particular complete?

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### Definition

Call a mapping  $\tau$  from AFs to AFs translation if  $\tau(F)$  can be computed in log space with respect to the size of F. Moreover, for semantics  $\sigma, \sigma', \tau$  is

- exact (for  $\sigma \Rightarrow \sigma'$ ) if for every AF F,  $\sigma(F) = \sigma'(\tau(F))$
- faithful (for  $\sigma \Rightarrow \sigma'$ ) if for every AF F = (A, R),  $\sigma(F) = \{E \cap A \mid E \in \sigma'(\tau(F))\}$  and  $|\sigma(F)| = |\sigma'(\tau(F))|$ .

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Gives another account on expressibility:

- more fine-grained than computational complexity
- equal signatures do not imply mutual translations

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# Issue 3: Translations

### Proposition

The transformation  $\tau$  mapping each AF (A, R) to (A', R'), with  $A' = A \cup \overline{A}$  and  $R' = R \cup \{(a, \overline{a}), (\overline{a}, a), (\overline{a}, \overline{a}) \mid a \in A\}$ , is an exact translation for *adm*  $\Rightarrow$  *comp*, *naive*  $\Rightarrow$  *stage* and *pref*  $\Rightarrow$  *sem*.

#### Example



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### Issue 3: Translations



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# Issue 4: Conflict-Explicit Conjecture

#### Definition

We call an AF F = (A, R) conflict-explicit under  $\sigma$  iff for each  $a, b \in A$ such that  $\{a, b\} \notin Pairs_{\sigma(F)}$ ,  $(a, b) \in R$  or  $(b, a) \in R$  (or both)

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### Conflict-Explicit Conjecture (Stable Case)

For each AF F = (A, R) there exists an AF F' = (A, R') which is conflict-explicit under the stable semantics and equivalent to F, i.e. stb(F) = stb(F')

### Conflict-Explicit Conjecture (Stable Case)

For each AF F = (A, R) there exists an AF F' = (A, R') which is conflict-explicit under the stable semantics and equivalent to F, i.e. stb(F) = stb(F')

Why important? Why tricky? ... attend our presentation on Friday!

#### Definition

Two AFs F, G are strongly equivalent wrt.  $\sigma$  (in symbols  $F \equiv_s^{\sigma} G$ ), if for any H,  $\sigma(F \cup H) = \sigma(G \cup H)$ 

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#### Proposition

$$(A, R) \equiv_{s}^{stb} (B, S)$$
 iff  $A = B$  and  $R^{k} = S^{k}$  where  $R^{k} = R \setminus \{(a, b) \in R \mid a \neq b, (a, a) \in R\}.$ 

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### Two AFs strongly equivalent under stable semantics





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#### Kernels

Given an AF F = (A, R). We can have different kernels  $F^{k\alpha} = (A, R^{k\alpha})$ :

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#### Kernels

Given an AF F = (A, R). We can have different kernels  $F^{k\alpha} = (A, R^{k\alpha})$ :

• 
$$R^{k1} = R \setminus \{(a, b) \mid a \neq b, (a, a), (b, b) \in R\}$$
  
•  $R^{k2} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R, \{(b, a), (b, b)\} \cap R \neq \emptyset\}$   
•  $R^{k3} = R \setminus \{(a, b) \mid a \neq b, (b, b) \in R, \{(a, a), (b, a)\} \cap R \neq \emptyset\}$ 

• 
$$R^{n+1} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}$$

#### Proposition

For any two AFs F and G, we have that  $F \equiv_s^{\sigma} G$  iff

• 
$$F = G$$
 for  $\sigma = cf2$ 

### Enforcing. Observation:

Let F = (A, R) be an AF. Then for any  $S \in cf(F)$ , there is an AF F' = (A', R') with  $A \subseteq A'$ ,  $R \subseteq R'$  such that  $S \in \sigma(F')$ ( $\sigma \in \{adm, comp, naive, stb, pref, stage, sem\}$ ).

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## Issue 5: Dynamics

#### Enforcing. Observation:

Let F = (A, R) be an AF. Then for any  $S \in cf(F)$ , there is an AF F' = (A', R') with  $A \subseteq A'$ ,  $R \subseteq R'$  such that  $S \in \sigma(F')$ ( $\sigma \in \{adm, comp, naive, stb, pref, stage, sem\}$ ).

#### Example: Enforcing $\{a, b\}$ under *stb*



# Issue 5: Dynamics

More general attempt:

- do not allow for any enhancement (use relation Φ between AFs); e.g. strong, normal, weak expansions
- what is the minimal number of attacks to be added?

#### Proposition

Let  $N_{\sigma,\Phi}^F(C)$  be the mininum number of attacks to be added (possibly infinite) to enforce C in F under semantics  $\sigma$ . Then, for any F and C,

$$N^{F}_{stb,\Phi}(C) \ge N^{F}_{sem,\Phi}(C) \ge N^{F}_{pref,\Phi}(C) = N^{F}_{comp,\Phi}(C) = N^{F}_{adm,\Phi}(C)$$

For several  $\Phi$  exact numbers  $N_{\sigma,\Phi}^F$  for *adm*, *comp*, *pref* and *stb* known *sem* is much harder to characterize due to missing local criteria!

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#### Definition

Given an AF (A, R), a function  $\mathcal{L} : A \to \{in, out, undec\}$  is a labeling iff the following conditions hold:

- $\mathcal{L}(a) = in$  iff for each b with  $(b, a) \in R$ ,  $\mathcal{L}(b) = out$
- $\mathcal{L}(a) = out$  iff there exists b with  $(b, a) \in R$ ,  $\mathcal{L}(b) = in$

Preferred labelings are those where  $\mathcal{L}_{in}$  is  $\subseteq$ -maximal among all labelings

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Preferred labelings are those where  $\mathcal{L}_{in}$  is  $\subseteq$ -maximal among all labelings

- 1-1 correspondence between preferred labelings and preferred extensions
- similar definitions for other semantics avaliable
- labellings provide additional information about arguments not contained in extension

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Example: Preferred Labelings



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Most we have discussed before, in particular

- Signatures
- Translations
- Strong Equivalence
- Enforcement
- is still unexplored for labellings.

- Modularization: which properties of semantics can be obtained locally and put together suitably
  - $\Rightarrow$  recall SCC recursive semantics
  - $\Rightarrow\,$  interesting relation to dynamic programming algorithms for AFs
- From weaker notions of equivalence (projection) to I-O-specifications.
- Does every finitary AF posses a *cf2* extension? ....

#### Summary

- Argumentation a highly active area in AI
- Dung's abstract frameworks a gold standard within the community
- AFs provide account of how to select acceptable arguments solely on basis of an attack relation between them
- Useful analytical tool with a variety of semantics and add-ons
- Huge body of theoretical results, but some surprisingly simple questions still unresolved.

# Conclusion

Isn't that all just graph theory?



# Conclusion

Isn't that all just graph theory?



#### No . . .

- Edges have different meaning (connection vs. attack)
- Paths have different meaning (reachability vs. defense)
- Different abstraction model
- Still,
  - stable extensions  $\Leftrightarrow$  independent dominating sets
  - several graph classes also important in Argu (acyclic, bipartite, ...)
  - and some of our problems/results might also be of interest for graph theory people

#### Future Research Directions



- Abstract away from concrete semantics
- Incorporate theoretical results to systems
- ... and solve the open problems!

#### Thanks and Credits go to:



... any many more!

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### See you at COMMA 2014. Sept 9 – 12, Scotland.



http://comma2014.arg.dundee.ac.uk/

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