

Trumpet Reincarnations: Abstract Argumentation¹

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Porgy & Bess: Lost & Found

April 24, 2017

FWF

Der Wissenschaftsfonds.



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LIVERPOOL

¹This research has been supported by FWF (project I1102).

May all beings be happy.
May they live in safety and joy.
All living beings,
Whether weak or strong,
Tall, stout, average or short,
Seen or unseen, near or distant,
Born or to be born,
May they all be happy.

Introduction

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safe

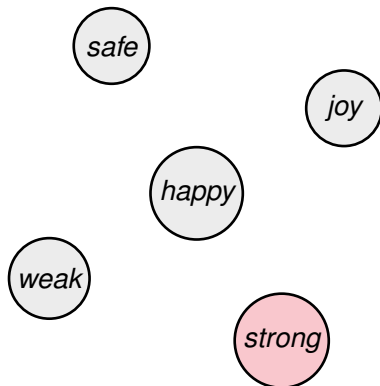
joy

happy

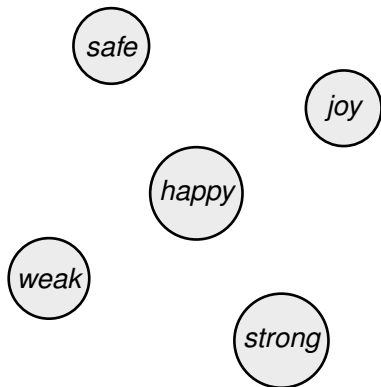
weak

Introduction

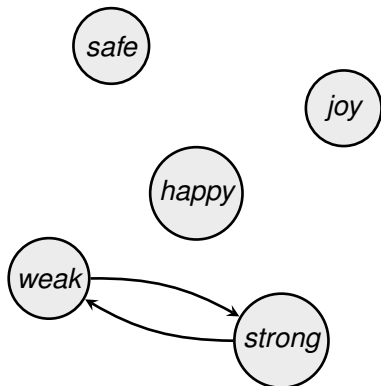
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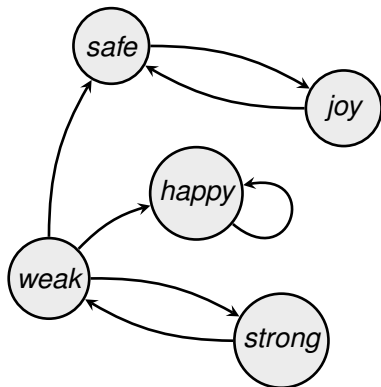
Introduction



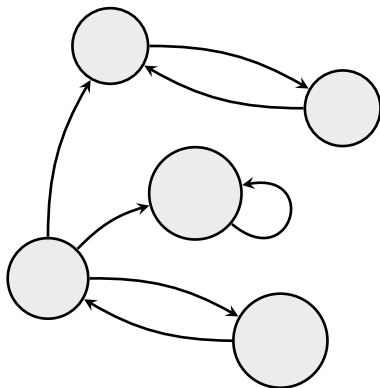
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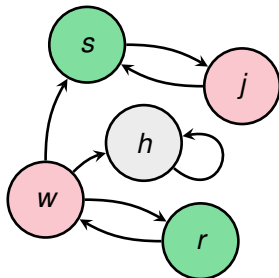
Introduction



Introduction



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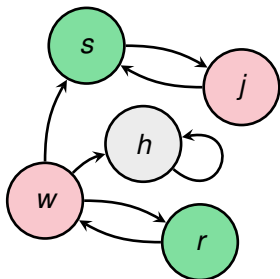


in: s,r

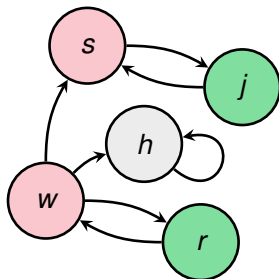
out: w,j

undec: h

Introduction

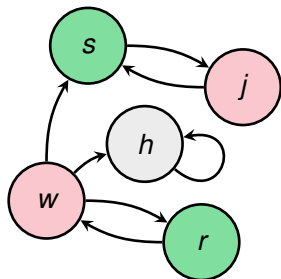


in: *s,r*
out: *w,j*
undec: *h*



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out: *s,w*
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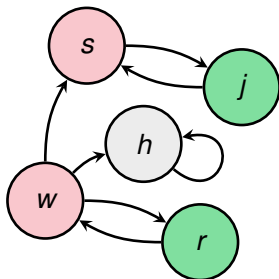
Introduction



