Computational Aspects of Abstract Argumentation

PhD Defense, TU Wien (Vienna)

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The Argumentation Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

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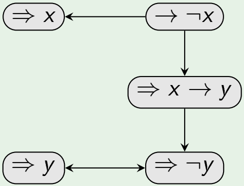

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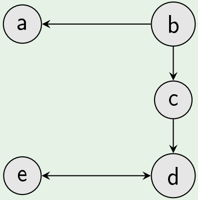

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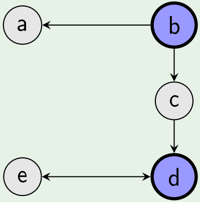

$$prf(\mathcal{F}_\Delta) = \{\{b, d\}, \{b, e\}\}$$

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The Argumentation Process

Remarks

- Main idea dates back to [Dung, 1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Abstraction allows to compare several Knowledge Representation (KR) formalisms on a conceptual level

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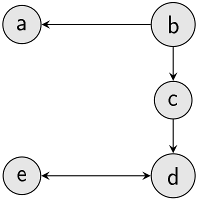
Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

Dung's Abstract Argumentation Frameworks



Main Properties

- Abstract from the concrete content of arguments and only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - "plethora of semantics"

Topics of the thesis

- Complexity Analysis
  - Complexity classification of standard reasoning tasks in abstract argumentation
- Towards Tractability
  - Graph classes as tractable fragments
  - Fixed-parameter tractability
- Intertranslatability of argumentation semantics
  - Translations between semantics as an reduction approach within argumentation

Dung's Abstract Argumentation Frameworks

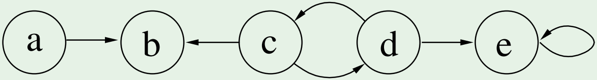
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example

F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})



Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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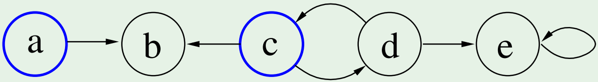

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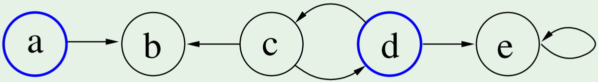

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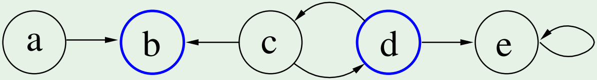

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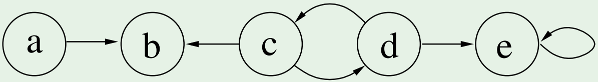

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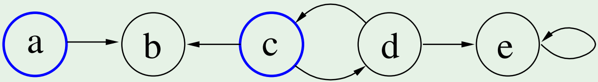

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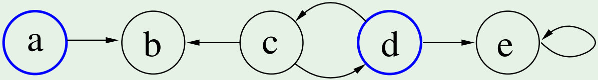

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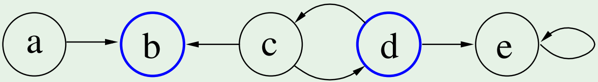

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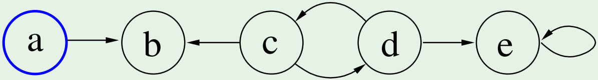

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Semantics

Definition

An extension-based semantics is a function σ mapping each AF $\mathcal{F}$ to a set of extensions $\sigma(\mathcal{F}) \subseteq 2^{A_{\mathcal{F}}}$.

If for each $\mathcal{F}$, $|\sigma(\mathcal{F})| = 1$ then we call σ a unique status semantics, otherwise multiple status semantics.

