FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

dbai

Compact Argumentation Frameworks

Ringo Baumann, Wolfgang Dvořák, Thomas Linsbichler,
Hannes Strass, Stefan Woltran

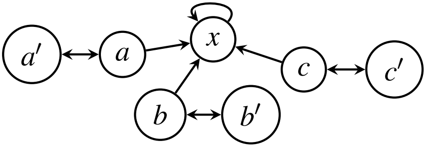
21st European Conference on Artificial Intelligence, Prague

August 22, 2014

FWF
Der Wissenschaftsfonds.

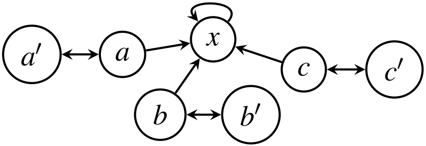
Introduction

- Abstract Argumentation Framework [Dung, 1995]:



Introduction

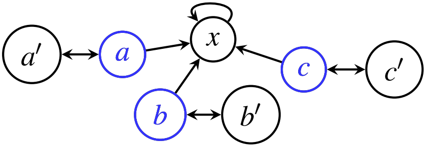
- Abstract Argumentation Framework [Dung, 1995]:



- Evaluation: Argumentation Semantics

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

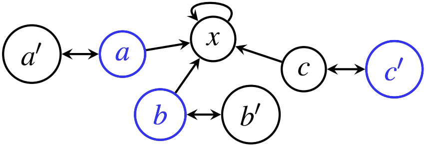


- Evaluation: Argumentation Semantics

$$stb(F) = \{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

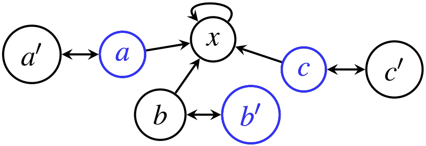


- Evaluation: Argumentation Semantics

$$\textit{stb}(F) = \big\{\{a,b,c\}, \{a,b,c'\}, \{a,b',c\}, \{a',b,c\}, \\ \{a,b',c'\}, \{a',b,c'\}, \{a',b',c\}\big\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

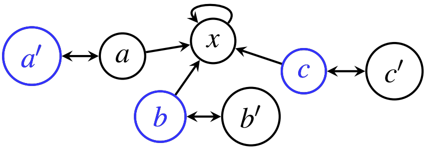


- Evaluation: Argumentation Semantics

$$\mathit{stb}(F) = \big\{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\big\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

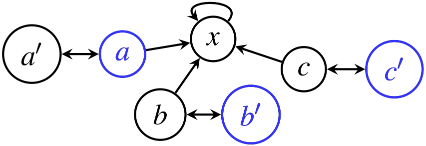


- Evaluation: Argumentation Semantics

$$\textit{stb}(F) = \big\{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\\ \{a,b',c'\},\{a',b,c'\},\{a',b',c\}\big\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

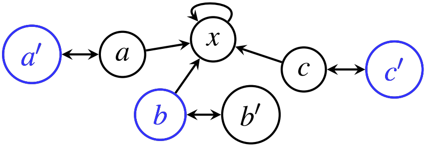


- Evaluation: Argumentation Semantics

$$stb(F) = \{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

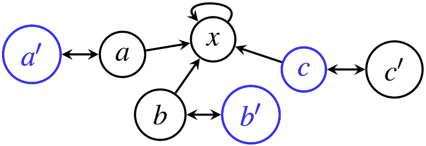


- Evaluation: Argumentation Semantics

$$\textit{stb}(F) = \big\{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\big\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:

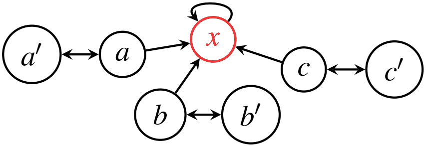


- Evaluation: Argumentation Semantics

$$\textit{stb}(F) = \{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\}.$$

Introduction

- Abstract Argumentation Framework [Dung, 1995]:



- Evaluation: Argumentation Semantics

$$\mathit{stb}(F) = \{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\}\}.$$

Problem

Can we find an equivalent AF F' without argument x?

