

# Expressibility of Argumentation Frameworks and its Relation to the Dynamics of Argumentation

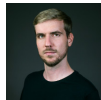
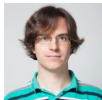
Stefan Woltran

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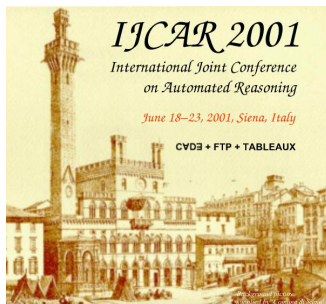
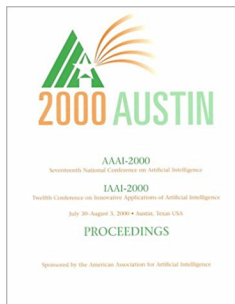
April 10th, 2018

Joint work with

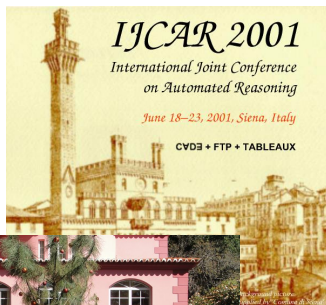
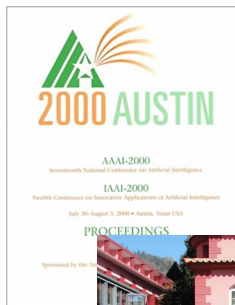
Martin Diller, Paul Dunne, Wolfgang Dvořák, Adrian Haret, Thomas Linsbichler, Stefan Rümmele.



# Prologue



# Prologue



## Dynamics of (Abstract) Argumentation

- Change in Argumentation Frameworks:
  - ▶ Boella, Kaci, van der Torre; Bisquert, Cayrol, Dupin de Saint-Cyr, Lagasque-Schiex; Baroni, Giacomin, Liao; Alfano, Greco, Parisi; Coste-Marquis, Devred, Konieczny, Lagasque-Schiex, Marquis; Doutre, Herzig, Perussel.
- Enforcement:
  - ▶ Baumann; Coste-Marquis, Konieczny, Mailly, Marquis; Järvisalo, Niskanen, Wallner; Kontarinis, Bonzon, Maudet, Perotti, van der Torre, Villata; Nouioua, Würbel; Booth, Kaci, Rienstra, van der Torre.
- AGM Belief Change applied to Argumentation Frameworks:
  - ▶ Baumann and Brewka; Coste-Marquis, Konieczny, Mailly, Marquis.
  - ▶ Dupin de Saint-Cyr, Bisquert, Cayrol, Lagasque-Schiex.
  - ▶ Dellobelle, Haret, Konieczny, Mailly, Rossit, W.
  - ▶ Moguillansky, Simari.

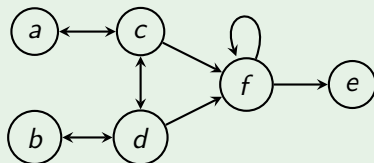
- Expressibility of Argumentation Frameworks
- Revision of Argumentation Frameworks
- Shifting from an Argument-Centric to a Claim-Centric View

# Expressibility of AFs

## Argumentation Frameworks

... abstract away from everything but attacks (calculus of opposition)

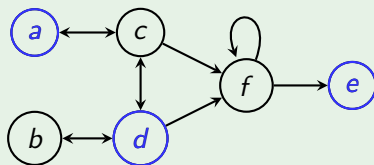
### Example



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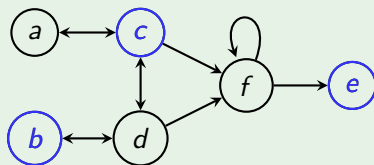


$$\text{naive}(F) = \{\{a, d, e\},$$

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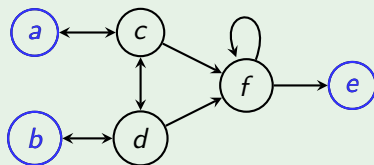
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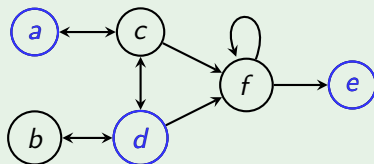