We consider 9 semantics, namely:

naive	grounded
stable	admissible
complete	resolution-based grounded
preferred	semi-stable
stage	

Semantics

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of F is defined as the outcome S of the following "algorithm":

1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
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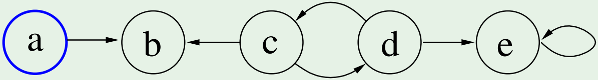

$grd(F) = \{\{a\}\}$

Semantics

Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

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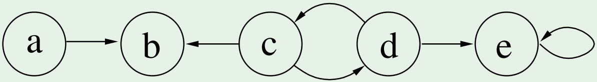

$prf(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

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Stable Extensions [Dung, 1995]

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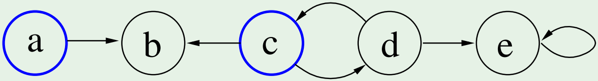

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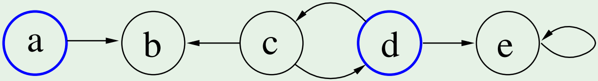

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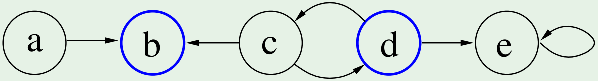

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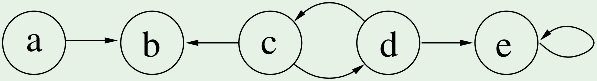

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Semantics

Semi-Stable Extensions [Caminada, 2006, Verheij, 1996]

Given an AF $F = (A, R)$. For a set $S \subseteq A$, define the range $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$.
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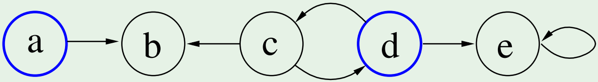

$sem(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{a\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$

Semantics

Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F.
2. Each stable extension of F is also a preferred one.
3. Each semi-stable extension of F is also a preferred one.
4. Each stable extension of F is also a semi-stable one.

$$stb(F) \subseteq sem(F) \subseteq prf(F) \subseteq adm(F) \subseteq cf(F)$$

Parametrised Ideal Semantics

Generalising [Dung et al., 2007, Caminada, 2007] we define:

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a ideal set w.r.t. base semantics σ of F, if

I1. $S \in adm(\mathcal{F})$

I2. $S \subseteq \bigcap_{E \in \sigma(\mathcal{F})} E$

We say that S is an ideal extension of $\mathcal{F}$ w.r.t. σ, if S is a $\subseteq$-maximal ideal set (of $\mathcal{F}$) w.r.t. σ.

For typical base semantics there is a unique ideal extension.

Complexity Analysis

Why doing Complexity Analysis?

- Complexity Theoretic View: To understand the Computational Costs that underlie a certain reasoning problem.
- Knowledge-Representation View: Measuring Expressivness of a formalism.
- Practitioners View: For applying the Reduction Approach, i.e. encoding a problem in other formalisms, the target formalism must be at least of the same complexity.

Decision Problems on AFs

Credulous Acceptance

$Cred_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in *at least one* σ-extension of F?

Skeptical Acceptance

$Skept_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in *every* σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.

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Ideal Acceptance

Ideal$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in the ideal extension (w.r.t. base-semantics σ) of F?

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

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Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Does there exist an extension?

$Exists_\sigma$: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Does there exist a nonempty extension?

$Exists_\sigma^{\neg\emptyset}$: Does there exist a non-empty σ-extension for F?

Complexity Landscape (State-of-the-Art)

σ	$Cred_\sigma$	$Skept_\sigma$	$Ideal_\sigma$	Ver_σ	$Exists_\sigma$	$Exists_\sigma^{\neg\emptyset}$
cf	in P	trivial	?	in P	trivial	in P
naive	in P	in P	?	in P	trivial	in P
grd	in P	in P	?	in P	trivial	in P
stb	NP-c	coNP-c	?	in P	NP-c	NP-c
adm	NP-c	trivial	?	in P	trivial	NP-c
com	NP-c	in P	?	in P	trivial	NP-c
resGr	NP-c	coNP-c	?	in P	trivial	in P
prf	NP-c	Π_2^P-c	in Θ_2^P	coNP-c	trivial	NP-c
sem	in Σ_2^P	in Π_2^P	?	coNP-c	trivial	NP-c
stg	?	?	?	?	?	?

