Complexity of Abstract Argumentation Under a Claim-centric View

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Instantiation-based Argumentation

A prominent approach to formal argumentation is *instantiation-based argumentation*:

1. start from a knowledge base (KB), which is potentially inconsistent;
2. from KB, all relevant arguments are constructed; an argument typically contains (a) a claim and (b) a support;
3. relationship between arguments is analysed;
4. abstract away from the contents of the arguments and only consider the remaining abstract argumentation framework (AF);
5. semantics for AFs deliver a collection of sets of arguments ("extensions") which are understood as jointly acceptable;
6. re-interpret extensions in terms of their claims.

Example: Instantiating AFs from Logic Programs

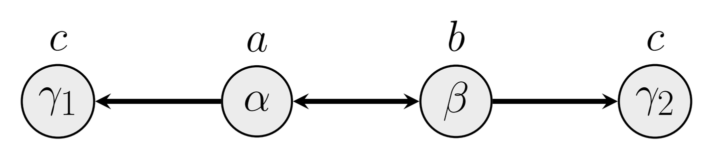
Consider the following logic program:

$$P = \{r_1 : a \leftarrow not\, b.;\quad r_2 : b \leftarrow not\, a.;\quad r_3 : c \leftarrow not\, a.;\quad r_4 : c \leftarrow not\, b.\}$$

The instantiation yields an AF $F_P = (A, R)$ with arguments $A = \{\alpha, \beta, \gamma_1, \gamma_2\}$, where

- α represents rule r_1 and has claim a;
- β represents rule r_2 with claim b;
- γ_1 and γ_2 represent rules r_3 and r_4 respectively, both have as claim c.

An argument representing rule r attacks an argument representing rule r' if the head of r occurs negated in the rule body of r'. Hence, $R = \{(\alpha, \beta), (\beta, \alpha), (\alpha, \gamma_1), (\beta, \gamma_2)\}$;



Stable model semantics of logic programs corresponds to stable extensions of AFs:

- the two stable models $S_1 = \{a, c\}$ and $S_2 = \{b, c\}$ of P are given via
- the two stable extensions $E_1 = \{\alpha, \gamma_2\}$ and $E_2 = \{\beta, \gamma_1\}$ of F_P;
- the claims of E_1 yield S_1 and those of E_2 yield S_2.

Reasoning Modes

- *Argument-centric Reasoning*: is a particular argument accepted w.r.t. the extensions?
- *Claim-centric Reasoning*: is a particular claim accepted w.r.t. the extensions?

Skeptical Acceptance: is a particular argument a / claim c covered by all extensions?

Example: Instantiating AFs from Logic Programs (ctd.)

With the extensions of F_P being $E_1 = \{\alpha, \gamma_2\}$ and $E_2 = \{\beta, \gamma_1\}$ of F_P, we have that

- no argument is skeptically accepted.

However, as the stable models of P are $S_1 = \{a, c\}$ and $S_2 = \{b, c\}$,

- claim c is a skeptical consequence of the program P.

Observation:

- Argument acceptance alone is insufficient to decide the acceptance of claims.

Reasoning about Claims

We consider AFs augmented by claims as a distinguished concept.

Claim-augmented Argumentation Frameworks

A *claim-augmented argumentation framework (CAF)* is a triple $(A, R, claim)$ where

- (A, R) is an AF with arguments A and attacks $R \subseteq A \times A$;
- $claim : A \to \mathcal{C}$ assigns a claim to each argument.

A CAF $(A, R, claim)$ is called *well-formed* if arguments with the same claim attack the same arguments.

- Different arguments can have the same claim.
- No further information about claims (like equivalence or contradict relation).
- The concept of well-formedness is satisfied by many (but not all) instantiations.

Semantics

For any CAF $CF = (A, R, claim)$ and semantics σ, we define its claim-based variant σ_c as:

$$\sigma_c(CF) = \{claim(S) \mid S \in \sigma((A, R))\}.$$

We consider conflict-free (cf), naive ($naive$), grounded (grd), stable (stb), admissible (adm), complete (com), and preferred (prf) semantics.

