Parametric Properties of Ideal Semantics◇

IJCAI 2011, Barcelona

Wolfgang Dvořák[1], Paul E. Dunne[2], Stefan Woltran[1]

[1]Institute of Information Systems,
Vienna University of Technology

[2]Department of Computer Science,
University of Liverpool, U.K.

July 22, 2011

◇ Supported by the Vienna Science and Technology Fund (WWTF) under grant ICT08-028 and by the Austrian Science Fund (FWF) under grant P20704-N18.

Motivation

"Ideal semantics" as an alternative basis for skeptical reasoning in abstract argumentation [Dung, Mancarella and Toni, 2007].

Motivation

"Ideal semantics" as an alternative basis for skeptical reasoning in abstract argumentation [Dung, Mancarella and Toni, 2007].

Ideal acceptance

Informally, ideal acceptance requires an argument to be in an admissible set all of whose arguments are also skeptically accepted.

Motivation

"Ideal semantics" as an alternative basis for skeptical reasoning in abstract argumentation [Dung, Mancarella and Toni, 2007].

Ideal acceptance

Informally, ideal acceptance requires an argument to be in an admissible set all of whose arguments are also skeptically accepted.

- Similar to the concept of prudent reasoning in nonmonotonic reasoning.
- The original proposal was couched in terms of preferred semantics.
- Has been applied to semi-stable semantics ($\Rightarrow$ eager semantics) [Caminada 2007].

Argumentation Frameworks

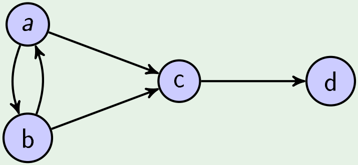
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing "attacks"

Example

$F = (\{a, b, c, d\}, \{(a, b), (b, a), (a, c), (b, c), (c, d)\})$

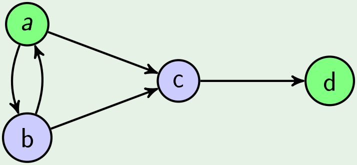


Argumentation Semantics

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



$\mathcal{E}_{cf}(F) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}\}$

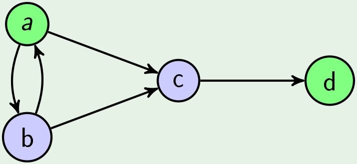
Argumentation Semantics (ctd.)

Admissible Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$\mathcal{E}_{adm}(F) = \{\emptyset, \{a\}, \{b\}, \sout{\{c\}}, \{a, d\}, \{b, d\}\}$

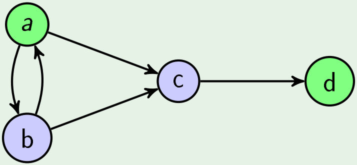
Argumentation Semantics (ctd.)

Preferred Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



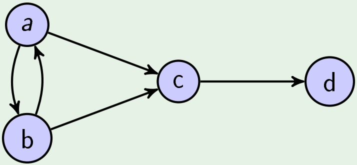
$\mathcal{E}_{pr}(F) = \{$~~$\emptyset, \{a\}, \{b\}$~~$, \{a, d\}, \{b, d\}\}$

Argumentation Semantics (ctd.)

Ideal Semantics

Given an AF $F = (A, R)$. The ideal extension is the $\subseteq$-maximal admissible set, that is contained in all preferred extensions.