Motivation

- Realizability [Dunne et al., 2014]
  - Structural analysis of the expressiveness of argumentation semantics.
  - Unlimited use of auxiliary arguments.

  $\Rightarrow$ Compact Realizability

- Compact Argumentation Frameworks
  - Each argument occurs in at least one extension.
  - “Semantic” subclass.
  - Attractive for normal-forms.

Background

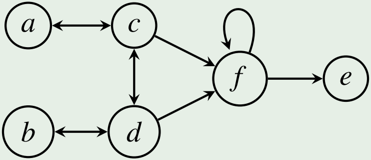
Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Example



$$F = \big(\{a, b, c, d, e, f\},$$
$$\{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d), (c, f), (d, f), (f, f), (f, e)\}\big)$$

Background

Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Definition

Given an AF $F = (A, R)$, a set $S \subseteq A$ is

- conflict-free if for each $a, b \in S$, $(a, b) \notin R$,
- naive extension if $S \in \mathit{cf}(F)$ and $\nexists T \in \mathit{cf}(F) : T \supset S$,
- stable extension if $S \in \mathit{cf}(F)$ and $\forall b \in A \setminus S\ \exists a \in S : (a, b) \in R$.

Signature of semantics σ: $\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF}\}$

Compact Realizability

Definition

Given a semantics σ, an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called compactly σ-realizable if there exists an AF $F = (\mathit{Args}_{\mathbb{S}}, R)$ such that $\sigma(F) = \mathbb{S}$.

C-Signature: $\Sigma_\sigma^c = \{\sigma(F) \mid F = (A, R) \text{ is an AF}, \mathit{Args}_{\sigma(F)} = A\}$.

Compact Realizability

Definition

Given a semantics σ, an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called compactly σ-realizable if there exists an AF $F = (\mathit{Args}_{\mathbb{S}}, R)$ such that $\sigma(F) = \mathbb{S}$.

C-Signature: $\Sigma_\sigma^c = \{\sigma(F) \mid F = (A, R)$ is an AF, $\mathit{Args}_{\sigma(F)} = A\}$.

Definition

Given an extension-set $\mathbb{S}$,

- $\mathit{Args}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, and
- $\mathit{Pairs}_{\mathbb{S}} = \{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$.

Compact Realizability

Definition

Given a semantics σ, an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called compactly σ-realizable if there exists an AF $F = (\mathit{Args}_{\mathbb{S}}, R)$ such that $\sigma(F) = \mathbb{S}$.

C-Signature: $\Sigma_\sigma^c = \{\sigma(F) \mid F = (A, R) \text{ is an AF}, \mathit{Args}_{\sigma(F)} = A\}$.

Definition

Given an extension-set $\mathbb{S}$,

- $\mathit{Args}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, and
- $\mathit{Pairs}_{\mathbb{S}} = \{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$.

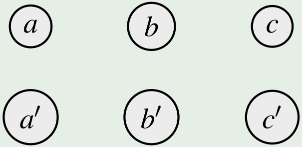
Canonical Argumentation Framework

$$F_{\mathbb{S}} = (\mathit{Args}_{\mathbb{S}}, (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}})$$

Compact Realizability: Naive Semantics

Example

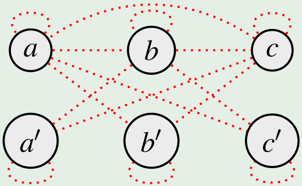
$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a, b, c\}\}$.



Compact Realizability: Naive Semantics

Example

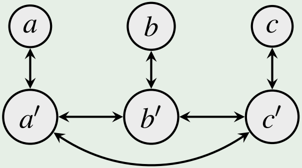
$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a, b, c\}\}$.