Table: State-of-the art complexity landscape for abstract argumentation.

Complexity Analysis - Contributions

We contribute in three directions:

- Exact complexity classifications for semi-stable and stage semantics
- Complexity analysis for ideal reasoning
  - Generic complexity results referring to the complexity of other reasoning tasks (membership and hardness results)
  - Exact complexity classifications for concrete base semantics
- P-completeness classification for tractable problems

Complexity of semi-stable and stage semantics

Theorem

$Cred_{sem}$ is Σ_2^P-complete and $Skept_{sem}$ is Π_2^P-complete.

Complexity of semi-stable and stage semantics

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$Cred_{sem}$ is Σ_2^P-complete and $Skept_{sem}$ is Π_2^P-complete.

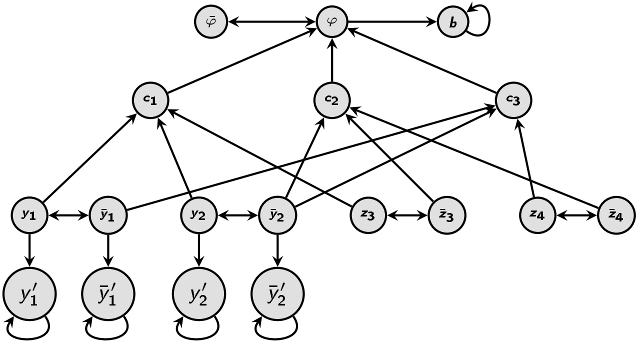
Hardness is via the following reduction:
Given a $QBF_\forall^2$ formula $\Phi = \forall Y \exists Z C$, we define $\mathcal{F}_\Phi = (A, R)$, where

$$\begin{array}{rcl} A & = & \{\varphi, \bar{\varphi}, b\} \cup C \cup Y \cup \bar{Y} \cup Y' \cup \bar{Y}' \cup Z \cup \bar{Z} \\ R & = & \{(c, \varphi) \mid c \in C\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi), (\varphi, b), (b, b)\} \cup \\ & & \{(x, \bar{x}), (\bar{x}, x) \mid x \in Y \cup Z\} \cup \\ & & \{(y, y'), (\bar{y}, \bar{y}'), (y', y'), (\bar{y}', \bar{y}') \mid y \in Y\} \cup \\ & & \{(l, c) \mid l \in C, c \in C\}. \end{array}$$

One can show that Φ is valid iff φ is skeptically accepted w.r.t. *sem*, iff $\bar{\varphi}$ is not credulously accepted w.r.t. *sem*.

Complexity of semi-stable and stage semantics

$$\Phi = \forall y_1, y_2 \; \exists z_3, z_4 \; (y_1 \vee y_2 \vee z_3) \wedge (\bar{y}_2 \vee \bar{z}_3 \vee \bar{z}_4) \wedge (\bar{y}_1 \vee \bar{y}_2 \vee z_4).$$



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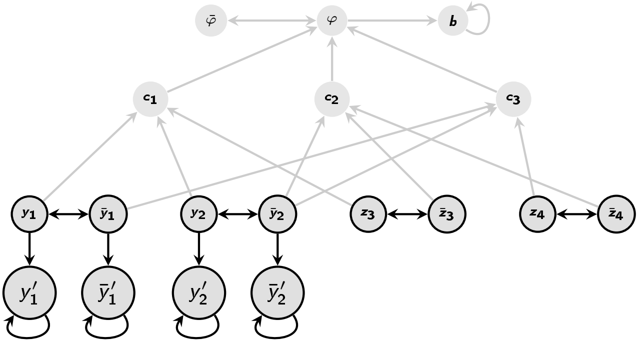

true assignment τ: $\tau(y_1) = f$, $\tau(y_2) = f$, $\tau(z_3) = f$, $\tau(z_4) = f$

