Complexity Theory

VU 181.142, WS 2020

2. Fundamental Notions and Results

(Short Recapitulation from “Formale Methoden der Informatik”)

Reinhard Pichler

Institute of Logic and Computation
DBAI Group
TU Wien

02 October, 2020

Outline

Computation and Computability

- Ability to read and formulate decision/optimization problems
- Several kinds of problems:
  decision, function, optimization, enumeration, counting problems
- Problem vs. problem instance
- Problem vs. algorithm vs. program
- Church-Turing thesis
- Halting problem
- Decidability vs. undecidability vs. semi-decidability
- Complement of a decision problem
- Properties of complementation

Complexity of Problems and Algorithms

- Asymptotic, worst-case complexity vs. other notions of complexity
- Basic understanding of growth rates (polynomial vs. exponential)
- The class P
- The class NP
- Tractability vs. intractability
- Optimization vs. decision problem

Reductions

- Two motivations for reducing one problem (or language) to another.
- Two kinds of reductions (Turing, many-one).
- Limiting the resources used by reductions.
- Cook / Karp reductions.
- Proving the correctness of problem reductions.
- The definitions of C-hard and C-complete problems for a complexity class C.
- Understanding the role of complete problems in complexity theory.
- Proving undecidability by reduction from the HALTING problem.
- Proving non-semi-decidability by reduction from the NON-HALTING problem.

NP-Completeness

- You should now be familiar with the intuition of NP-completeness (and recognize NP-complete problems).
- Two fundamental NP-complete problems: **SAT** and **3-SAT**.
- Difference between logical equivalence and sat-equivalence.
- Many more examples of NP-complete problems, e.g.: **CLIQUE**, **INDEPENDENT SET**, **VERTEX COVER**, **3-COLORABILITY**, **HAMILTON-PATH**, **HAMILTON-CYCLE**, **TSP(D)**, etc.
- Usefulness of reductions to **SAT**.

Other Important Complexity Classes

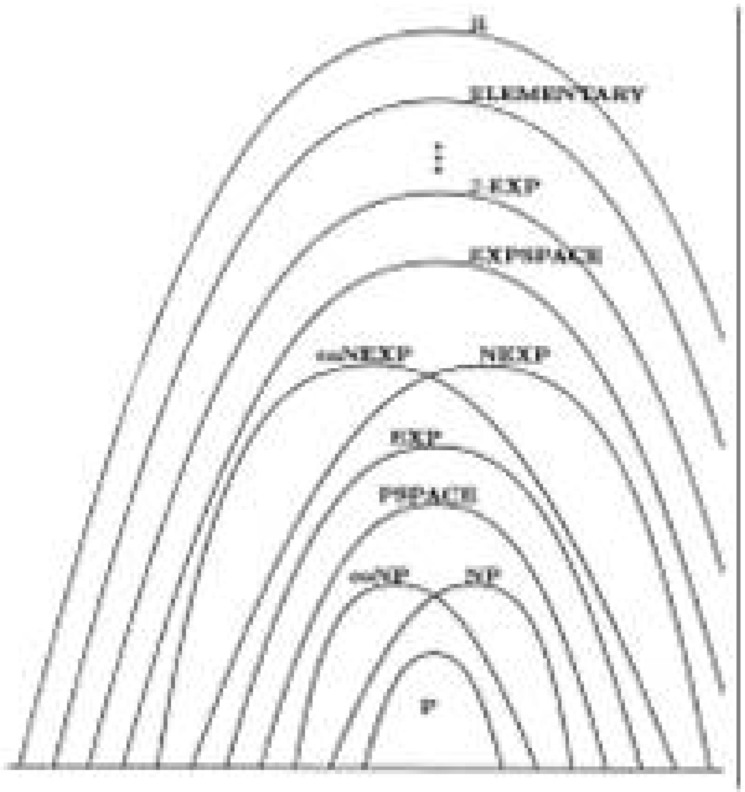
- Understanding the definitions of L, PSPACE and EXPTIME
- Being aware of the main inclusions between P, NP, and the three classes above.

Turing Machines

- Definition of Turing machines.
- Turing machines as a reasonable model of computation.
- Formal definition of "deterministic" complexity classes P, EXPTIME, L, PSPACE, EXPSPACE.
- Solving problems with Turing machines.
  (Decision problems can be considered as languages!)
- (Strengthening of) the Church-Turing Thesis
- Nondeterministic Turing machines. Differences between deterministic and nondeterministic TMs
- Nondeterminism as "guess and check" algorithms
- Definitions of NL, NP, NEXPTIME via nondeterministic TMs.
- The definition of complementary problems.
- Summary of important complexity classes: L, NL, co-NL, P, NP, co-NP, PSPACE, EXPTIME, NEXPTIME, co-NEXPTIME, EXPSPACE.

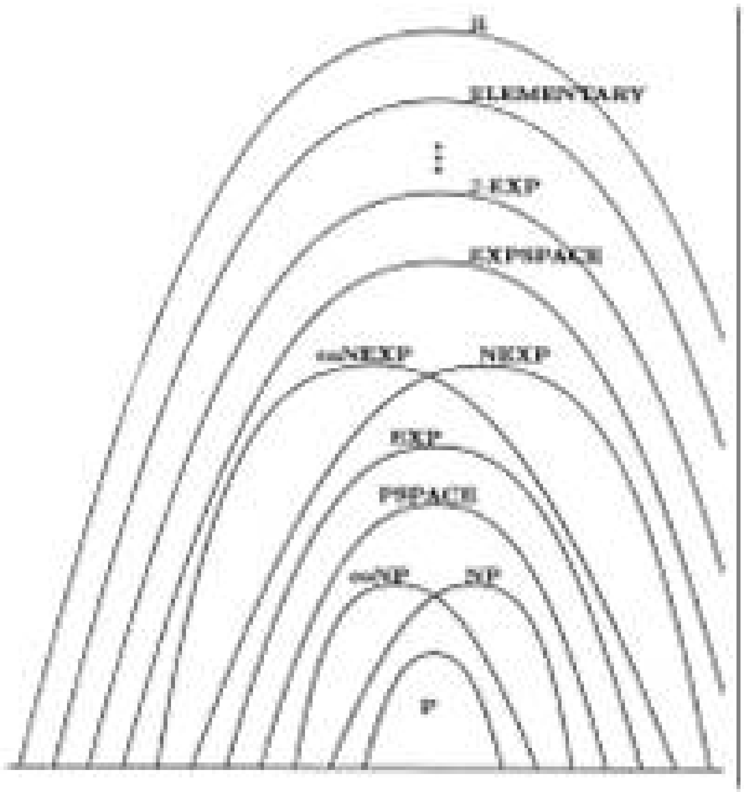
Overview of Complexity Classes

Recursive Problems



Overview of Complexity Classes

Recursive Problems



Inside PSPACE