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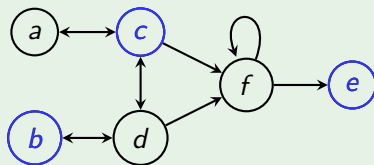


$$\begin{aligned} \text{naive}(F) &= \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\} \\ \text{stb}(F) &= \{\{a, d, e\}, \end{aligned}$$

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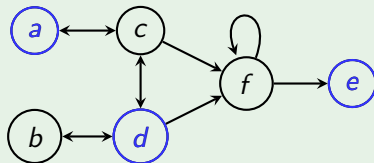
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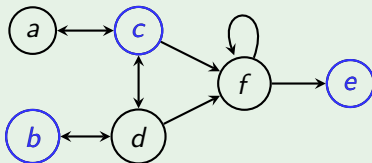
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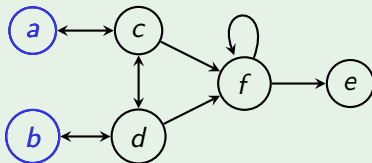
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# Expressibility of AFs

## Definition

The **signature** of a semantics  $\sigma$  is defined as

$$\Sigma_\sigma = \{ \sigma(F) \mid F \text{ is an AF} \}.$$

Thus signatures capture all what a semantics can express.

# Expressibility of AFs

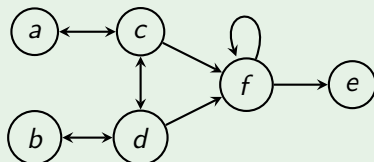
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## Example



- $\mathcal{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{pref}$
- Question: Can we change the AF, such that  $\mathcal{S}' = \{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$  become preferred extensions; in other words does  $\mathcal{S}' \in \Sigma_{pref}$  hold?
- $\mathcal{S} \in \Sigma_{stb}$ ?



# Expressibility of AFs

## Some Notation

Call a set of sets of arguments  $\mathcal{S}$  extension-set. Moreover,

- $Args_{\mathcal{S}} = \bigcup_{S \in \mathcal{S}} S$
- $Pairs_{\mathcal{S}} = \{\{a, b\} \mid \exists E \in \mathcal{S} \text{ with } \{a, b\} \subseteq E\}$

## Definition

An extension-set  $\mathcal{S}$  is called **conflict-sensitive** if for each  $A, B \in \mathcal{S}$  such that  $A \cup B \notin \mathcal{S}$  it holds that  $\exists a, b \in A \cup B : \{a, b\} \notin Pairs_{\mathcal{S}}$ .

## Example

Given  $\mathcal{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ :

$Args_{\mathcal{S}} = \{a, b, c, d, e\}$

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Observation:  $\mathcal{S}$  is conflict-sensitive;  $\{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$  is not!

## Proposition

For any AF  $F$ ,  $\text{pref}(F)$  is conflict-sensitive.

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Recall:  $\mathcal{S}$  is **conflict-sensitive** if for each  $A, B \in \mathcal{S}$  such that  $A \cup B \notin \mathcal{S}$  it holds that  $\exists a, b \in A \cup B : (a, b) \notin \text{Pairs}_{\mathcal{S}}$ .

Proof:

- 1 Let  $F$  be an AF.  $\text{adm}(F)$  is conflict-sensitive: Suppose  $B, C \in \text{adm}(F)$  such that  $B \cup C \notin \text{adm}(F)$ , but for all  $b, c \in B \cup C$ ,  $(b, c) \in \text{Pairs}_{\text{adm}(F)}$ .  $B \cup C$  defends itself in  $F$ . Thus,  $(b, c) \in R_F$  for some pair  $\{b, c\} \subseteq B \cup C$ . But then, for all  $D \in \text{adm}(F)$ ,  $\{b, c\} \not\subseteq D$ . Hence,  $\{b, c\} \notin \text{Pairs}_{\text{adm}(F)}$ , a contradiction.
- 2 For any conflict-sensitive  $\mathcal{S}$ , its subset-maximal elements form a set  $\mathcal{S}'$  that is conflict-sensitive, too (follows from  $\text{Pairs}_{\mathcal{S}} = \text{Pairs}_{\mathcal{S}'}$ ).