Example



- $\mathcal{E}_{pr}(F) = \{\{a, d\}, \{b, d\}\}$
- Credulous accepted arguments: $\{a, b, d\}$
- Skeptical accepted arguments: $\{d\}$
- Ideal extension: $\emptyset$

Argumentation Semantics (landscape)

Definition

For an AF $F = \langle \mathcal{X}, \mathcal{A} \rangle$ we define the following semantics:

$$
\begin{aligned}
\mathcal{E}_{cf}(F) &= \{S \subseteq \mathcal{X} \mid \forall x, y \in S,\ \langle x, y \rangle \notin \mathcal{A}\} \\
\mathcal{E}_{adm}(F) &= \{S \in \mathcal{E}_{cf}(F) \mid S \subseteq \mathcal{F}(S)\} \\
\mathcal{E}_{comp}(F) &= \{S \in \mathcal{E}_{adm}(F) \mid \mathcal{F}(S) \subseteq S\} \\
\mathcal{E}_{gr}(F) &= \mathcal{F}^k(\emptyset),\ \text{for } k \text{ such that } \mathcal{F}^k(\emptyset) = \mathcal{F}^{k+1}(\emptyset) \\
\mathcal{E}_{naive}(F) &= \{S \in \mathcal{E}_{cf}(F) \mid S \subset T \Rightarrow T \notin \mathcal{E}_{cf}(F)\} \\
\mathcal{E}_{pr}(F) &= \{S \in \mathcal{E}_{adm}(F) \mid S \subset T \Rightarrow T \notin \mathcal{E}_{adm}(F)\} \\
\mathcal{E}_{sst}(F) &= \{S \in \mathcal{E}_{adm}(F) \mid S \cup S^+ \subset T \cup T^+ \Rightarrow T \notin \mathcal{E}_{adm}(F)\} \\
\mathcal{E}_{stage}(F) &= \{S \in \mathcal{E}_{cf}(F) \mid S \cup S^+ \subset T \cup T^+ \Rightarrow T \notin \mathcal{E}_{cf}(F)\} \\
\mathcal{E}_{gr*}(F) &= \min \bigcup_{\beta \in \gamma(\langle \mathcal{X}, \mathcal{A} \rangle)} \{\mathcal{E}_{gr}(\langle \mathcal{X}, \mathcal{A} \setminus \beta \rangle)\}
\end{aligned}
$$

Parameterised Ideal Semantics

Let $\langle \mathcal{X}, \mathcal{A} \rangle$ be an AF and σ a semantics that for every AF promises at least one extension.

Definition

$S \subseteq \mathcal{X}$ is an ideal set w.r.t. base semantics σ of $\langle \mathcal{X}, \mathcal{A} \rangle$ iff:

- $S \in \mathcal{E}_{adm}(\langle \mathcal{X}, \mathcal{A} \rangle)$
- $S \subseteq \bigcap_{T \in \mathcal{E}_{\sigma}(\langle \mathcal{X}, \mathcal{A} \rangle)} T$

S is an ideal extension wrt σ, if S is a $\subseteq$-maximal ideal set wrt σ.

Parameterised Ideal Semantics

Let $\langle \mathcal{X}, \mathcal{A} \rangle$ be an AF and σ a semantics that for every AF promises at least one extension.

Definition

$S \subseteq \mathcal{X}$ is an ideal set w.r.t. base semantics σ of $\langle \mathcal{X}, \mathcal{A} \rangle$ iff:

- $S \in \mathcal{E}_{adm}(\langle \mathcal{X}, \mathcal{A} \rangle)$
- $S \subseteq \bigcap_{T \in \mathcal{E}_\sigma(\langle \mathcal{X}, \mathcal{A} \rangle)} T$

S is an ideal extension wrt σ, if S is a $\subseteq$-maximal ideal set wrt σ.

Some Notation:

- E_σ^{ie} denotes an ideal extension wrt σ.
- σ^{ie} denotes the corresponding semantics.

Parameterised Ideal Semantics - Basic Properties

We show that standard properties of classical ideal semantics continue to hold for any "reasonable" extension-based base-semantics σ.

Parameterised Ideal Semantics - Basic Properties

We show that standard properties of classical ideal semantics continue to hold for any "reasonable" extension-based base-semantics σ.

Theorem

If every σ-extension is conflict-free then σ^{ie} is a unique status semantics.

Parameterised Ideal Semantics - Basic Properties

We show that standard properties of classical ideal semantics continue to hold for any "reasonable" extension-based base-semantics σ.