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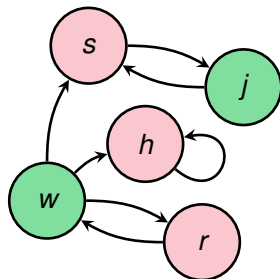
undec: h



in: j,r

out: s,w

undec: h



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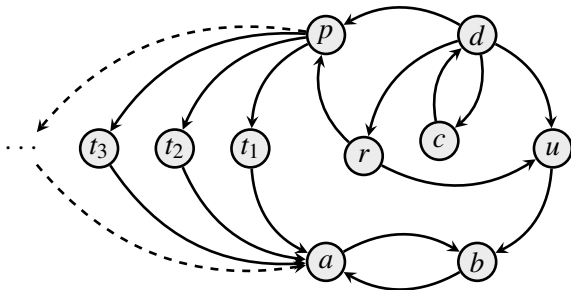
out: s,h,r

undec:

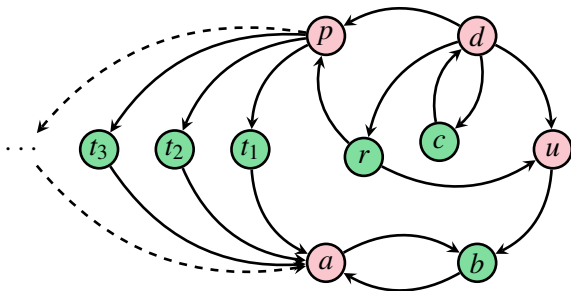
Outline

- 1 Introduction
- 2 Stable Extensions and Conflict**
- 3 Variations of Stable Semantics
- 4 Collapse
- 5 Perfection
- 6 References

What are the stable extensions?



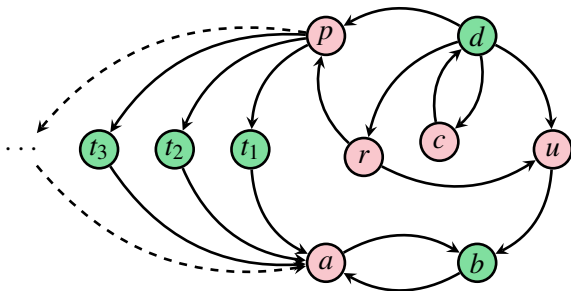
What are the stable extensions?



Stable Extensions:

- $\{c, r, b, t_1, t_2, t_3, \dots\}$

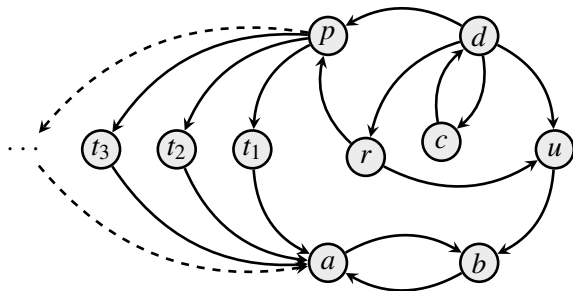
What are the stable extensions?



Stable Extensions:

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- $\{d, b, t_1, t_2, t_3, \dots\}$

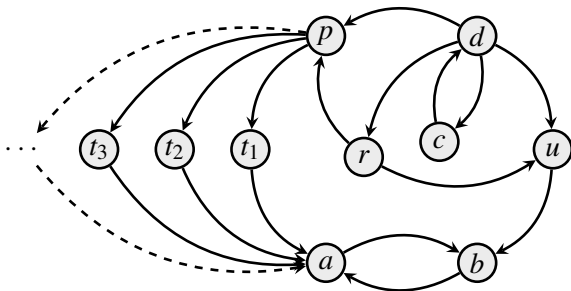
Relation between arguments c and u ?