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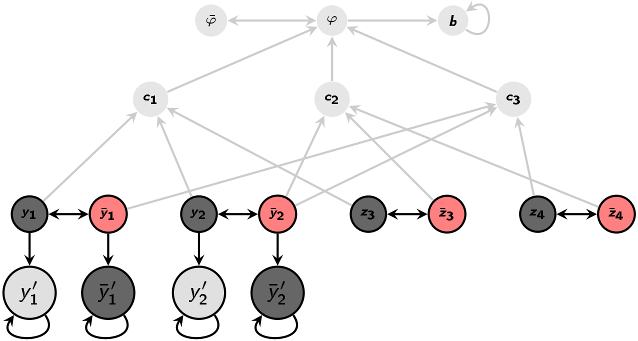
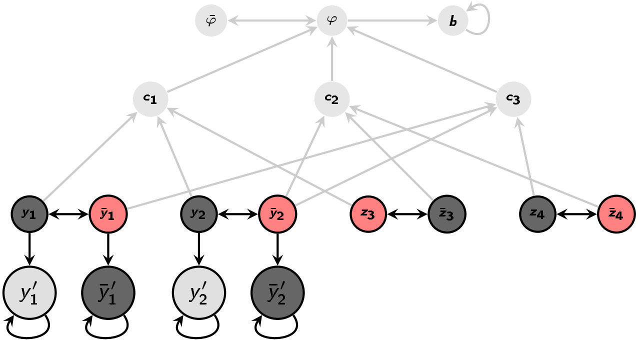


true assignment τ: $\tau(y_1) = f$, $\tau(y_2) = f$, $\tau(z_3) = f$, $\tau(z_4) = f$

Complexity of semi-stable and stage semantics

$$\Phi = \forall y_1, y_2 \ \exists z_3, z_4 \ (y_1 \vee y_2 \vee z_3) \wedge (\bar{y}_2 \vee \bar{z}_3 \vee \bar{z}_4) \wedge (\bar{y}_1 \vee \bar{y}_2 \vee z_4).$$



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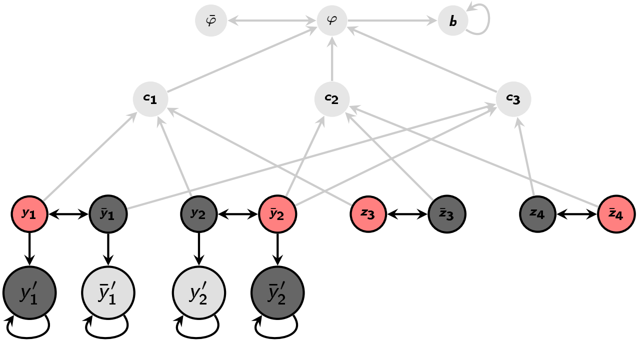

true assignment τ: $\tau(y_1) = t$, $\tau(y_2) = f$, $\tau(z_3) = t$, $\tau(z_4) = f$

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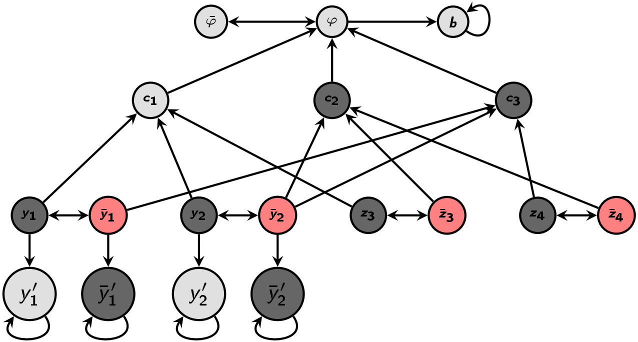