Theorem

If every σ-extension is conflict-free then σ^{ie} is a unique status semantics.

Theorem

If σ satisfies the reinstatement property [a] then the ideal extension E_σ^{ie} is a complete extension.

[a] A semantics σ satisfies reinstatement iff for every AF $\langle \mathcal{X}, \mathcal{A} \rangle$ and $E \in \mathcal{E}_\sigma(\mathcal{X}, \mathcal{A})$, we have that if E defends $x \in \mathcal{X}$ then $x \in E$.

Parameterised Ideal Semantics - Algorithmic Aspects

Algorithms

We provide two algorithms for computing ideal extensions:

- A generalisation of the algorithm presented by Dunne (2009) that uses a proof procedure for CA_σ.
- A new algorithm using proof procedures for SA_σ.

Computational Complexity

- We give generic upper bounds for the complexity of several decision problems associated with ideal semantics.
- Moreover we provide generic hardness results for some of the decision problems.

$\subseteq$ - Relations between Ideal Extensions

We study several instantiations of parametric ideal semantics and the relations between those.

Theorem

For any AF $F = \langle \mathcal{X}, \mathcal{A} \rangle$ the following $\subseteq$-relations hold:

$$\begin{array}{ccccc} E^{IE}_{comp}(F) \subseteq E^{IE}_{gr*}(F) \subseteq & E^{IE}_{pr}(F) & \subseteq E^{IE}_{sst}(F) \\ & \cup\!\mid & \\ & E^{IE}_{naive}(F) & \subseteq E^{IE}_{stage}(F) \end{array}$$

We have that:

- $E^{IE}_{comp}(F)$ is the the grounded semantics.
- $E^{IE}_{pr}(F)$ is the the standard ideal semantics.
- $E^{IE}_{sst}(F)$ is the the eager semantics.

Complexity Landscape

σ	$\mathrm{VER}^{\mathrm{idl}}_{\sigma}$	$\mathrm{CA}^{\mathrm{idl}}_{\sigma}$	$\mathrm{NE}^{\mathrm{idl}}_{\sigma}$	$\mathrm{VER}^{\mathrm{ie}}_{\sigma}$	$\mathrm{CONS}^{\mathrm{ie}}_{\sigma}$
comp	P-c	P-c	in L	P-c	in FP
pr	co-NP-c	in Θ_2^P	in Θ_2^P	in Θ_2^P	$\mathrm{FP}_{\parallel}^{\mathrm{NP}}$-c
sst	Π_2^p-c	Π_2^p-c	Π_2^p-c	DP_2-c	$\mathrm{FP}_{\parallel}^{\Sigma_2^p}$-c
stage	Π_2^p-c	Π_2^p-c	Π_2^p-c	DP_2-c	$\mathrm{FP}_{\parallel}^{\Sigma_2^p}$-c
gr$*$	co-NP-c	co-NP-c	co-NP-c	DP-c	$\mathrm{FP}_{\parallel}^{\mathrm{NP}}$-c
naive	in L	P-c	P-c	P-c	in FP

Conclusion

In this work we:

- Argue that the notion of "ideal acceptability" is applicable to arbitrary semantics.
- Justify this claim by showing that standard properties of classical ideal semantics continue to hold.
- Categorise the relationship between the divers concepts of "ideal extension wrt semantics σ".
- Give a comprehensive analysis of algorithmic and complexity issues.

Conclusion

In this work we:

- Argue that the notion of "ideal acceptability" is applicable to arbitrary semantics.
- Justify this claim by showing that standard properties of classical ideal semantics continue to hold.
- Categorise the relationship between the divers concepts of "ideal extension wrt semantics σ".
- Give a comprehensive analysis of algorithmic and complexity issues.

Future research directions:

- Ideal Reasoning in generalizations of AFs (VAF, EAF, AFRA)
- in particular: Uncontested Semantics for Value-based Argumentation