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Parametric Properties of Ideal Semantics

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Motivation

The concept of *"ideal semantics"* [Dung et al. 2007] has been promoted as an alternative basis for skeptical reasoning in abstract argumentation.

Informally, *ideal acceptance* not only requires an argument to be skeptically accepted in the traditional sense but further insists that the argument is in an admissible set all of whose arguments are also skeptically accepted.

The original proposal was couched in terms of the so-called preferred semantics for abstract argumentation.
In this work we:

- Argue that the notion of "ideal acceptability" is applicable to arbitrary semantics.
- Justify this claim by showing that standard properties of classical ideal semantics continue to hold.
- Categorise the relationship between the divers concepts of "ideal extension wrt semantics σ".
- Present a comprehensive analysis of algorithmic and complexity-theoretic issues.

Preliminaries

$F = \langle \mathcal{X}, \mathcal{A} \rangle$ is a (finite) Argumentation Framework (AF) with argument set $\mathcal{X}$ and attack relation $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{X}$. An argument, $x \in \mathcal{X}$ is *acceptable w.r.t.* a set $S \subseteq \mathcal{X}$ if for any $y \in \mathcal{X}$ for which $\langle y, x \rangle \in \mathcal{A}$ there is some $z \in S$ such that $\langle z, y \rangle \in \mathcal{A}$. The *characteristic function* $\mathcal{F}$ reports the set of arguments that are acceptable w.r.t. a given set.

Argumentation Semantics

$$\mathcal{E}_{cf}(F) = \{S \subseteq \mathcal{X} \mid \forall x, y \in S,\ \langle x, y \rangle \notin \mathcal{A}\}$$
$$\mathcal{E}_{adm}(F) = \{S \in \mathcal{E}_{cf}(F) \mid S \subseteq \mathcal{F}(S)\}$$
$$\mathcal{E}_{comp}(F) = \{S \in \mathcal{E}_{adm}(F) \mid \mathcal{F}(S) \subseteq S\}$$
$$\mathcal{E}_{gr}(F) = \mathcal{F}^k(\emptyset),\ \text{for } k \text{ such that } \mathcal{F}^k(\emptyset) = \mathcal{F}^{k+1}(\emptyset)$$
$$\mathcal{E}_{naive}(F) = \{S \in \mathcal{E}_{cf}(F) \mid S \subset T \Rightarrow T \notin \mathcal{E}_{cf}(F)\}$$
$$\mathcal{E}_{pr}(F) = \{S \in \mathcal{E}_{adm}(F) \mid S \subset T \Rightarrow T \notin \mathcal{E}_{adm}(F)\}$$
$$\mathcal{E}_{sst}(F) = \{S \in \mathcal{E}_{adm}(F) \mid S \cup S^+ \subset T \cup T^+ \Rightarrow T \notin \mathcal{E}_{adm}(F)\}$$
$$\mathcal{E}_{stage}(F) = \{S \in \mathcal{E}_{cf}(F) \mid S \cup S^+ \subset T \cup T^+ \Rightarrow T \notin \mathcal{E}_{cf}(F)\}$$

where $S^+ = \{x \mid \exists y \in S \text{ s.t. } \langle y, x \rangle \in \mathcal{A}\}$.

Parameterised Ideal Semantics

Let $\langle \mathcal{X}, \mathcal{A} \rangle$ be an AF and σ a semantics that for every AF promises at least one extension.

We exclude stable semantics as a base semantics

Definition

The *ideal sets* w.r.t. *base semantics* σ of $\langle \mathcal{X}, \mathcal{A} \rangle$ are those sets $S \subseteq \mathcal{X}$ that satisfy the following two conditions:

$$S \in \mathcal{E}_{adm}(\langle \mathcal{X}, \mathcal{A} \rangle)$$

$$S \subseteq \bigcap_{T \in \mathcal{E}_\sigma(\langle \mathcal{X}, \mathcal{A} \rangle)} T$$

S is an ideal *extension* wrt σ, if S is a $\subseteq$-maximal ideal set wrt σ.

$\mathcal{E}_\sigma^{\mathsf{IDL}}$ denotes the collection of ideal *sets* wrt σ and E_σ^{IE} denotes an ideal extension wrt σ. σ^{IDL} and σ^{IE} denote the corresponding semantics.

Parameterised Ideal Semantics

Properties

Properties of parameterised ideal semantics:

- If $\sigma(S)$ implies $cf(S)$ then σ^{IE} is a unique status semantics.
- If σ satisfies the reinstatement property then the ideal extension wrt σ is a complete extension.

We provide *two algorithms for computing ideal extensions*:

- A generalisation of the algorithm presented by Dunne (2009) that uses a proof procedure for CA_σ.
- A new algorithm that uses proof procedures for SA_σ.

We give generic *upper bounds*, i.e. upper bounds that are based on the complexity of the base semantics, for the complexity of several *decision problems associated with ideal semantics*.

Moreover we also provide *generic hardness results* for some of these decision problems.

Instantiations

Here we consider concrete instantiations of the base semantics σ and investigate how these different ideal semantics are related to each other.

For any AF $F = \langle \mathcal{X}, \mathcal{A} \rangle$ the following $\subseteq$-relations hold:

$$E^{IE}_{comp}(F) \subseteq E^{IE}_{gr*}(F) \subseteq E^{IE}_{pr}(F) \subseteq E^{IE}_{sst}(F)$$
$$E^{IE}_{pr}(F) \supseteq E^{IE}_{naive}(F) \subseteq E^{IE}_{stage}(F)$$

gr∗ denotes the resolution based grounded semantics [Baroni et al., 2011].

We obtain the following complexity results:

Complexity Landscape

σ	$\mathsf{VER}_\sigma^{\mathsf{IDL}}$	$\mathsf{CA}_\sigma^{\mathsf{IDL}}$	$\mathsf{NE}_\sigma^{\mathsf{IDL}}$	$\mathsf{VER}_\sigma^{\mathsf{IE}}$	$\mathsf{CONS}_\sigma^{\mathsf{IE}}$
comp	**P**-c	**P**-c	in **L**	**P**-c	in FP
pr	**coNP**-c	in Θ_2^P	in Θ_2^P	in Θ_2^P	$\mathrm{FP}_{\parallel}^{\mathbf{NP}}$-c
sst	Π_2^P-c	Π_2^P-c	Π_2^P-c	D_2^P-c	$\mathrm{FP}_{\parallel}^{\Sigma_2^P}$-c
stage	Π_2^P-c	Π_2^P-c	Π_2^P-c	D_2^P-c	$\mathrm{FP}_{\parallel}^{\Sigma_2^P}$-c
gr∗	**coNP**-c	**coNP**-c	**coNP**-c	D^P-c	$\mathrm{FP}_{\parallel}^{\mathbf{NP}}$-c
naive	in **L**	**P**-c	**P**-c	**P**-c	in FP

σ^{IE}_{comp} is the grounded semantics and σ^{IE}_{sst} is the eager semantics [Caminada, 2007].

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