

An Extension-Based Approach to Belief Revision in Abstract Argumentation

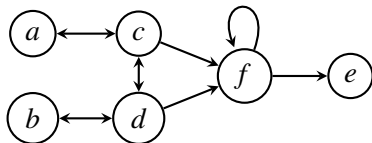
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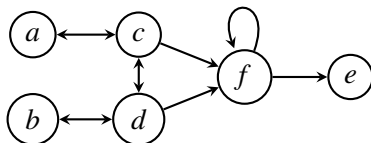
July 30, 2015



- Abstract Argumentation Framework (AF) [Dung, 1995]:

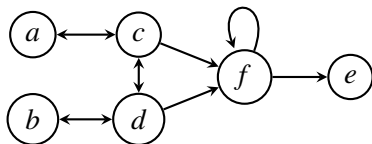


- Abstract Argumentation Framework (AF) [Dung, 1995]:



- Evaluation: argumentation **semantics**
- **Extension**: set of jointly acceptable arguments

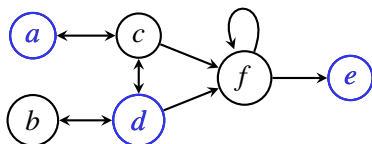
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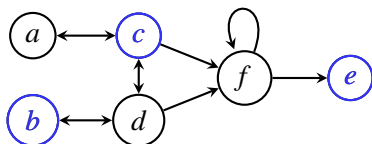
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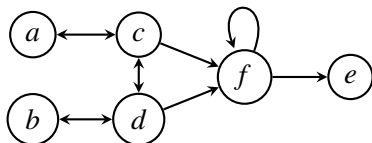
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- Further semantics: preferred, complete, semi-stable, stage, ...

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Introduction

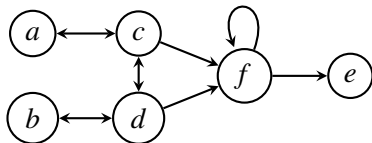
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Model-based revision	Extension-based revision
Knowledge base Model	Argumentation framework Extension wrt. σ
Revision formula	1. Formula / 2. AF
Knowledge base	Argumentation framework

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- Here: Revision results in a single AF

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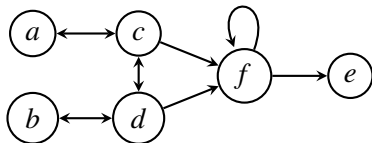
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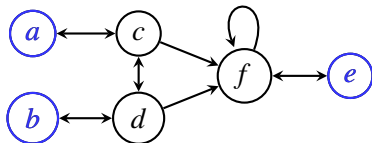
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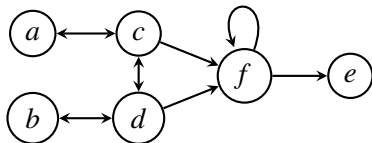
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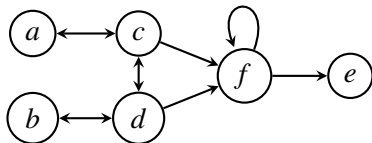
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There exists no argumentation framework having this extension-set under stable semantics!

Main Contributions

- **Representation theorems:** Correspondence between revision operators captured by rankings and revision operators given by (extended) set of AGM postulates.
- Revision by **propositional formulas**
 - $\star_{\sigma} : AF_{\mathcal{A}} \times \mathcal{P}_{\mathcal{A}} \mapsto AF_{\mathcal{A}}$
- Revision by **argumentation frameworks**
 - $*_{\sigma} : AF_{\mathcal{A}} \times AF_{\mathcal{A}} \mapsto AF_{\mathcal{A}}$

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Tool-Kit:

- **Realizability** results for AF semantics [Dunne et al., 2014]
 - Exact characterization of realizable extension-sets Σ_{σ}
- **Horn** belief revision [Delgrande and Peppas, 2015]
 - How to modify postulates and rankings in order to stay in the fragment

Definition (Proper I-maximal Semantics)

A semantics σ is called **proper I-maximal** if for each $\mathcal{S} \in \Sigma_\sigma$:

- 1 for all $S_1, S_2 \in \mathcal{S}$: $S_1 \subseteq S_2$ implies $S_1 = S_2$
- 2 for all $\emptyset \neq \mathcal{S}' \subseteq \mathcal{S}$: $\mathcal{S}' \in \Sigma_\sigma$
- 3 for all \subseteq -incomparable extensions S_1, S_2 : $\{S_1, S_2\} \in \Sigma_\sigma$

Examples:

- **stable** semantics
- **preferred** semantics
- **semi-stable** semantics
- **stage** semantics

1. Revision by Propositional Formula

$\star_\sigma: AF_{\mathcal{A}} \times \mathcal{P}_{\mathcal{A}} \mapsto AF_{\mathcal{A}}$:

(P*1) $\sigma(F \star_\sigma \varphi) \subseteq [\varphi]$.

(P*2) If $\sigma(F) \cap [\varphi] \neq \emptyset$ then $\sigma(F \star_\sigma \varphi) = \sigma(F) \cap [\varphi]$.

(P*3) If $[\varphi] \neq \emptyset$ then $\sigma(F \star_\sigma \varphi) \neq \emptyset$.

(P*4) If $\varphi \equiv \psi$ then $\sigma(F \star_\sigma \varphi) = \sigma(F \star_\sigma \psi)$.

(P*5) $\sigma(F \star_\sigma \varphi) \cap [\psi] \subseteq \sigma(F \star_\sigma (\varphi \wedge \psi))$.