Compact Realizability: Naive Semantics

Example

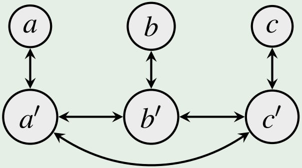
$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a, b, c\}\}$.



Compact Realizability: Naive Semantics

Example

$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a, b, c\}\}$.

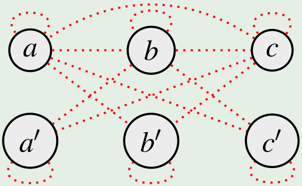


naive$(F_{\mathbb{T}}) = \mathbb{T}$.

Compact Realizability: Naive Semantics

Example

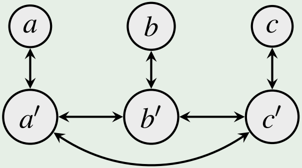
$\mathbb{U} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\},$ ~~$\{a, b, c\}$~~$\}$.



Compact Realizability: Naive Semantics

Example

$\mathbb{U} = \{\{a,b,c'\},\{a,b',c\},\{a',b,c\},\sout{\{a,b,c\}}\}$.

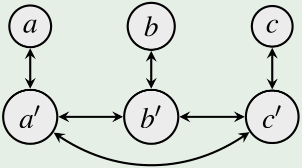


naive$(F_{\mathbb{U}}) = \mathbb{T} \neq \mathbb{U}$.

Compact Realizability: Naive Semantics

Example

$\mathbb{U} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \sout{\{a, b, c\}}\}$.



naive$(F_{\mathbb{U}}) = \mathbb{T} \neq \mathbb{U}$.

- $\mathbb{S}^+ = naive(F_{\mathbb{S}}) = stb(F_{\mathbb{S}})$
- $\mathbb{S}^- = \mathbb{S}^+ \setminus \mathbb{S}$

Theorem

$\Sigma^c_{naive} = \{\mathbb{S} \subseteq 2^{\mathfrak{A}} \mid \mathbb{S} \neq \emptyset, \mathbb{S} = \mathbb{S}^+\} = \Sigma_{naive}$.

Compact Realizability: Stable Semantics

Proposition

For every extension-set $\mathbb{S} \in \Sigma_{stb}$ it holds that if $|\mathbb{S}| \leq 3$ then $\mathbb{S} \in \Sigma^{c}_{stb}$.

Compact Realizability: Stable Semantics

Proposition

For every extension-set $\mathbb{S} \in \Sigma_{stb}$ it holds that if $|\mathbb{S}| \leq 3$ then $\mathbb{S} \in \Sigma^c_{stb}$.

Proposition

For every extension-set $\mathbb{S} \in \Sigma_{stb}$ such that for each $S \in \mathbb{S}$ there is an $a \in S$ with $\forall T \in (\mathbb{S} \setminus \{S\}) : a \notin T$ then $\mathbb{S} \in \Sigma^c_{stb}$.

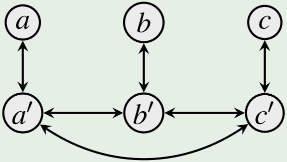
Compact Realizability: Stable Semantics

Proposition

$\Sigma^c_{naive} = \Sigma_{naive} \subset \Sigma^c_{stb} \subset \Sigma_{stb}$

Example

$\mathbb{U} = \{\{a,b,c'\},\{a,b',c\},\{a',b,c\},\cancel{\{a,b,c\}}\} \subset \mathbb{U}^+$



$\mathbb{U}^- = \{\{a,b,c\}\}$.

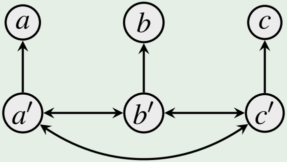
Compact Realizability: Stable Semantics

Proposition

$\Sigma^c_{naive} = \Sigma_{naive} \subset \Sigma^c_{stb} \subset \Sigma_{stb}$

Example

$\mathbb{U} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \cancel{\{a, b, c\}}\} \subset \mathbb{U}^+$



$\mathbb{U}^- = \{\{a, b, c\}\}$.