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Relation between arguments c and u ?

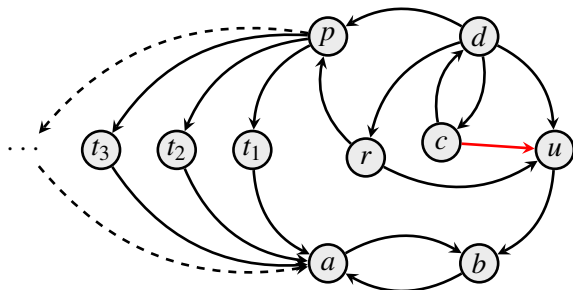


Stable Extensions:

- $\{c, r, b, t_1, t_2, t_3, \dots\}$
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Arguments c and u are implicitly in conflict.

Relation between arguments c and u ?

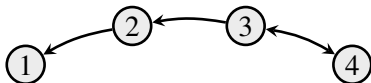


Stable Extensions:

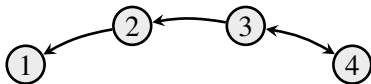
- $\{c, r, b, t_1, t_2, t_3, \dots\}$
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Arguments c and u are implicitly in conflict. An attack (c, u) does not change the extension sets.

Implicit Conflicts made Explicit in a Simpler Case



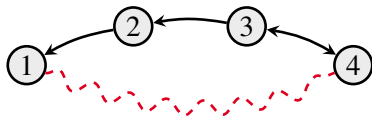
Implicit Conflicts made Explicit in a Simpler Case



Stable Extensions:

- $\{3, 1\}$
- $\{4, 2\}$

Implicit Conflicts made Explicit in a Simpler Case



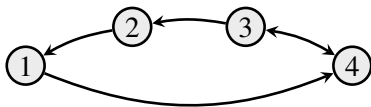
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Only Implicit Conflict:

- 1 vs. 4

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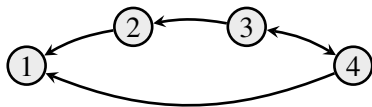
Only Implicit Conflict:

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Resultion:

$(1, 4)$

Implicit Conflicts made Explicit in a Simpler Case



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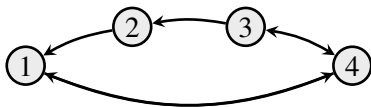
Resultion:

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or

$(4, 1)$

Implicit Conflicts made Explicit in a Simpler Case



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Only Implicit Conflict:

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Resultion:

