Disjunctive Datalog with Strong and Weak Constraints

Representational and Computational Issues

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Disjunctive Datalog

Function-free First-order Language

Basic Syntax: $\mathtt{H}_1 \lor \ldots \lor \mathtt{H}_m \leftarrow \mathtt{B}_1, \ldots, \mathtt{B}_n.$ $(m \geq 1, n \geq 0)$

H_i: Atoms

B_i: Negation-as-failure Literals

Extension: explicit literals instead of atoms ($\approx$ classical negation)

Example: 3-Colorability

- **Input:** A graph represented by `node/1` and `edge/2`
- **Problem:** Assign each node one of three colors,
  so that no two adjacent nodes have the same color

$$\texttt{col}(\texttt{X},\texttt{r}) \vee \texttt{col}(\texttt{X},\texttt{g}) \vee \texttt{col}(\texttt{X},\texttt{b}) \leftarrow \texttt{node}(\texttt{X}).$$

$$\texttt{stable} \leftarrow \texttt{edge}(\texttt{X},\texttt{Y}), \texttt{col}(\texttt{X},\texttt{C}), \texttt{col}(\texttt{Y},\texttt{C}), \texttt{not stable}.$$

Disjunctive Datalog

- **Merits**
  - Simple representation of problems
  - Capability to represent incomplete knowledge
  - High expressive power (Σ_2^P)
  - Strong relations to non-monotonic formalisms
- **Drawbacks**
  - Expressing desiderata is not intuitive

Strong and Weak (Integrity) Constraints

- Strong Constraints: $\leftarrow$ $\mathtt{B}_1$, ..., $\mathtt{B}_n$.
  - $\mathrm{B}_1, \ldots,$ and B_n must not be true in any model
  - Special case of a rule
- Weak Constraints: $\Leftarrow$ $\mathtt{B}_1$, ..., $\mathtt{B}_n$. `[L:W]`
  - Generalisation of Strong Constraints
  - `L` partitions Weak Constraints into priority layers
  - `W` assigns weights among priority layers
  - `L` and `W` induce a preference relation onto models

Example: Minimum Spanning Tree

- **Input:** An undirected graph represented by `arc/2`, associated costs $\mathtt{c}_{XY}$, an arbitrary node in `root/1`

- **Problem:** Find a partial graph with minimum cost

$$\mathtt{st(nil, X)} \leftarrow \mathtt{root(X)}.$$

$$\mathtt{st(X, Y)} \leftarrow \mathtt{reached(X), not\ outside(Y, X), arc(X, Y)}.$$

$$\mathtt{outside(Y, X)} \leftarrow \mathtt{st(Z, Y), arc(X, Y), Z} \neq \mathtt{X}.$$

$$\Leftarrow \mathtt{st(X, Y)}.\ [\ :\mathtt{c_{XY}}]$$

Computation of Preferred Models

`dlv`: efficient algorithm for computing stable models

- Objective function which is minimal for preferred models
- Modification of `dlv` incorporating this function
- Greedy algorithm
- Computation branches may be cut off early

Conclusion

- Presentation of an extension of disjunctive datalog
  - Formal definition of syntax and semantics
- Study of knowledge representation capabilities
- Design of an efficient algorithm