Compact Realizability: Stable Semantics

Definition

Given an extension-set $\mathbb{S}$, an exclusion-mapping is the set

$$\mathfrak{R}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}^-} \{(s, \mathfrak{f}_{\mathbb{S}}(S)) \mid s \in S \text{ s.t. } (s, \mathfrak{f}_{\mathbb{S}}(S)) \notin \mathit{Pairs}_{\mathbb{S}}\}$$

where $\mathfrak{f}_{\mathbb{S}} : \mathbb{S}^- \to \mathit{Args}_{\mathbb{S}}$ is a function with $\mathfrak{f}_{\mathbb{S}}(S) \in (\mathit{Args}_{\mathbb{S}} \setminus S)$.

An extension-set $\mathbb{S}$ is called independent if there exists an exclusion-mapping $\mathfrak{R}_{\mathbb{S}}$ such that

- $\mathfrak{R}_{\mathbb{S}}$ is antisymmetric, and
- $\forall S \in \mathbb{S} \forall a \in (\mathit{Args}_{\mathbb{S}} \setminus S) : \exists s \in S : (s, a) \notin (\mathfrak{R}_{\mathbb{S}} \cup \mathit{Pairs}_{\mathbb{S}})$.

Theorem

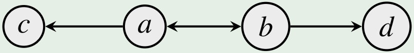
For every independent extension-set $\mathbb{S} \in \Sigma_{stb}$ it holds that $\mathbb{S} \in \Sigma^c_{stb}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \mathit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



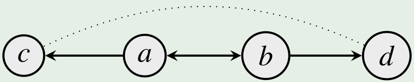
$\mathit{stb}(F) = \{\{a, d\}, \{b, c\}\}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \mathit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



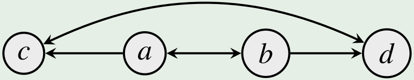
$\mathit{stb}(F) = \{\{a, d\}, \{b, c\}\}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \mathit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



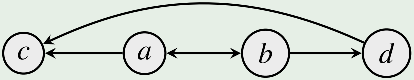
$\mathit{stb}(F) = \{\{a, d\}, \{b, c\}\}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \mathit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



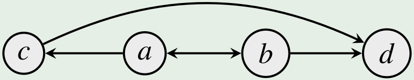
$\mathit{stb}(F) = \{\{a, d\}, \{b, c\}\}$.

Compact Realizability: Stable Semantics

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \textit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



$\textit{stb}(F) = \{\{a, d\}, \{b, c\}\}$.

Compact Realizability: Stable Semantics

Explicit-Conflict-Conjecture

For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ which is conflict-explicit under the stable semantics such that $\mathit{stb}(F) = \mathit{stb}(F')$.

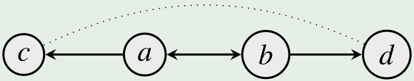
Theorem

Under the assumption that the EC-conjecture holds,

$$\Sigma^c_{\mathit{stb}} = \{\mathbb{S} \in \Sigma_{\mathit{stb}} \mid \mathbb{S} \text{ is independent}\}.$$

Explicit-Conflict-Conjecture

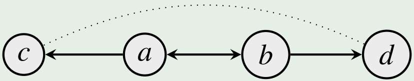
Example



$stb(F) = \{\{a, d\}, \{b, c\}\}$.

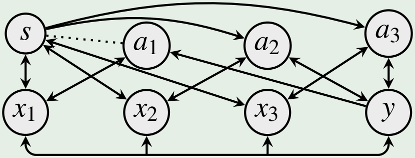
Explicit-Conflict-Conjecture

Example



stb$(F) = \{\{a, d\}, \{b, c\}\}$.