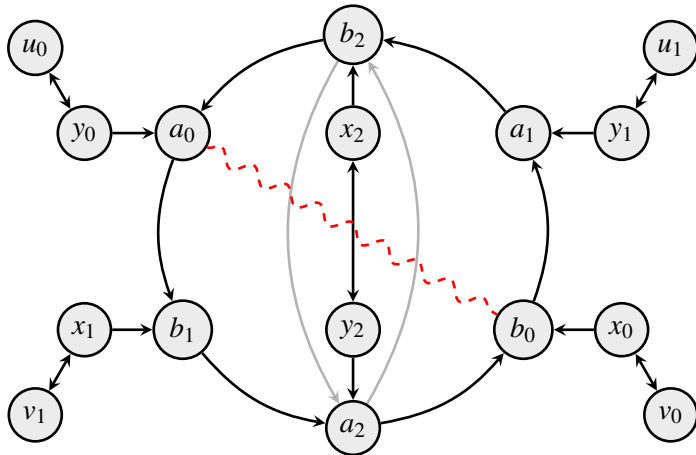
$(1, 4)$

or

$(4, 1)$

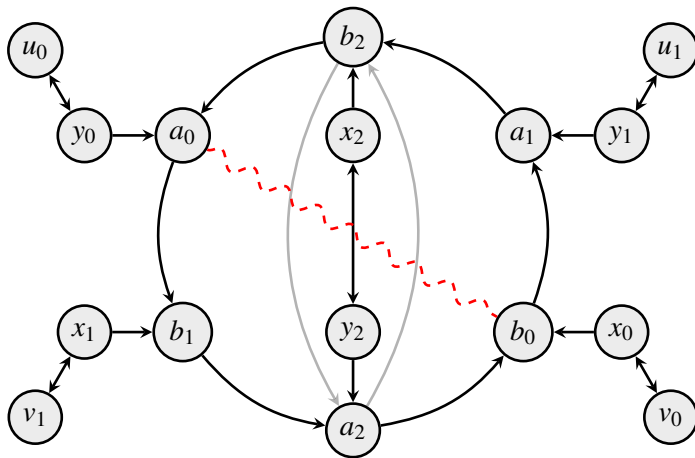
or both

Implicit Conflicts vs. Explicit Conflicts III



Argument set of interest: $\{a_0, a_1, y_2, u_0, u_1, v_0, v_1\}$

Implicit Conflicts vs. Explicit Conflicts III



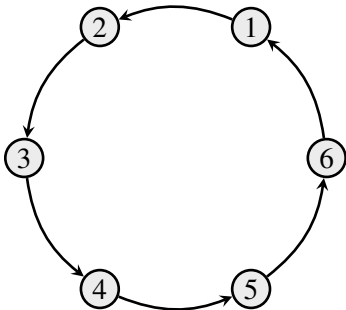
Theorem

Some implicit conflicts can not be made explicit for stable semantics.

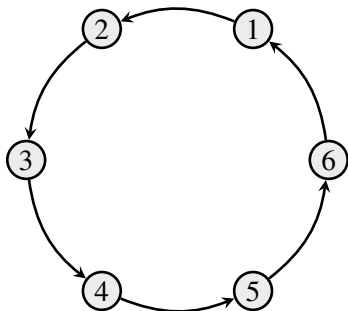
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What are the stable extensions?



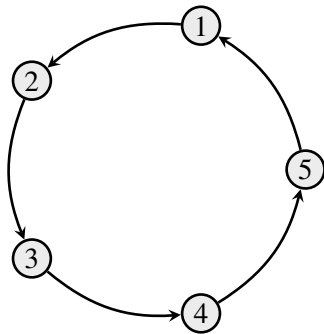
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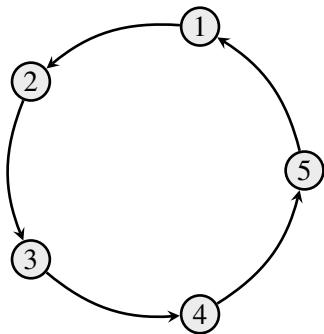
Stable Extensions:

- $\{1, 3, 5\}$ and $\{2, 4, 6\}$

What are the stable extensions?



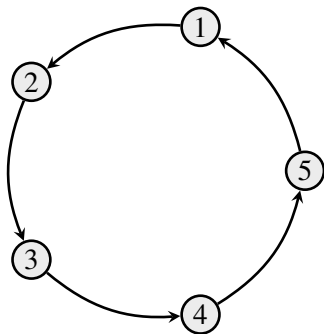
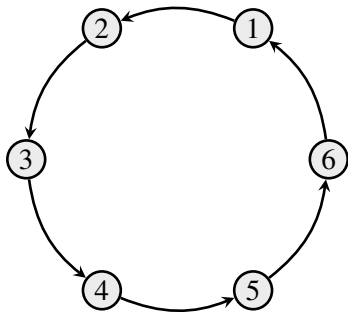
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Stable Extensions:

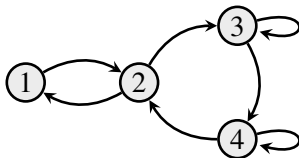
- There are none!!!

What are the stable extensions?



Why is collapse a problem?

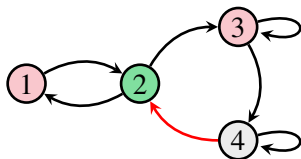
Collapse Solutions



Example

Collapse of Stable Semantics.

Collapse Solutions



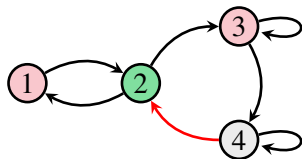
Definition (Stage)

Consider argument sets where undecided arguments are minimized.