## Proposition

For any non-empty, incomparable conflict-sensitive extension set  $\mathcal{S}$ , there exists an AF  $F$ , such that  $\text{pref}(F) = \mathcal{S}$ .

## Theorem

$\Sigma_{\text{pref}} = \{ \mathcal{S} \mid \mathcal{S} \neq \emptyset \text{ is incomparable and conflict-sensitive} \}$ .

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## Proposition (Limitation)

There exist incomparable sets  $\mathcal{S}$ , such that  $\mathcal{S} \notin \Sigma_{\text{pref}}$ .

Examples:  $\mathcal{S} = \{ \{a, b\}, \{a, c\}, \{b, c\} \}$   
 $\mathcal{S} = \{ \{a, d, e\}, \{b, c, e\}, \{a, b, d\} \}$

## Important Consequence

For any semantics  $\sigma$  satisfying, for any AF  $F = (A, R)$ ,

- (i)  $\sigma(F) \neq \emptyset$ ;
- (ii)  $\sigma(F) \subseteq cf(F)$ ;
- (iii)  $\sigma(F)$  is incomparable; and
- (iv) for all  $S_1, S_2 \in \sigma(F)$  ( $S_1 \neq S_2$ ) there exist  $a, b \in S_1 \cup S_2$  with  $(a, b) \in R$ .

it holds that  $\Sigma_\sigma \subseteq \Sigma_{pref}$ .

# Expressibility of AFs

## Definition

Given a collection  $\mathcal{S}$  of sets of arguments, define

$$\text{Confs}_{\mathcal{S}} = \{\{a, b\} \subseteq \text{Args}_{\mathcal{S}} \mid \nexists S \in \mathcal{S} : a, b \in S\}, \text{ and}$$

$$\text{bd}(\mathcal{S}) = \{T \subseteq \text{Args}_{\mathcal{S}} \mid b \in \text{Args}_{\mathcal{S}} \setminus T \text{ iff } \exists a \in T : \{a, b\} \in \text{Confs}_{\mathcal{S}}\}.$$

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$$\Sigma_{\text{stb}} = \{\mathcal{S} \mid \mathcal{S} \subseteq \text{bd}(\mathcal{S})\}$$



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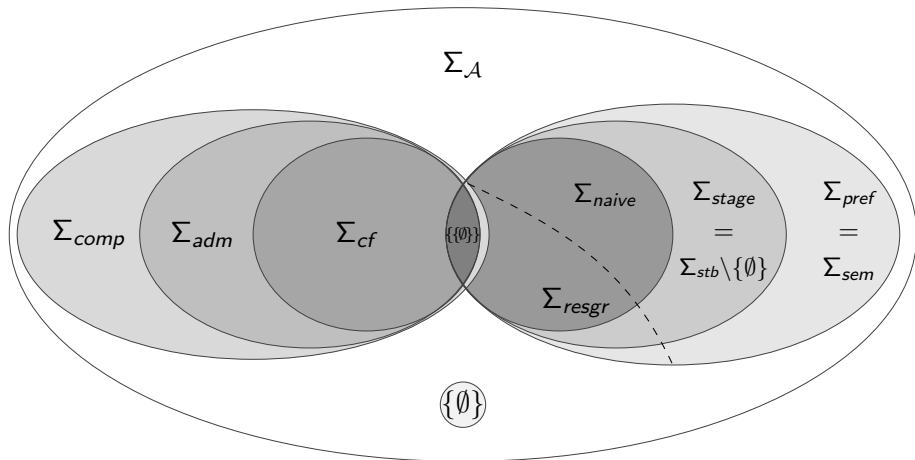
For  $\mathcal{S} = \{\{a, b\}, \{a, c, e\}, \{b, d, e\}\} \in \Sigma_{\text{pref}}$ , we have

$$\text{Confs}_{\mathcal{S}} = \{\{a, d\}, \{b, c\}, \{c, d\}\}$$

$$\text{bd}(\mathcal{S}) = \{\{a, b, e\}, \{a, c, e\}, \{b, d, e\}\}$$

Hence,  $\mathcal{S} \notin \Sigma_{\text{stb}}$ !