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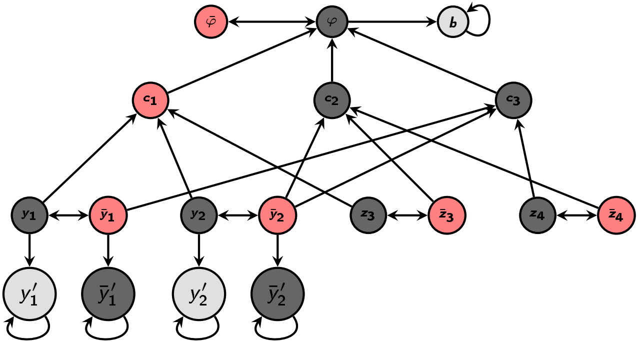

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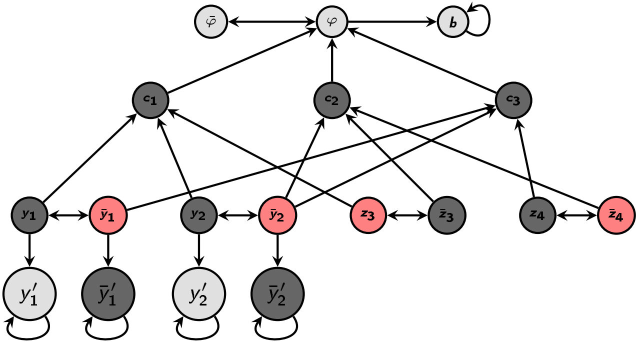

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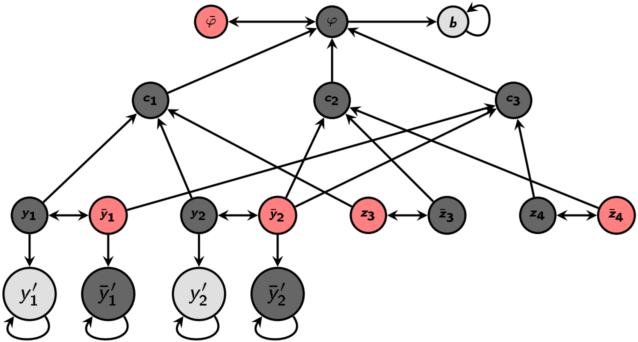

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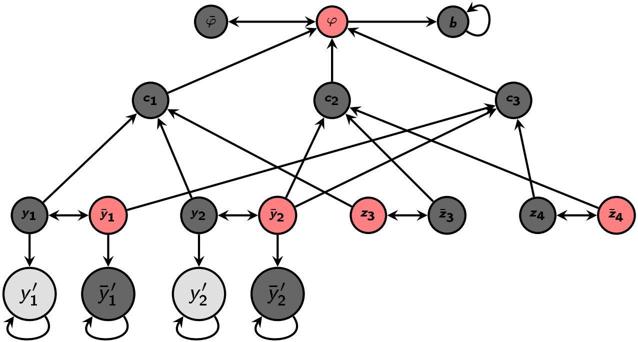

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Complexity Landscape

σ	$Cred_\sigma$	$Skept_\sigma$	$Ideal_\sigma$	Ver_σ	$Exists_\sigma$	$Exists_\sigma^{\neg\emptyset}$
cf	**in L**	trivial	trivial	**in L**	trivial	**in L**
naive	**in L**	**in L**	**P-c**	**in L**	trivial	**in L**
grd	**P-c**	**P-c**	**P-c**	**P-c**	trivial	**in L**
stb	NP-c	coNP-c	$\mathbf{D^P}$**-c**	**in L**	NP-c	NP-c
adm	NP-c	trivial	trivial	**in L**	trivial	NP-c
com	NP-c	**P-c**	**P-c**	**in L**	trivial	NP-c
resGr	NP-c	coNP-c	**coNP-c**	**P-c**	trivial	in P
prf	NP-c	Π_2^P-c	in Θ_2^P	coNP-c	trivial	NP-c
sem	$\mathbf{\Sigma_2^P}$**-c**	$\mathbf{\Pi_2^P}$**-c**	$\mathbf{\Pi_2^P}$**-c**	coNP-c	trivial	NP-c
stg	$\mathbf{\Sigma_2^P}$**-c**	$\mathbf{\Pi_2^P}$**-c**	$\mathbf{\Pi_2^P}$**-c**	**coNP-c**	trivial	**in L**

Table: Complexity of abstract argumentation.