Stage Extensions:

$\{2\}$

Collapse Solutions

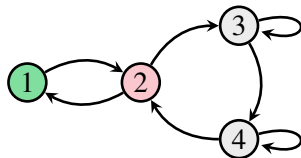


Definition (Stage)

Consider argument sets where undecided arguments are minimized.

Stage Extensions:

$\{2\}$



Definition (Semi-stable)

Consider **self-defending** argument sets where undecided arguments are minimized.

Semi-stable Extensions:

$\{1\}$

Theorem

If there only finitely many arguments then stage and semi-stable never collapse.

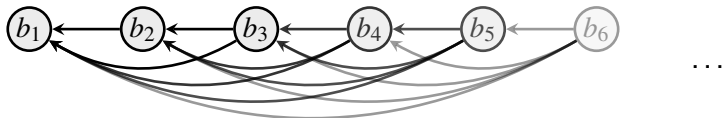
Theorem

If stage does not collapse for any induced sub-structure, then it does still not collapse if we add any finite amount of arguments and induced attacks.

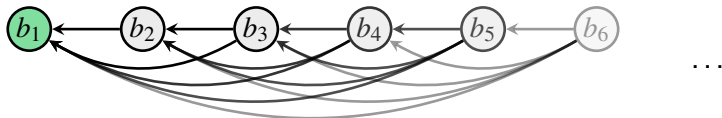
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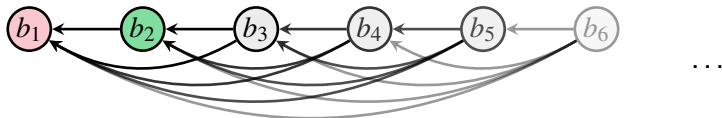
Stage Collapse



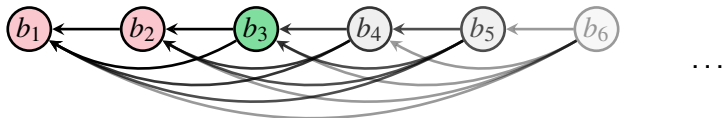
Stage Collapse



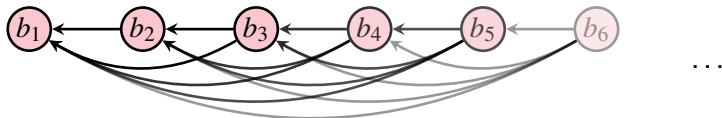
Stage Collapse



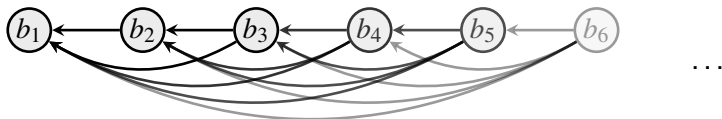
Stage Collapse



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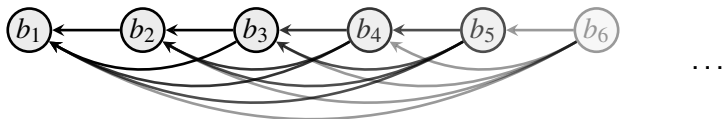
Stage Collapse



Example

There is no biggest natural number...