# Expressibility of AFs



## Definition ( $\cap$ -closure)

For any AFs  $F_1, F_2$  such that  $\mathcal{S} = \sigma(F_1) \cap \sigma(F_2) \neq \emptyset$  there exists an AF  $F$  with  $\sigma(F) = \mathcal{S}$ .

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## Definition ( $\subseteq$ -closure)

For any AF  $F$  and non-empty set  $\mathcal{S} \subseteq \sigma(F)$  there exists an AF  $G$  with  $\sigma(G) = \mathcal{S}$ .

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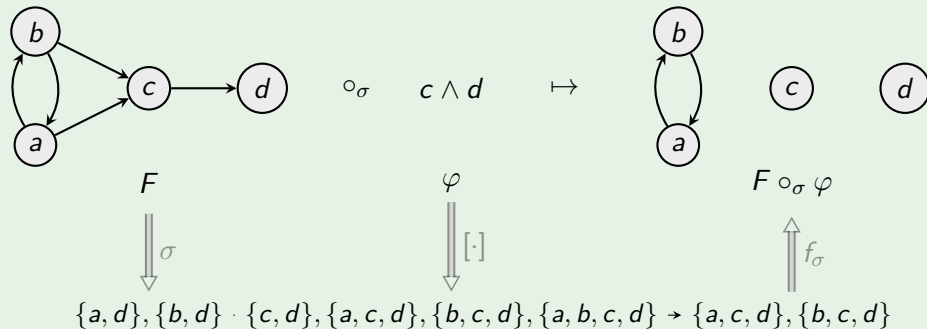
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	<i>cf</i>	<i>adm</i>	<i>comp</i>	<i>naive</i>	<i>pref</i>	<i>stb</i>	<i>stage</i>	<i>sem</i>
$\supseteq$	<b>x</b>	<b>x</b>	<b>x</b>	<b>x</b>	✓	✓	✓	✓
$\cap$	✓	✓	<b>x</b>	✓	✓	✓	✓	✓

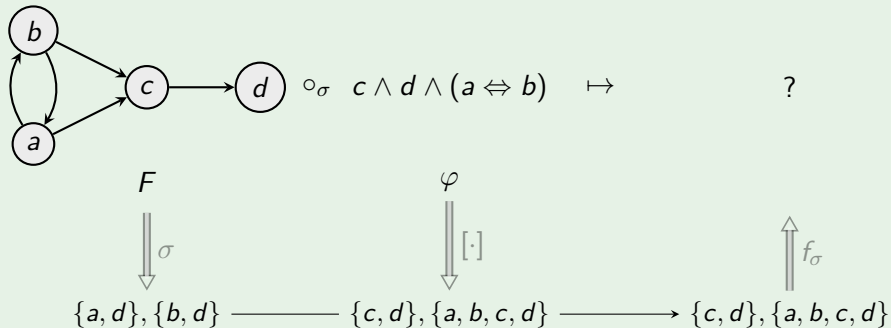
- In this talk: purely semantic approach of revision
  - ▶ AFs play the role of knowledge bases and their extensions express an agent's beliefs
  - ▶ a revision formula  $\varphi$  encodes desired changes in the status of some arguments
  - ▶ a revision operator yields a result that satisfies  $\varphi$  and preserves as much useful information from the AF as possible.
- Main Goal: Representation Theorems
  - ▶ Correspondence between revision operators given by postulates and revision operators captured by rankings.

# Revision of AFs

## Example



## Example





## Variant 1 [Coste-Marquis et al]

$$\star_\sigma: AF_{\mathcal{A}} \times \mathcal{P}_{\mathcal{A}} \mapsto 2^{AF_{\mathcal{A}}}$$

(For a set  $S$  of AFs  $\sigma(S) = \bigcup_{F \in S} \sigma(F)$ .)

