Comparing the Expressiveness of Argumentation Semantics

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Motivation

“Plethora” of Argumentation Semantics

Comparison of semantics still relates to
- basic properties,
- computational aspects,
but do not provide satisfying answers about expressiveness.
“Plethora” of Argumentation Semantics

Comparison of semantics still relates to
- basic properties,
- computational aspects,
but do not provide satisfying answers about expressiveness.

Intertranslatability

A translation function transforms Argumentation Frameworks s.t. one can switch from one semantics to another.
- Intertranslatability w.r.t. efficiency has been studied for several semantics and gives a clear hierarchy [Dvořák and Woltran, 2011].
- Considering expressiveness we no longer care about efficiency.
We consider 9 semantics: conflict-free, naive, grounded, admissible, stable, complete, preferred, semi-stable and stage.

We present consider two kinds of translations (faithful and exact), and provide full hierarchies of expressiveness.

Semi-stable and preferred are of same expressiveness (although they have different complexity).
2. Background

Argumentation Frameworks

**Definition**

An argumentation framework (AF) is a pair \((A, R)\) where

- \(A\) is a non-empty set of arguments
- \(R \subseteq A \times A\) is a relation representing "attacks" ("defeats")

**Example**

\[ F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}) \]
Transitions

Definition
A *Translation* $Tr$ is a function mapping (finite) AFs to (finite) AFs.
Translational Tr is a function mapping (finite) AFs to (finite) AFs.
Translating "Levels of Faithfulness" (for semantics $\sigma, \sigma'$)

- exact: for every AF $F$, $\sigma(F) = \sigma'(\text{Tr}(F))$
- faithful: for every AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(\text{Tr}(F))\}$ and $|\sigma(F)| = |\sigma'(\text{Tr}(F))|$. 

Comparing the Expressiveness of Argumentation Semantics
2. Background

Translations

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- **exact**: for every AF $F$, $\sigma(F) = \sigma'(Tr(F))$
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Example (An exact translation: $cf \Rightarrow adm$)

$\{b, d\} \in cf(F)$
2. Background

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- **exact**: for every AF \( F \), \( \sigma(F) = \sigma'(\text{Tr}(F)) \)
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**Example (An exact translation: \( cf \Rightarrow adm \))**

\[
\{b, d\} \in cf(\mathcal{F}) \quad \{b, d\} \in adm(\text{Tr}(\mathcal{F}))
\]
Translators

“Levels of Faithfulness” (for semantics $\sigma, \sigma'$)

- **exact**: for every AF $F$, $\sigma(F) = \sigma'(\text{Tr}(F))$
- **faithful**: for every AF $F$, $\sigma(F) = \{ E \cap A_F \mid E \in \sigma'\left(\text{Tr}(F)\right)\}$ and $|\sigma(F)| = |\sigma'(\text{Tr}(F))|$. 
2. Background

Translations

“Levels of Faithfulness” (for semantics $\sigma, \sigma'$)

- **exact**: for every AF $F$, $\sigma(F) = \sigma'(Tr(F))$
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Example (A faithful translation: $comp \Rightarrow stable$)

$$\{a\} \in comp(F)$$
2. Background

Translations

“Levels of Faithfulness” (for semantics $\sigma, \sigma'$)

- **exact**: for every AF $F$, $\sigma(F) = \sigma'(\text{Tr}(F))$
- **faithful**: for every AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(\text{Tr}(F))\}$ and $|\sigma(F)| = |\sigma'(\text{Tr}(F))|$.

Example (A faithful translation: $\text{comp} \Rightarrow \text{stable}$)

$$\{a\} \in \text{comp} (\mathcal{F}) \quad \{a, a^*, c^*, d^*, e^*\} \in \text{stable} (\text{Tr} (\mathcal{F}))$$
2. Background

Translations

“Levels of Faithfulness” (for semantics $\sigma, \sigma'$)

- **exact**: for every AF $F$, $\sigma(F) = \sigma'(\text{Tr}(F))$
- **faithful**: for every AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(\text{Tr}(F))\}$ and $|\sigma(F)| = |\sigma'(\text{Tr}(F))|$
- **weakly exact**: there is a fixed $S$ of sets of arguments, such that for any AF $F$, $\sigma(F) = \sigma'(\text{Tr}(F)) \setminus S$;
- **weakly faithful**: there is a fixed $S$ of sets of arguments, such that for any AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(\text{Tr}(F)) \setminus S\}$ and $|\sigma(F)| = |\sigma'(F) \setminus S|$