Stage Collapse



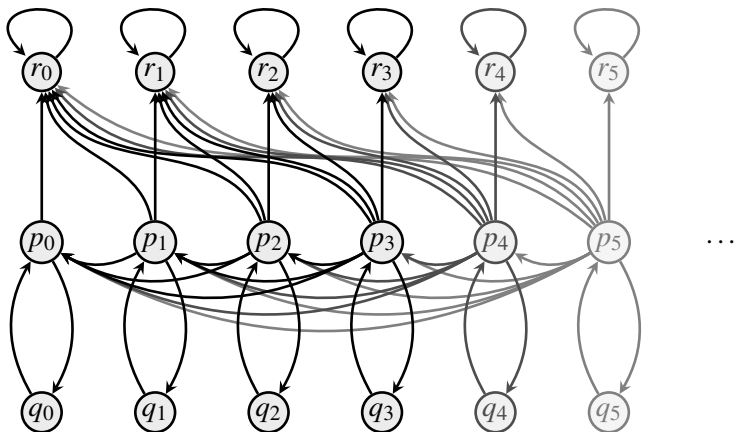
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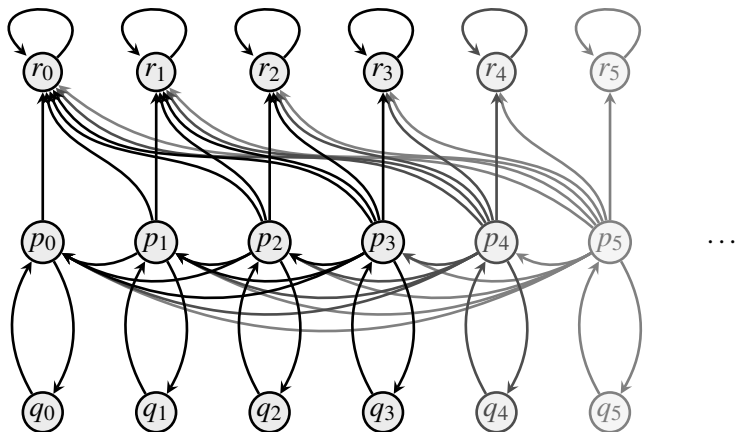
Example

But the empty set is a semi-stable extension!

Collapse or not?



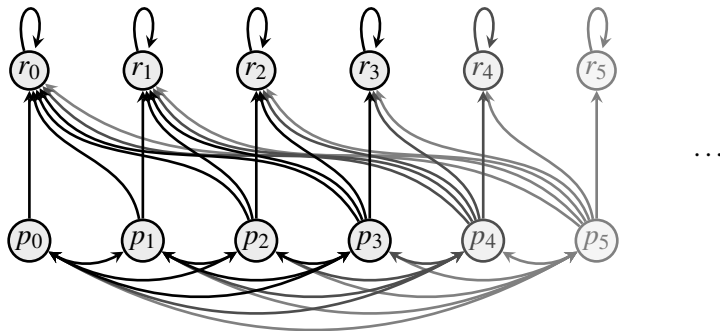
Collapse or not?



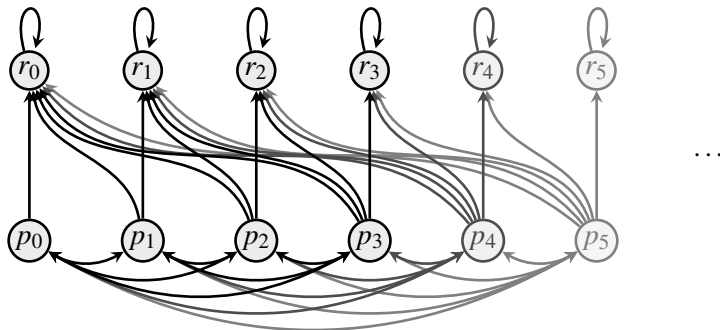
Example

Stage and semi-stable collapse.

Collapse or not?



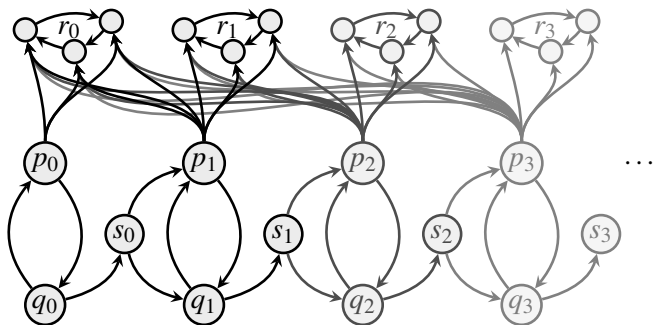
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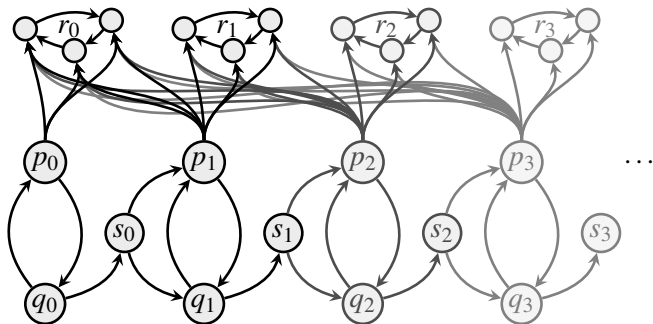
Example

Stage and semi-stable collapse.