(P*6) If $\sigma(F \star_\sigma \varphi) \cap [\psi] \neq \emptyset$ then $\sigma(F \star_\sigma (\varphi \wedge \psi)) \subseteq \sigma(F \star_\sigma \varphi) \cap [\psi]$.

[Alchourrón et al., 1985, Katsuno and Mendelzon, 1991,
Coste-Marquis et al., 2014]

1. Revision by Propositional Formula

Definition (σ -compliance)

A pre-order \preceq is σ -compliant if for every formula φ it holds that $\min([\varphi], \preceq)$ is realizable under σ .

Example ($\sigma \in \{stable, preferred, stage, semi-stable\}$)

- $\varphi = \neg(a \wedge b \wedge c)$
- $\{a, b, c\} \prec \{a, b\} \approx \{a, c\} \approx \{b, c\} \prec \{a\} \approx \{b\} \approx \{c\} \prec \emptyset$
 - $\min([\varphi], \preceq) = \{\{a, b\}, \{a, c\}, \{b, c\}\} \notin \Sigma_\sigma$
 - \preceq is not σ -compliant
- $\{a, b, c\} \prec' \{a\} \approx' \{b\} \approx' \{c\} \prec' \{a, b\} \prec' \{a, c\} \prec' \{b, c\} \prec' \emptyset$
 - \preceq' is σ -compliant
 - For instance, $\min([\varphi], \preceq') = \{\{a\}, \{b\}, \{c\}\} \in \Sigma_\sigma$

1. Revision by Propositional Formula

Definition

Given semantics σ and AF F , a pre-order \preceq_F is a **faithful ranking** if it is total and for any sets E_1, E_2 and AFs F, F_1, F_2 :

- (i) if $E_1, E_2 \in \sigma(F)$, then $E_1 \approx_F E_2$,
- (ii) if $E_1 \in \sigma(F)$ and $E_2 \notin \sigma(F)$, then $E_1 \prec_F E_2$,
- (iii) if $\sigma(F_1) = \sigma(F_2)$, then $\preceq_{F_1} = \preceq_{F_2}$.

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Theorem

An operator \star_σ satisfies postulates $P\star 1 - P\star 6$ for proper I -maximal semantics σ

iff

*there exists an assignment mapping each AF F to a **faithful and σ -compliant** ranking \preceq_F such that $\sigma(F \star_\sigma \varphi) = \min([\varphi], \preceq_F)$.*

2. Revision by Argumentation Framework

$*_{\sigma}: AF_{\mathfrak{A}} \times AF_{\mathfrak{A}} \mapsto AF_{\mathfrak{A}}:$

- (A*1) $\sigma(F *_{\sigma} G) \subseteq \sigma(G)$.
- (A*2) If $\sigma(F) \cap \sigma(G) \neq \emptyset$, then $\sigma(F *_{\sigma} G) = \sigma(F) \cap \sigma(G)$.
- (A*3) If $\sigma(G) \neq \emptyset$, then $\sigma(F *_{\sigma} G) \neq \emptyset$.
- (A*4) If $\sigma(G) = \sigma(H)$, then $\sigma(F *_{\sigma} G) = \sigma(F *_{\sigma} H)$.
- (A*5) $\sigma(F *_{\sigma} G) \cap \sigma(H) \subseteq \sigma(F *_{\sigma} f_{\sigma}(\sigma(G) \cap \sigma(H)))$.
- (A*6) If $\sigma(F *_{\sigma} G) \cap \sigma(H) \neq \emptyset$, then
 $\sigma(F *_{\sigma} f_{\sigma}(\sigma(G) \cap \sigma(H))) \subseteq \sigma(F *_{\sigma} G) \cap \sigma(H)$.
- (Acyc) If for $0 \leq i \leq n$ we have $\sigma(F *_{\sigma} G_{i+1}) \cap \sigma(G_i) \neq \emptyset$ and
 $\sigma(F *_{\sigma} G_0) \cap \sigma(G_n) \neq \emptyset$ then $\sigma(F *_{\sigma} G_n) \cap \sigma(G_0) \neq \emptyset$.

2. Revision by Argumentation Framework

Definition

Given semantics σ and AF F , a pre-order \preceq_F is an **I-faithful ranking** if it is **I-total** and for any **\subseteq -incomparable** sets E_1, E_2 and AFs F, F_1, F_2 :

- (i) if $E_1, E_2 \in \sigma(F)$, then $E_1 \approx_F E_2$,
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2. Revision by Argumentation Framework

Definition

Given semantics σ and AF F , a pre-order \preceq_F is an **l-faithful ranking** if it is l-total and for any **\subseteq -incomparable** sets E_1, E_2 and AFs F, F_1, F_2 :

- (i) if $E_1, E_2 \in \sigma(F)$, then $E_1 \approx_F E_2$,
- (ii) if $E_1 \in \sigma(F)$ and $E_2 \notin \sigma(F)$, then $E_1 \prec_F E_2$,
- (iii) if $\sigma(F_1) = \sigma(F_2)$, then $\preceq_{F_1} = \preceq_{F_2}$.

Theorem

An operator $*_\sigma$ satisfies postulates $A*1 - A*6 + (Acyc)$ for proper l-maximal semantics σ

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there exists an assignment mapping each AF F to an l-faithful ranking \preceq_F such that $\sigma(F *_\sigma \varphi) = \min([\varphi], \preceq_F)$.

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\Rightarrow standard model-based revision operators (e.g. [Dalal, 1988]) work.



Summary:

- Extension-based revision resulting in a single AF
- Combining recent results in argumentation and belief revision
- Different representation theorems:
 - Revision by propositional formulas
 - Revision by argumentation frameworks

Future work:

- Concrete operators
- Other semantics
- Minimal-change criteria for the realizing AFs
- Iterated revision of AFs

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