We further consider translations w.r.t. the properties efficient, covering, embedding, monotone, and modular.
### State of the Art

**Table:** Faithful / exact intertranslatability (efficient).

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### State of the Art

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Comparing the Expressiveness of Argumentation Semantics
Summarized Results

Table: Faithful / exact intertranslatability

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Main Contributions

The Paper

For the 9 Semantics under our considerations we
- provide exact / faithful translations whenever possible, and
- prove that no such translation exists otherwise.
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The Paper

For the 9 Semantics under our considerations we
- provide exact / faithful translations whenever possible, and
- prove that no such translation exists otherwise.

The Talk

In the following we give examples for both kind of results.
- Translation 8: exact for semi-stable to stage semantics.
- Theorem 3: There is no weakly faithful translation for preferred to naive semantics.
**Definition**

For $\mathcal{F} = (A, R)$ an Argumentation Framework and a set $S \subseteq A$ we call

$$S^+ = S \cup \{ a \in A \mid \exists b \in A, b \rightarrow a \}$$

the range of $S$.

**Definition**

Let $\mathcal{F} = (A, R)$ be an Argumentation Framework. For $S \subseteq A$ it holds that

- $S \in cf(\mathcal{F})$ if there are no $a, b \in S$, such that $(a, b) \in R$;
- $S \in adm(\mathcal{F})$, if each $a \in S$ is defended by $S$;
- $S \in pref(\mathcal{F})$, if $S \in adm(\mathcal{F})$ and there is no $T \in adm(\mathcal{F})$ with $T \supset S$;
- $S \in semi(\mathcal{F})$, if $S \in adm(\mathcal{F})$ and there is no $T \in adm(\mathcal{F})$ with $T^+_R \supset S^+_R$. 
Translation 8, $semi \Rightarrow pref$

Example

- $pref(F) = \{\{a, c\}, \{a, d\}\}$
- $semi(F) = \{\{a, d\}\}$
Translation 8, $\text{semi} \Rightarrow \text{pref}$

**Definition**

- $\text{Tr}(A, R) = (A', R')$
- $A' = A \cup \{E | E \in \text{pref}(\mathcal{F}) \setminus \text{semi}(\mathcal{F})\}$
- $R' = R \cup \{(a, E), (E, E), (E, b) | a \in A \setminus E, b \in E\}$

**Example**

- $\text{pref}(\mathcal{F}) = \{\{a, c\}, \{a, d\}\}$
- $\text{semi}(\mathcal{F}) = \{\{a, d\}\}$
Translation 8, \( semi \Rightarrow pref \)

**Definition**

- \( Tr(A, R) = (A', R') \)
- \( A' = A \cup \{ E \mid E \in pref(F) \setminus semi(F) \} \)
- \( R' = R \cup \{(a, E), (E, E), (E, b) \mid a \in A \setminus E, b \in E \} \)

**Example**

- \( pref(F) = \{\{a, c\}, \{a, d\}\} \)
- \( semi(F) = \{\{a, d\}\} \)
Translation 8, $semi \Rightarrow pref$

**Definition**

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![Diagram](a -> b <-> c -> d -> e)

**Example**

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- $semi(F) = \{\{a, d\}\}$
Translation 8, $\text{semi} \Rightarrow \text{pref}$

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![Graph showing the transition between states]

**Example**

- $pref(\mathcal{F}) = \{\{a, c\}, \{a, d\}\}$
- $semi(\mathcal{F}) = \{\{a, d\}\}$
Translation 8, \( \text{semi} \Rightarrow \text{pref} \)

**Definition**

- \( \text{Tr}(A, R) = (A', R') \)
- \( A' = A \cup \{E \mid E \in \text{pref}(\mathcal{F}) \setminus \text{semi}(\mathcal{F})\} \)
- \( R' = R \cup \{(a, E), (E, E), (E, b) \mid a \in A \setminus E, b \in E\} \)

**Example**

- \( \text{pref}(\mathcal{F}) = \{\{a, c\}, \{a, d\}\} \)
- \( \text{semi}(\mathcal{F}) = \{\{a, d\}\} \)
- \( \text{pref}(\text{Tr}(\mathcal{F})) = \{\{a, d\}\} \)
Definition

Let $\mathcal{F} = (A, R)$ be an Argumentation Framework. For $S \subseteq A$ it holds that