Towards Tractability

Tractability for Abstract Argumentation

- Increasing interest for reasoning in argumentation frameworks (AFs).
- Many reasoning tasks are computationally intractable.
- As AFs can be considered as graphs,
  - there are several graph classes where some in general hard problems have been shown to be tractable (Tractable Fragments)
  - there is broad range of graph parameters we can consider to identify tractable fragments (Fixed-Parameter Tractability)

Tractable Fragments

We study four tractable fragments proposed by the literature:

- acyclic AFs [Dung, 1995]
- AFs without even length cycles (noeven) [Dunne and Bench-Capon, 2001]
- symmetric AFs [Coste-Marquis et al., 2005]
- bipartite AFs [Dunne, 2007]

We complement existing results by

- generalising them to all semantics under our considerations,
- classifying them w.r.t. P-completeness,
- solving an open problem concerning resolution-based grounded semantics and bipartite AFs.

Tractable Fragments

The P-hardness for acyclic, noeven, and bipartite is by the following:

Theorem

$Cred_{grd}$ is P-complete even for acyclic bipartite AFs.

Tractable Fragments

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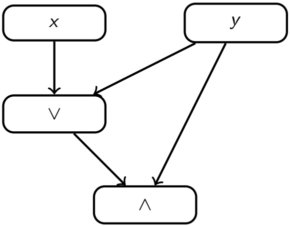
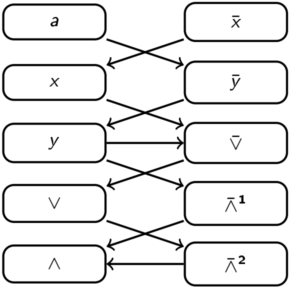
Hardness is by a reduction from the Mon. Circuit Value Problem (β, a)



Monotone Boolean Circuit β



AF $\mathcal{F}_{\beta,\mathbf{a}}$, with $a(x) = 0$, $a(y) = 1$

Fixed-Parameter Tractability

- Often computational costs primarily depend on some problem parameters rather than on the mere size of the instances.
- Many hard problems become tractable if some problem parameter is fixed or bounded by a fixed constant.
- In the arena of graphs important parameters are tree-width and clique-width. They have served as the key to many fixed-parameter tractability (FPT) results.
- We are looking for algorithms with a worst case runtime that might be exponential in the parameter but is polynomial in the size of the instance.

Fixed-Parameter Tractability

Positive Results:

We show FPT results for the parameters

- tree-width and
- clique-width

via meta-theorems by Courcelle (1987) and Courcelle, Makowsky & Rotics (2000), and MSO encodings of the argumentation semantics.

Negative Results:

- We show that typical reasoning tasks remain intractable if we bound the parameter cycle-rank.
- We extend this result to the parameters directed path-width, DAG-width, Kelly-width, and directed tree-width.

Negative Results

Definition

An AF $F = (A, R)$, has cycle rank 0 ($\mathrm{cr}(F) = 0$) iff F is acyclic, and $\mathrm{cr}(F) \leq 1$ iff each strongly connected component of F can be made acyclic by removing one argument.

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Theorem

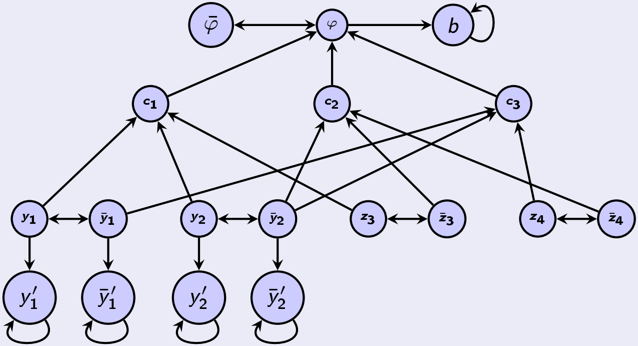
When restricted to AFs which have a cycle-rank of 1

1. $Cred_{sem}$ *remains* Σ_2^P*-hard, and*
2. $Skept_{sem}$ *remains* Π_2^P*-hard.*

Negative Results

Proof.