Loop-free: collapse or not?



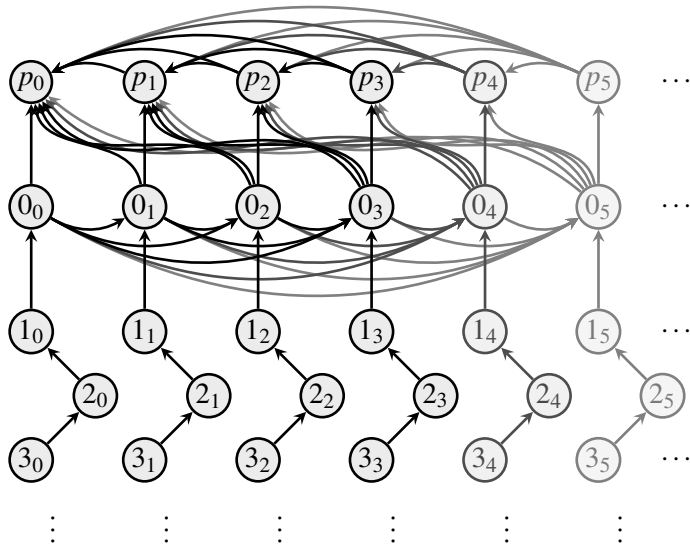
Loop-free: collapse or not?



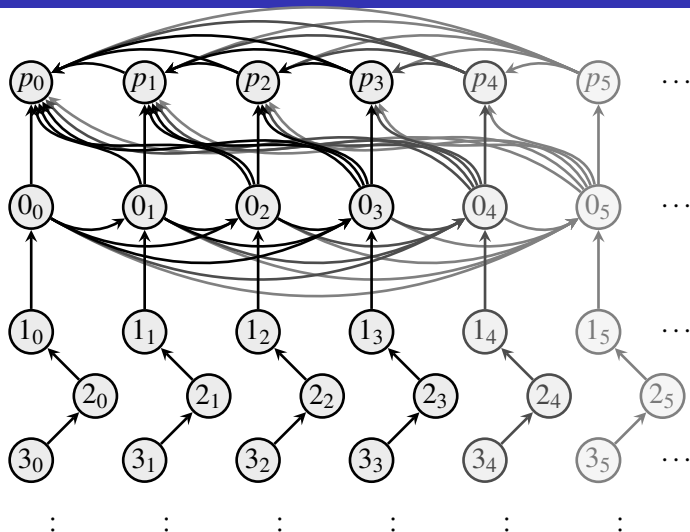
Example

Stage and semi-stable collapse.

Cycle-free: collapse or not?



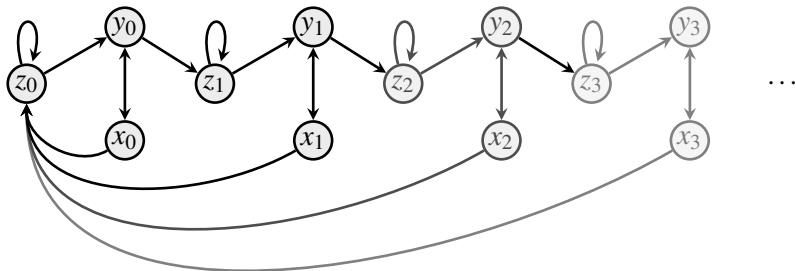
Cycle-free: collapse or not?