$$P\star 1: \sigma(F \star_\sigma \varphi) \subseteq [\varphi].$$

$$P\star 2: \text{If } \sigma(F) \cap [\varphi] \neq \emptyset \text{ then } \sigma(F \star_\sigma \varphi) = \sigma(F) \cap [\varphi].$$

$$P\star 3: \text{If } [\varphi] \neq \emptyset \text{ then } \sigma(F \star_\sigma \varphi) \neq \emptyset.$$

$$P\star 4: \text{If } \varphi \equiv \psi \text{ then } \sigma(F \star_\sigma \varphi) = \sigma(F \star_\sigma \psi).$$

$$P\star 5: \sigma(F \star_\sigma \varphi) \cap [\psi] \subseteq \sigma(F \star_\sigma (\varphi \wedge \psi)).$$

$$P\star 6: \text{If } \sigma(F \star_\sigma \varphi) \cap [\psi] \neq \emptyset \text{ then } \sigma(F \star_\sigma (\varphi \wedge \psi)) \subseteq \sigma(F \star_\sigma \varphi) \cap [\psi].$$

## Definition

Given semantics  $\sigma$  and AF  $F$ , a pre-order  $\preceq_F$  is a **faithful ranking** if it is total and for any sets  $E_1, E_2$  and AFs  $F, F_1, F_2$ :

- (i) if  $E_1, E_2 \in \sigma(F)$ , then  $E_1 \approx_F E_2$ ,
- (ii) if  $E_1 \in \sigma(F)$  and  $E_2 \notin \sigma(F)$ , then  $E_1 \prec_F E_2$ ,
- (iii) if  $\sigma(F_1) = \sigma(F_2)$ , then  $\preceq_{F_1} = \preceq_{F_2}$ .

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## Theorem

Let  $\sigma$  a semantics such that, for each  $S \subseteq \mathcal{A}$ ,  $\{S\} \in \Sigma_\sigma$ .

An operator  $\star_\sigma$  satisfies postulates  $P\star 1 - P\star 6$  for  $\sigma$   
iff

there exists an assignment mapping each AF  $F$  to a **faithful ranking**  $\preceq_F$   
such that  $\sigma(F \star_\sigma \varphi) = \min([\varphi], \preceq_F)$ .

## Variant 2 [Diller et al]

$\circ_\sigma: AF_{\mathcal{A}} \times \mathcal{P}_{\mathcal{A}} \mapsto AF_{\mathcal{A}}$

Po1:  $\sigma(F \circ_\sigma \varphi) \subseteq [\varphi]$ .

Po2: If  $\sigma(F) \cap [\varphi] \neq \emptyset$  then  $\sigma(F \circ_\sigma \varphi) = \sigma(F) \cap [\varphi]$ .

Po3: If  $[\varphi] \neq \emptyset$  then  $\sigma(F \circ_\sigma \varphi) \neq \emptyset$ .

Po4: If  $\varphi \equiv \psi$  then  $\sigma(F \circ_\sigma \varphi) = \sigma(F \circ_\sigma \psi)$ .

Po5:  $\sigma(F \circ_\sigma \varphi) \cap [\psi] \subseteq \sigma(F \circ_\sigma (\varphi \wedge \psi))$ .

Po6: If  $\sigma(F \circ_\sigma \varphi) \cap [\psi] \neq \emptyset$  then  $\sigma(F \circ_\sigma (\varphi \wedge \psi)) \subseteq \sigma(F \circ_\sigma \varphi) \cap [\psi]$ .

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A pre-order  $\preceq$  is  $\sigma$ -compliant if for every formula  $\varphi$  it holds that  $\min([\varphi], \preceq) \in \Sigma_\sigma$ .

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## Theorem

Let  $\sigma$  a semantics which is (i)  $I$ -maximal, (ii)  $\subseteq$ -closed, and (iii) such that for all incomparable  $S_1, S_2 \subseteq \mathcal{A}$ ,  $\{S_1, S_2\} \in \Sigma_\sigma$ .

An operator  $\circ_\sigma$  satisfies postulates  $P_{\circ 1} - P_{\circ 6}$  for  $\sigma$  iff

there exists an assignment mapping each AF  $F$  to a *faithful and  $\sigma$ -compliant* ranking  $\preceq_F$  such that  $\sigma(F \circ_\sigma \varphi) = \min([\varphi], \preceq_F)$ .