- $S \in cf(\mathcal{F})$ if there are no $a, b \in S$, such that $(a, b) \in R$;
- $S \in naive(\mathcal{F})$, if there is no $T \in cf(\mathcal{F})$ with $T \supset S$;
- $S \in adm(\mathcal{F})$, if each $a \in S$ is defended by $S$;
- $S \in pref(\mathcal{F})$, if $S \in adm(\mathcal{F})$ and there is no $T \in adm(\mathcal{F})$ with $T \supset S$;
Theorem 3, $\text{pref} \Rightarrow \text{naive}$

Theorem

There is no weakly faithful translation for $\text{pref} \Rightarrow \text{naive}$. 
Theorem 3, $\text{pref} \Rightarrow \text{naive}$

**Theorem**

*There is no weakly faithful translation for $\text{pref} \Rightarrow \text{naive}$.*

**Counterexample**

![Diagram showing a counterexample](image-url)
Theorem 3, $\text{pref} \Rightarrow \text{naive}$

**Theorem**

*There is no weakly faithful translation for $\text{pref} \Rightarrow \text{naive}$.***

**Counterexample**

$\text{pref}(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \{b_1, a_2, b_3\}, \{b_1, b_2, a_3\}\}$
Theorem 3, \( \text{pref} \Rightarrow \text{naive} \)

**Theorem**

*There is no weakly faithful translation for \( \text{pref} \Rightarrow \text{naive} \).*

**Counterexample**

\[
\text{pref}(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \\{b_1, a_2, b_3\}, \\{b_1, b_2, a_3\}\}
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Theorem 3, \( \text{pref} \Rightarrow \text{naive} \)

Theorem

There is no weakly faithful translation for \( \text{pref} \Rightarrow \text{naive} \).

Counterexample

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\text{pref}(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \{b_1, a_2, b_3\}, \{b_1, b_2, a_3\}\}
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Theorem 3, $\text{pref} \Rightarrow \text{naive}$

**Theorem**

*There is no weakly faithful translation for $\text{pref} \Rightarrow \text{naive}$.***

**Counterexample**

$\text{pref}(\mathcal{F}) = \{\{a_1, b_2, b_3\},$
$\{b_1, a_2, b_3\},$
$\{b_1, b_2, a_3\}\}$

$\subseteq \text{naive}(\text{Tr}(\mathcal{F}))$
3. Contribution

Theorem 3, \( \text{pref} \Rightarrow \text{naive} \)

**Theorem**

*There is no weakly faithful translation for \( \text{pref} \Rightarrow \text{naive} \).*

**Counterexample**

\[
\text{pref}(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \{b_1, a_2, b_3\}, \{b_1, b_2, a_3\}\} \\
\subseteq \text{naive}(\text{Tr}(\mathcal{F}))
\]

\[
\Rightarrow \{b_1, b_2, b_3\} \in \text{cf}(\text{Tr}(\mathcal{F}))
\]
Theorem 3, \( \text{pref} \Rightarrow \text{naive} \)

There is no weakly faithful translation for
\( \{ \text{stage}, \text{stable}, \text{semi}, \text{pref}, \text{comp}, \text{adm} \} \Rightarrow \{ \text{cf}, \text{naive} \} \).

Counterexample

\[
\text{pref} (\mathcal{F}) = \{ \{ a_1, b_2, b_3 \}, \\
\{ b_1, a_2, b_3 \}, \\
\{ b_1, b_2, a_3 \} \} \\
\subseteq \text{naive} (\text{Tr} (\mathcal{F}))
\]

\[
\Rightarrow \{ b_1, b_2, b_3 \} \in \text{cf} (\text{Tr} (\mathcal{F}))
\]
4. Conclusion

Results

(weakly) exact

(comp, adm, cf, stable, ground, semi, pref)

(weakly) faithful

(stage, stable, semi, pref, comp, adm)
Almost finished... 

Achievements

- Full hierarchy of expressiveness for the selected semantics.
- Extended existing investigations on intertranslatability
  - to naive extensions and conflict-free sets, and
  - to the case of inefficient translations.
- Improved an existing translation w.r.t. size of transformed Argumentation Frameworks.

Open Questions

- More semantics for investigation
- Labeling-preserving translations
4. Conclusion

Finished.

Achievements

- Full hierarchy of expressiveness for the selected semantics.
- Extended existing investigations on intertranslatability
  - to naive extensions and conflict-free sets, and
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- Improved an existing translation w.r.t. size of transformed Argumentation Frameworks.

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