Hunt for the Collapse of Semantics in Infinite Abstract Argumentation Frameworks

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some argument
Fact Check II

some attack
some infinite argumentation framework
Let’s get to the real stuff

Theorem ([Baumann and Spanring, 2015, Weydert, 2011])

Any finitary (no argument with infinitely many attackers) argumentation framework provides semi-stable and stage extensions.

Theorem (Not yet published)

For any framework-property that is subframework-valid and guarantees existence of stage extensions, we can have any finite amount of arguments violating this property without loosing the guarantee for the existence of stage extensions.

Corollary (Conjecture from this paper)

If for some argumentation framework there is no stage extension, then there is an infinite amount of arguments with infinitely many attackers.
Outline

1 Introduction
   - Fact Checks
   - Real Stuff ;)

2 Background
   - Examples
   - Definitions

3 Real Real Stuff
   - More Examples
   - Theorems
Stable, Stage and Semi-Stable Semantics

\( stb : \{\{b\}\} \)
\( sem : \{\{b\}\} \)
\( stg : \{\{b\}\} \)

\( stb : \emptyset \)
\( sem : \{\{a\}\} \)
\( stg : \{\{a\}, \{b\}\} \)

\( stb : \emptyset \)
\( sem : \{\emptyset\} \)
\( stg : \{\{b\}\} \)

\( stb : \emptyset \)
\( sem : \{\{a\}\} \)
\( stg : \{\{a\}\} \)
Stable, Stage and Semi-Stable Semantics ctd.

\[
\begin{align*}
stb : \{\{a\}\} \\
sem : \{\{a\}\} \\
stg : \{\{a\}\}
\end{align*}
\]

\[
\begin{align*}
stb : \emptyset \\
sem : \{\emptyset\} \\
stg : \{\emptyset\}
\end{align*}
\]

\[
\begin{align*}
stb : \emptyset \\
sem : \{\{a\}\} \\
stg : \{\{a\}\}
\end{align*}
\]
Stage and Semi-Stable Semantics

\[
\text{sem} : \{\{a\}\} \\
\text{stg} : \{\{b\}\}
\]
Definitions

**Definition ([Dung, 1995])**

An argumentation framework is a pair $F = (A, R)$ of arguments $A$ and attacks $R \subseteq A \times A$. The range of a set of arguments $S$ is given as $S^+ = S \cup \{a \in A, S \hookrightarrow a\}$.

**Definition ([Verheij, 2003, Caminada and Verheij, 2010])**

A set $S \subseteq A$ is called conflict conflict-free, $S \in cf(F)$, if $S \times S \cap R = \emptyset$. $S \in cf(F)$ is called

- admissible, $S \in adm(F)$, if $a \hookrightarrow S$ implies $S \hookrightarrow a$;
- a stable extension, $S \in stb(F)$, if $S^+ = A$;
- a stage extension, $S \in stg(F)$, if it is maximal in range.

An set $S \in adm(A)$ is called

- a semi-stable extension, $S \in sem(F)$, if it is maximal in range.
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Collapse of Semi-Stable and Stage Semantics
[Verheij, 2003]
Collapse of Semi-Stable Semantics
Collapse of Semi-Stable and Stage Semantics
Collapse of Semi-Stable Semantics

\[ \begin{align*}
q_0 & \rightarrow r_5 \\
x_0 & \rightarrow y_0 \\
z_0 & \rightarrow x_0 \\
y_0 & \rightarrow y_1 \\
z_1 & \rightarrow y_1 \\
x_1 & \rightarrow y_1 \\
y_1 & \rightarrow y_2 \\
z_2 & \rightarrow y_2 \\
x_2 & \rightarrow y_2 \\
y_2 & \rightarrow y_3 \\
z_3 & \rightarrow y_3 \\
x_3 & \rightarrow y_3 \\
\end{align*} \]
Insights

Theorem ([Baumann and Spanring, 2015, Weydert, 2011])

Any finitary (no argument with infinitely many attackers) argumentation framework provides semi-stable and stage extensions.

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