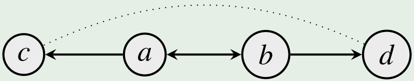
Example



stb$(F) = \{\{a_1, a_2, x_3\}, \{a_1, a_3, x_2\}, \{a_2, a_3, x_1\}, \{s, y\}\}$.

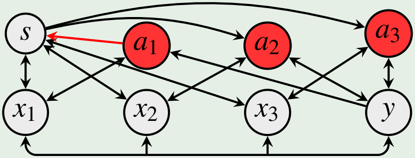
Explicit-Conflict-Conjecture

Example



stb$(F) = \{\{a, d\}, \{b, c\}\}$.

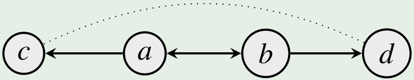
Example



stb$(F) = \{\{a_1, a_2, x_3\}, \{a_1, a_3, x_2\}, \{a_2, a_3, x_1\}, \{s, y\}, \{a_1, a_2, a_3\}\}$.

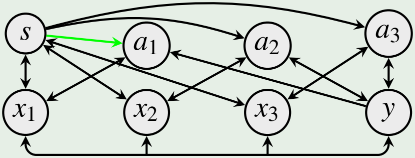
Explicit-Conflict-Conjecture

Example



stb$(F) = \{\{a, d\}, \{b, c\}\}$.

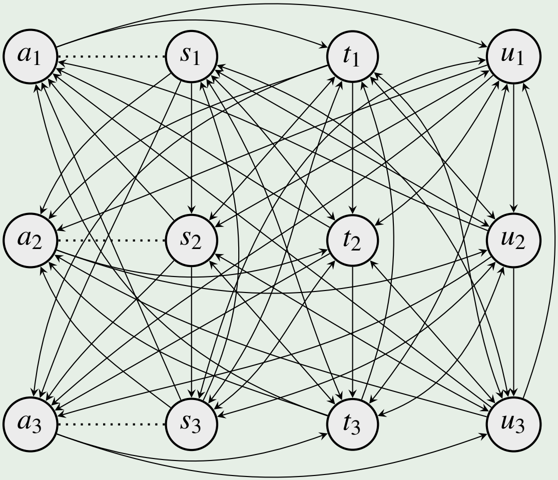
Example



stb$(F) = \{\{a_1, a_2, x_3\}, \{a_1, a_3, x_2\}, \{a_2, a_3, x_1\}, \{s, y\}$ ~~$, \{a_1, a_2, a_3\}$~~$\}$.