## Example

- $\sigma(F) = \{\{a, b, c\}\}$ .
- $\varphi = \neg(a \wedge b \wedge c)$
- $\{a, b, c\} \prec \{a, b\} \approx \{a, c\} \approx \{b, c\} \prec \{a\} \approx \{b\} \approx \{c\} \prec \emptyset$ 
  - ▶  $\min([\varphi], \preceq) = \{\{a, b\}, \{a, c\}, \{b, c\}\} \notin \Sigma_\sigma$
  - ▶  $\preceq$  is not  $\sigma$ -compliant
- $\{a, b, c\} \prec' \{a\} \approx' \{b\} \approx' \{c\} \prec' \{a, b\} \prec' \{a, c\} \prec' \{b, c\} \prec' \emptyset$ 
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  - ▶  $\preceq$  is not  $\sigma$ -compliant
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- finding concrete AF revision operators comes down to defining appropriate (i.e. faithful and  $\sigma$ -compliant) rankings on extensions
- compliance leads to quite discriminating choices
- Example operator (indexed preorder) ranks extensions by cardinality plus a form of tie-breaking using lexicographic information



# Argumentation Frameworks with Claims

- Abstract argumentation frameworks introduced as part of an argumentation process
- arguments and conflicts are constructed from a given knowledge base
- arguments typically consist of a claim and a support
- hence, in this context claims are the central objects of interest (rather than arguments)

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- hence, in this context claims are the central objects of interest (rather than arguments)
- how does this affect expressibility and dynamic aspects?

## Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
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$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

# Argumentation Frameworks with Claims

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- Starting point: knowledge-base
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$$\langle \{w, w \rightarrow \neg s\}, \neg s \rangle$$

$$\langle \{s, s \rightarrow \neg r\}, \neg r \rangle$$

$$\langle \{r, r \rightarrow \neg w\}, \neg w \rangle$$

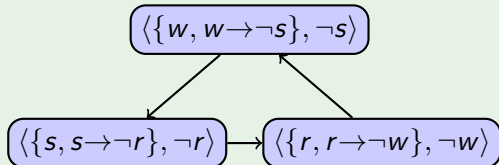
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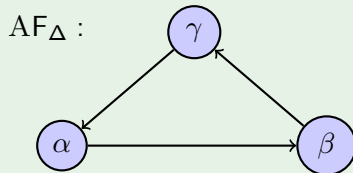
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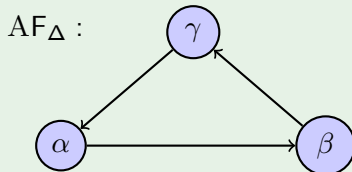
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$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$



$$\begin{aligned} \text{pref}(AF_{\Delta}) &= \{\emptyset\} \\ \text{naive}(AF_{\Delta}) &= \{\{\alpha\}, \{\beta\}, \{\gamma\}\} \end{aligned}$$



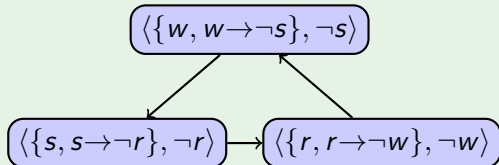
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$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$



$$Cn_{pref}(AF_{\Delta}) = Cn(\top)$$

$$Cn_{naive}(AF_{\Delta}) = Cn(\neg r \vee \neg w \vee \neg s)$$

## Definition

A **Claim-augmented Argumentation Framework (CAF)** is a triple  $(A, R, \gamma)$  where  $(A, R)$  is an AF and  $\gamma : A \rightarrow C$  maps arguments to claims.

A CAF  $(A, R, \gamma)$  is called **well-formed** if, for any  $a, b$  with  $\gamma(a) = \gamma(b)$ ,  $\{c \mid (a, c) \in R\} = \{c \mid (b, c) \in R\}$ .

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Given a set  $A$  of arguments and  $\gamma : A \rightarrow C$ , let  $\gamma(A) = \{\gamma(a) \mid a \in A\}$ .

## Definition

For a semantics  $\sigma$ , we define its claim-based variant as follows:

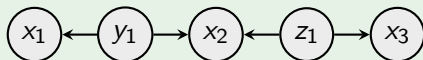
$$\sigma_c((A, R, \gamma)) = \{\gamma(S) \mid S \in \sigma((A, R))\}.$$

Immediate Consequence: For each (well-formed) CAF  $CF$ ,

- $stb_c(CF) \subseteq pref_c(CF)$ ;
- $stb_c(CF) \subseteq naive_c(CF)$ .