Recall the reduction from the hardness proof:



every framework of the form $\mathcal{F}_\Phi$ has cycle-rank 1. $\square$

Tractability Results

	stb	*adm*	*com*	*resGr*	*prf*	*sem*	*stg*
acyclic	✓	✓	✓	✓	✓	✓	✓
noeven	✓	✓	✓	✓	✓	✓	✗
bipartite	✓	✓	✓	✗	✓	✓	✓
symmetric	✗	✓	✓	✓	✓	✗	✗
bounded tree-width	✓	✓	✓	✓	✓	✓	✓
bounded clique-width	✓	✓	✓	✓	✓	✓	✓
bounded cycle-rank	✗	✗	✗	✗	✗	✗	✗
bounded directed path-width	✗	✗	✗	✗	✗	✗	✗
bounded Kelly-width	✗	✗	✗	✗	✗	✗	✗
bounded DAG-width	✗	✗	✗	✗	✗	✗	✗
bounded directed tree-width	✗	✗	✗	✗	✗	✗	✗

Intertranslatability of Argumentation Semantics

Why consider translations between Argumentation Semantics ?

- "Plethora" of Argumentation Semantics
- Reduction approach within argumentation:

  Given a translation for semantics σ to semantics σ' we can reuse sophisticated solver for σ' for semantics σ.

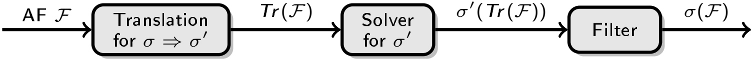


Figure: Generalising Argumentation Systems via Translations

Translations

Definition

A Translation Tr is a function mapping (finite) AFs to (finite) AFs.

Translations

Definition

A Translation Tr is a function mapping (finite) AFs to (finite) AFs.

We want translations to satisfy certain properties:

Basic Properties of a Translation Tr

- efficient: for every AF F, $Tr(F)$ can be computed using logarithmic space wrt. to $|F|$

Translations

Next we connect translations with semantics.

"Levels of Faithfulness" (for semantics σ, σ')

- exact: for every AF F, $\sigma(F) = \sigma'(Tr(F))$
- faithful: for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.

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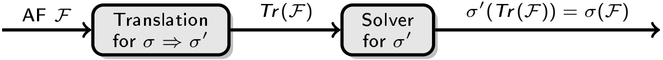

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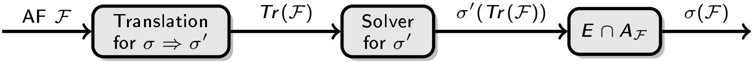
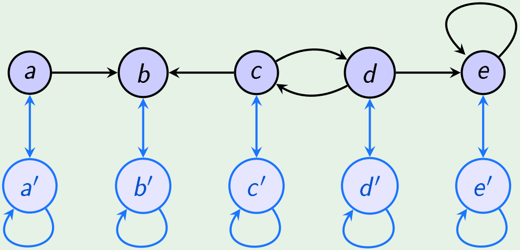


Example Translation 1

Definition

For AF F, let $Tr_1(F) = (A^*, R^*)$ where $A^* = A_F \cup A'_F$ and $R^* = R_F \cup \{(a, a'), (a', a), (a', a') \mid a \in A_F\}$, with $A'_F = \{a' \mid a \in A_F\}$.

Example



Result:

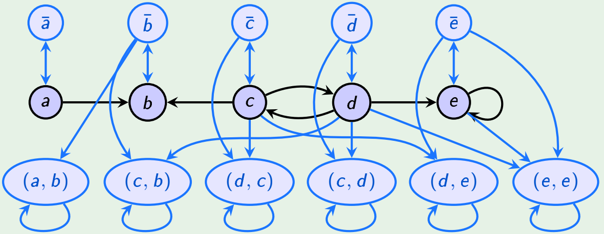
Tr_1 is an exact translation for $prf \Rightarrow sem$.