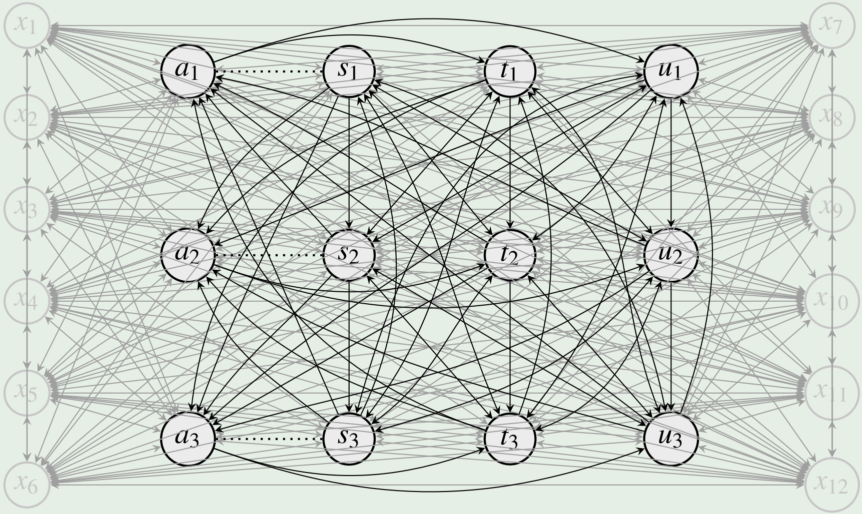
Explicit-Conflict-Conjecture

Example



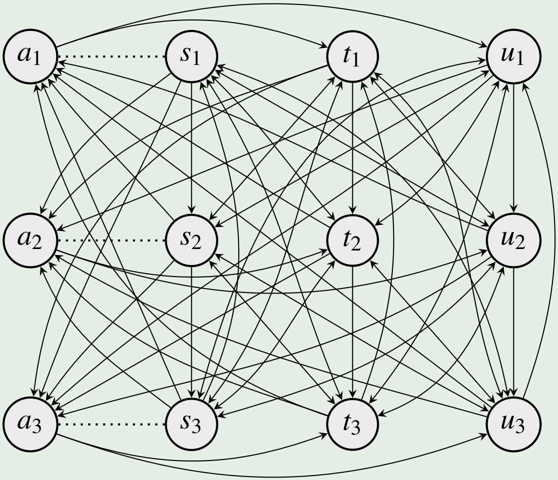
Explicit-Conflict-Conjecture

Example



Explicit-Conflict-Conjecture

Example



Impossibility Results

- Decision procedure for compact realizability supposed to be hard.
- Shortcuts can be achieved by impossible numbers.
- Maximal numbers for non-compact frameworks: [Baumann and Strass, 2014].
- Based on results for maximal independent sets [Griggs et al., 1988].

- Subsequent results hold for $\sigma \in \{\textit{stb}, \textit{sem}, \textit{pref}, \textit{stage}, \textit{naive}\}$.

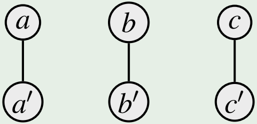
Impossibility Results

Proposition

Given an extension-set $\mathbb{S}$, the component-structure $\mathcal{K}(\mathbb{S})$ of any AF F compactly realizing $\mathbb{S}$ under σ is given by the equivalence classes of the transitive closure of $\overline{\textit{Pairs}_{\mathbb{S}}}$, i.e. $\left(\overline{\textit{Pairs}_{\mathbb{S}}}\right)^*$.

Example

$\mathbb{S}=\{\{a,b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\}\}$.



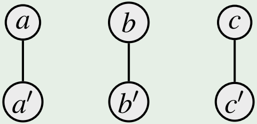
Impossibility Results

Proposition

Given an extension-set $\mathbb{S}$, the component-structure $\mathcal{K}(\mathbb{S})$ of any AF F compactly realizing $\mathbb{S}$ under σ is given by the equivalence classes of the transitive closure of $\overline{Pairs_{\mathbb{S}}}$, i.e. $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$.

Example

$\mathbb{S}=\{\{a,b,c\},\{a,b',c'\},\{a',b,c'\},\{a',b',c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\}\}$.



Proposition

Given an extension-set $\mathbb{S}$ where $|\mathbb{S}|$ is odd, it holds that if $\exists K \in \mathcal{K}(\mathbb{S}) : |K| = 2$ then $\mathbb{S}$ is not compactly realizable under semantics σ.

Impossibility Results

$\sigma^{\text{con}}_{\text{max}}(n) = \max\{|\sigma(F)| \mid F \in \text{AF}_n, F \text{ connected}\}$

Theorem

$$\sigma^{\text{con}}_{\text{max}}(n) = \begin{cases} n, & \text{if } n \leq 5, \\ 2 \cdot 3^{s-1} + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s, \\ 3^s + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s+1, \\ 4 \cdot 3^{s-1} + 3 \cdot 2^{s-2}, & \text{if } n \geq 6 \text{ and } n = 3s+2. \end{cases}$$

Definition

We denote the set of possible numbers of σ-extensions of a compact and connected AF with n arguments as $\mathcal{P}^c(n)$.