Claim-centric Complexity Analysis

Claim-centric Reasoning Problems

Given semantics σ, a CAF $CF = (A, R, claim)$, claim $c \in \mathcal{C}$, and claims $C \subseteq \mathcal{C}$:

- $Cred^{CAF}_\sigma$: Does $c \in S$ hold for at least one $S \in \sigma_c(CF)$?
- $Skept^{CAF}_\sigma$: Does $c \in S$ hold for all $S \in \sigma_c(CF)$?
- Ver^{CAF}_σ: Does $C \in \sigma_c(CF)$ hold?
- $NEmpty^{CAF}_\sigma$: Does $S \neq \emptyset$ hold for some $S \in \sigma_c(CF)$?

Complexity of CAFs

σ	$Cred^{CAF}_\sigma$	$Skept^{CAF}_\sigma$	Ver^{CAF}_σ	$NEmpty^{CAF}_\sigma$
cf	in P	trivial	**NP**-c	in P
$naive$	in P	**coNP**-c	**NP**-c	in P
grd	P-c	P-c	P-c	in P
stb	NP-c	coNP-c	**NP**-c	NP-c
adm	NP-c	trivial	**NP**-c	NP-c
com	NP-c	P-c	**NP**-c	NP-c
prf	NP-c	Π^P_2-c	$\boldsymbol{\Sigma^P_2}$-c	NP-c

(Results that deviate from the corresponding results for AFs are highlighted in **blue**.)

Complexity of well-formed CAFs

σ	$Cred^{wf}_\sigma$	$Skept^{wf}_\sigma$	Ver^{wf}_σ	$NEmpty^{wf}_\sigma$
cf	in P	trivial	in **P**	in P
$naive$	in P	coNP-c	in **P**	in P
grd	P-c	P-c	P-c	in P
stb	NP-c	coNP-c	in **P**	NP-c
adm	NP-c	trivial	in **P**	NP-c
com	NP-c	P-c	in **P**	NP-c
prf	NP-c	Π^P_2-c	**coNP**-c	NP-c

(Results that deviate from general CAFs are highlighted in **red**.)

Coincides with results for argument-centric reasoning except for $Skept^{wf}_{naive}$.

Analysing the Tractability Frontier

We follow three directions towards tractability results:

Exploiting Special Graph Classes

Some results are in contrast to argument-centric reasoning:

- $Skept^{CAF}_{naive}$, $Skept^{wf}_{naive}$, Ver^{CAF}_{naive}, Ver^{CAF}_{cf} remain coNP/NP-hard for acyclic CAFs.
- For $\sigma \in \{naive, stb, prf\}$, $Skept^{CAF}_\sigma$ is coNP-complete for bipartite well-formed CAFs.

Exploiting the Number of Claims

We parameterize the problems with the number k of different claims that appear in the CAF and obtain a **Fixed-Parameter Tractability Result:**

- $Cred^{wf}_\sigma$, $Skept^{wf}_\sigma$, and Ver^{wf}_{prf} can be solved in time $O(2^k \cdot poly(n))$ for $\sigma \in \{naive, stb, adm, com, prf\}$.

Exploiting (Incidence) Tree-Width of CAFs

We introduce the parameter *incidence tree-with* of well-formed CAFs which measures the structure of the interplay between claims and arguments and is complementary to tree-width.

Main Results (for $\sigma \in \{naive, stb, adm, com, prf\}$):

- $Cred^{CAF}_\sigma$ and $Skept^{CAF}_\sigma$ are fixed-parameter tractable w.r.t. the tree-width;
- Ver^{CAF}_σ is NP-hard for CAFs of tree-width 1;
- $Cred^{wf}_\sigma$, $Skept^{wf}_\sigma$, and Ver^{wf}_σ are fixed-parameter tractable w.r.t. incidence tree-width.

Main References

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