## Example

Let  $CF = (A, R, \gamma)$  with  $(A, R)$  given as

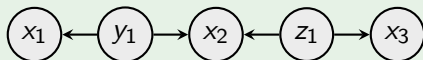


and with  $\gamma(x_1) = \gamma(x_2) = \gamma(x_3) = x$ ,  $\gamma(y_1) = y$  and  $\gamma(z_1) = z$ .

Note that  $CF$  is a well-formed CAF.

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Note that  $CF$  is a well-formed CAF.

We have

- $pref_c(CF) = stb_c(CF) = \{\{y, z\}\}$ .
- $naive_c(CF) = \{\{x\}, \{x, y\}, \{x, z\}, \{y, z\}\}$ .

# Argumentation Frameworks with Claims

We define signatures for well-formed CAFs as follows:

$$\Sigma_{\sigma}^c = \{ \sigma_c(CF) \mid CF \text{ is a well-formed CAF} \}.$$

## Lemma

For any well-formed CAF  $F = (A, R, \gamma)$ ,  $\text{pref}_c(F)$  is incomparable.

Proof: Let  $S, T \in \text{pref}((A, R))$ . Then, there exists an  $s \in S$  attacking some  $t \in T$ . It follows that  $\gamma(s) \notin \gamma(T)$  (otherwise the argument  $t' \in T$  with  $\gamma(t') = \gamma(s)$  also attacks  $t$  due to well-formedness; since  $T$  is conflict-free, this cannot be the case). By symmetry, the claim follows.

## Corollary

For any well-formed CAF  $F = (A, R, \gamma)$ ,  $\text{stb}_c(F)$  is incomparable.

# Argumentation Frameworks with Claims

## Theorem

$$\Sigma_{stb}^c = \{\mathcal{S} \subseteq 2^C \mid \mathcal{S} \text{ is incomparable}\}; \quad \Sigma_{pref}^c = \Sigma_{stb}^c \setminus \{\emptyset\}.$$

Proof Sketch: Given  $\mathcal{S} = \{S_1, \dots, S_n\}$ , let  $CF = (A, R, \gamma)$  be as follows:

- $A = \{a_i \mid a \in S_i, 1 \leq i \leq n\}$ ;
- $R = \{(a_i, b_j) \mid 1 \leq i, j \leq n, a \notin S_j\}$ ;
- $\gamma(a_i) = a$  for all  $1 \leq i \leq n$ .

Then,  $stb((A, R)) = pref((A, R)) = \{\{a_i \mid a \in S_i\} \mid S_i \in \mathcal{S}\}$ .

# Argumentation Frameworks with Claims

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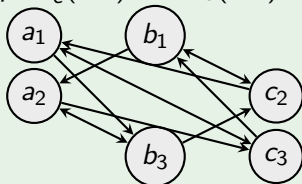
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## Example

A well-formed CAF with  $pref_c(CF) = stb_c(CF) = \{\{a, b\}, \{a, c\}, \{b, c\}\}$ :





# Towards Claim-based Revision

Revision now shall take place on the level of the knowledge base.

- CAFs represent the status of a knowledge base and their **claim-based** extensions express an agent's beliefs
- a revision formula  $\varphi$  encodes desired changes in the status of **claims**
- a revision operator yields a result that satisfies  $\varphi$  and preserves as much useful information from the KB as possible

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Most ingredients already available:

- $stb_c$  and  $pref_c$  satisfy the necessary properties to revise CAFs
- even more flexibility for concrete operators (card-based revision)
- open issue: how to obtain revised KB from revised CAF

# Conclusion

- Dynamic aspects applied to argumentation frameworks important and vibrant research field
- Understanding expressibility of argumentation formalisms key for extension-based change operations
- Results on the level of abstract frameworks available; less is known for structured argumentation
- Here: first step towards bridging this gap

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- Understanding expressibility of argumentation formalisms key for extension-based change operations
- Results on the level of abstract frameworks available; less is known for structured argumentation
- Here: first step towards bridging this gap
- Open Issues:
  - ▶ combination of compliance and principle of minimal change
  - ▶ is revision on the level of extension always appropriate?
  - ▶ missing pieces for revision in instantiation-based argumentation

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