- $\forall p \in \mathcal{P}^c(n) : p \leq \sigma^{\text{con}}_{\text{max}}(n)$.
- Exact contents of $\mathcal{P}^c(n)$ unknown.

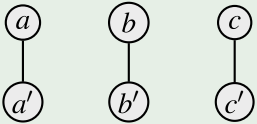
Impossibility Results

Proposition

Let $\mathbb{S}$ be an extension-set that is compactly realizable under semantics σ where $\mathcal{K}_{\geq 2}(\mathbb{S}) = \{K_1, \dots, K_n\}$. Then for each $1 \leq i \leq n$ there is a $p_i \in \mathcal{P}^c(|K_i|)$ such that $|\mathbb{S}| = \prod_{i=1}^{n} p_i$.

Example

$\mathbb{V} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a', b', c'\}\}$.



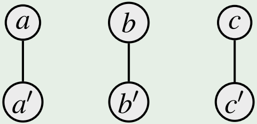
Impossibility Results

Proposition

Let $\mathbb{S}$ be an extension-set that is compactly realizable under semantics σ where $\mathcal{K}_{\geq 2}(\mathbb{S}) = \{K_1, \ldots, K_n\}$. Then for each $1 \leq i \leq n$ there is a $p_i \in \mathcal{P}^c(|K_i|)$ such that $|\mathbb{S}| = \prod_{i=1}^n p_i$.

Example

$\mathbb{V} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a', b', c'\}\}$.



Corollary

Let extension-set $\mathbb{S}$ with $|Args_{\mathbb{S}}| = n$ be compactly realizable under σ. If $|\mathbb{S}|$ is a prime number, then $|\mathbb{S}| \leq \sigma_{\max}^{\text{con}}(n)$.

Compact Argumentation Frameworks

Theorem

1. $\mathit{CAF}_{\mathit{sem}} \subset \mathit{CAF}_{\mathit{pref}}$
2. $\mathit{CAF}_{\mathit{stb}} \subset \mathit{CAF}_{\sigma} \subset \mathit{CAF}_{\mathit{naive}}$ for $\sigma \in \{\mathit{pref}, \mathit{sem}, \mathit{stage}\}$
3. $\mathit{CAF}_{\theta} \not\subseteq \mathit{CAF}_{\mathit{stage}}$ and $\mathit{CAF}_{\mathit{stage}} \not\subseteq \mathit{CAF}_{\theta}$ for $\theta \in \{\mathit{pref}, \mathit{sem}\}$

Theorem

For $\sigma \in \{\mathit{pref}, \mathit{sem}, \mathit{stage}\}$, AF $F = (A, R) \in \mathit{CAF}_{\sigma}$ and $E \subseteq A$, it is coNP-complete to decide whether $E \in \sigma(F)$.

Conclusion

Summary

- Compact realizability
  - Exact characterizations hard to find
  - Missing step for stable semantics: EC-Conjecture
- Shortcuts via impossible numbers of extensions
- Full picture of relations between compact AFs under the considered semantics

Future Work

- Exact characterizations of compact signatures.
- Closing the gap between general and compact realizability with fragments of ADFs.
- Explicit-Conflict-Conjecture.

References I

Baumann, R. and Strass, H. (2014).
On the Maximal and Average Numbers of Stable Extensions.
In TAFA 2013, volume 8306 of LNAI, pages 111–126.

Dung, P. M. (1995).
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
AIJ, 77(2):321–357.

Dunne, P. E., Dvořák, W., Linsbichler, T., and Woltran, S. (2014).
Characteristics of multiple viewpoints in abstract argumentation.
In KR, pages 72–81.

Griggs, J. R., Grinstead, C. M., and Guichard, D. R. (1988).
The number of maximal independent sets in a connected graph.
Discrete Mathematics, 68(2–3):211–220.

$\Sigma_{\textit{stage}} \not\subseteq \Sigma_{\textit{sem}}$