Example Translation 2

Definition

For AF F, $Tr_6(F) = (A^*, R^*)$ where $A^* = A_F \cup \bar{A}_F \cup R_F$ and
$R^* = R_F \cup \{(a, \bar{a}), (\bar{a}, a) \mid a \in A_F\} \cup \{(r, r) \mid r \in R_F\} \cup$
$\{(\bar{a}, r) \mid r = (y, a) \in R_F\} \cup \{(a, r) \mid r = (z, y) \in R_F, (a, z) \in R_F\}$.

Example



Result:

Tr_6 is a faithful translation for $adm \Rightarrow stb$.

Impossibility Results

Proposition

There is no exact translation for

- $adm \Rightarrow \sigma$ with $\sigma \in \{stb, prf, sem\}$
- $com \Rightarrow adm$

Impossibility Results

Proposition

There is no exact translation for

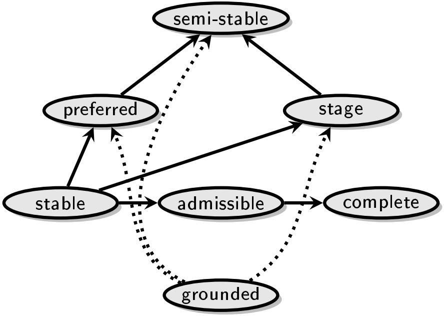
- $adm \Rightarrow \sigma$ with $\sigma \in \{stb, prf, sem\}$
- $com \Rightarrow adm$

Proposition

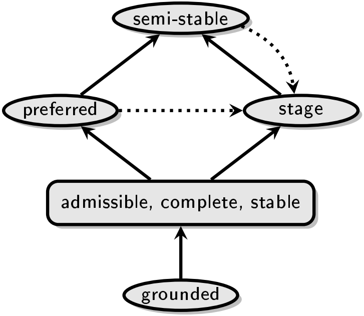
There is no efficient faithful translation for $sem \Rightarrow \sigma$, $\sigma \in \{adm, stb\}$, unless $\Sigma_2^P = \mathrm{NP}$.

Follows from complexity results.

Hierarchies of intertranslatability



Exact Intertranslatability



Faithful Intertranslatability

Summary

- We complemented existing Complexity Analysis by
  - exact classifications for semi-stable and stage semantics
  - our studies on ideal reasoning
  - P-completeness classifications
- Towards tractable instances we studied Tractable Fragments as well as Fixed-Parameter Tractability.
  - We complemented studies of Tractable Fragments
  - Fixed-Parameter Tractability results for tree-width and clique-width.
- By the Intertranslatability of semantics we applied the reduction approach within abstract argumentation presenting
  - translations between argumentation semantics
  - negative results showing that certain translations are impossible

Publications

Complexity Analysis:

- Dvořák, W. and Woltran, S. (2010). Complexity of semi-stable and stage semantics in argumentation frameworks. *Inf. Process. Lett.*, 110(11):425–430.
- Dvořák, W., Dunne, P. E., and Woltran, S. (2011). Parametric properties of ideal semantics. *IJCAI 2011*

Towards Tractability:

- Dvořák, W., Pichler, R., and Woltran, S. (2012). Towards fixed-parameter tractable algorithms for abstract argumentation. *Artificial Intelligence*, 186(0):1 – 37.
- Dvořák, W., Ordyniak, S., and Szeider, S. (2012). Augmenting tractable fragments of abstract argumentation. *Artificial Intelligence*, in press.

Intertranslatability:

- Dvořák, W. and Woltran, S. (2011). On the intertranslatability of argumentation semantics. *J. Artif. Intell. Res. (JAIR)*, 41:445–475.

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