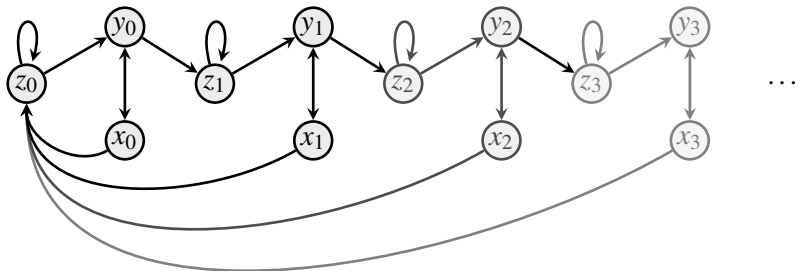
Example

Stage and semi-stable collapse.

Only one argument with finitely many attackers: collapse or not?



Only one argument with finitely many attackers: collapse or not?



Example

Semi-stable collapses, stage not.

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Collapse Solutions II

Theorem

If there is no infinite downwards path with no starting point (such as (x_1, x_0) , (x_2, x_1) , . . . , or any cycle) then stable does not collapse.

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In ZFC if all attacks are symmetric and not self-attacking, then stage and semi-stable do not collapse.

Collapse Solutions II

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Collapse Solutions II

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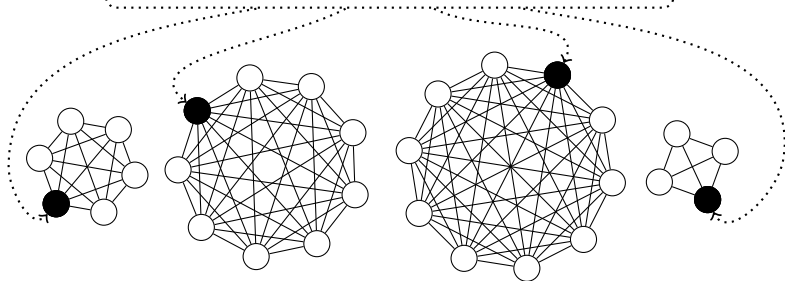
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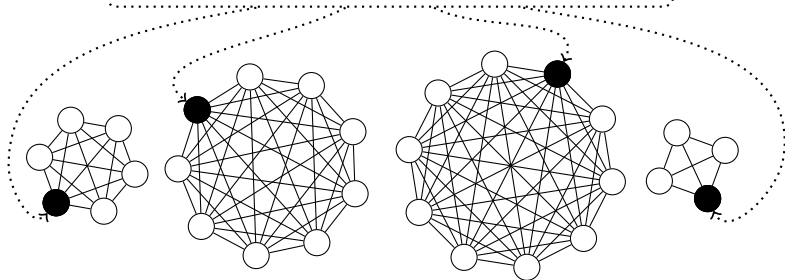
What is ZFC and why bother about Choice?

Selecting Nodes/Elements: a choice function



What is ZFC and why bother about Choice?

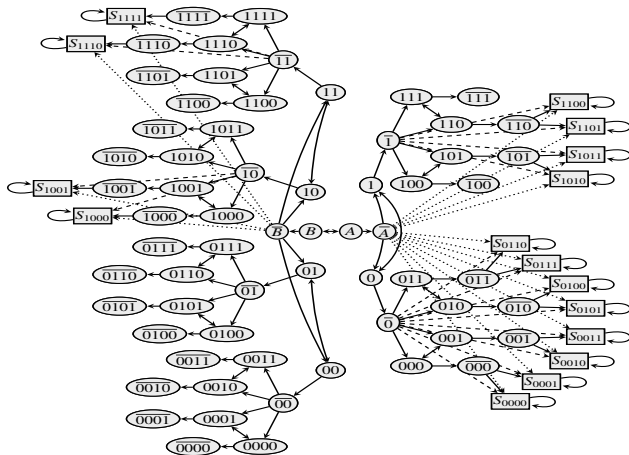
Selecting Nodes/Elements: a choice function



Example

Consider a bucket of water. You know that there is one O-atom in each molecule. But can you select exactly one O-atom from each H₂O molecule?

Downsides of Choice: Axiom of Determinacy



Theorem

The 2^ω generalization of this framework does not collapse for stable semantics if and only if we do not assume Choice.

References



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... may we all be happy ...