Complexity of Semi-Stable and Stage Semantics in Argumentation Frameworks◇

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Outline

Argumentation in AI

- Very general idea: representation of an argument
- Different views: modeling the process, verifying the correctness, analyzing the conflicts,...etc.
- Thus, representation of arguments came in many different flavors

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Abstract Argumentation

- Arguments are "atomic"
- Argumentation frameworks (AFs) formalize relations (rebuttals) between arguments
- Semantics gives an abstract handle to solve the inherent conflicts between statements by selecting acceptable subsets

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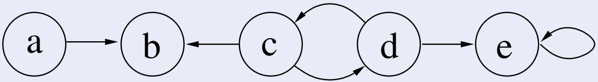
Example

AF=({a,b,c,d,e},{(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})



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Given an AF (A, R).
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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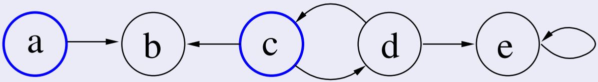

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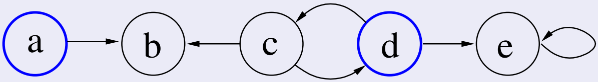

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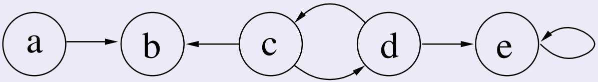

$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

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Given an AF (A, R). A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
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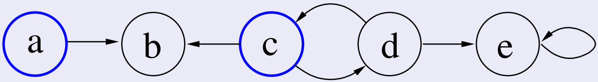

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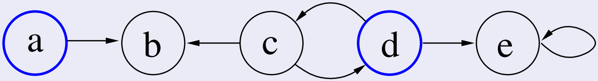

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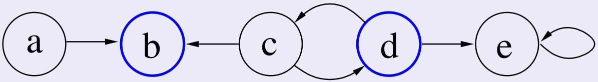

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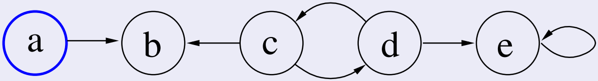

$adm(F) = \{\{a, c\}, \{a, d\}, \sout{\{b, d\}}, \{a\}, \sout{\{b\}}, \{c\}, \{d\}, \emptyset\}$

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Given an AF (A, R). A set $S \subseteq A$ is stable in F, if

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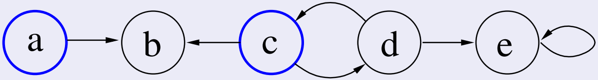

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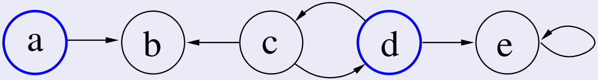

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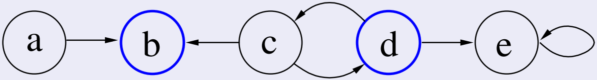

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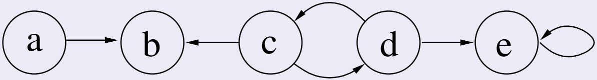

stable$(F) = \{$ ~~$\{a, c\}$~~, $\{a, d\}$, ~~$\{b, d\}$~~, ~~$\{a\}$~~, ~~$\{b\}$~~, ~~$\{c\}$~~, ~~$\{d\}$~~, ~~$\emptyset$~~, $\}$

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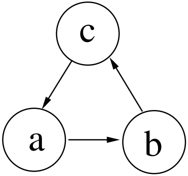

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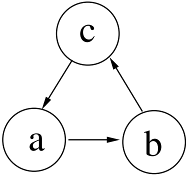

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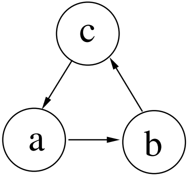

Idea: Using extensions minimizing the unattacked arguments in $A \setminus S$.

- For $S \subseteq A$ we define $S^+ = S \cup \{a : \exists b \in S : (b, a) \in R\}$
- minimizing $A \setminus S^+ \Leftrightarrow$ maximizing S^+
- If S is a stable extension then $S^+ = A$

Stage/Semi-Stable Extension

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Given an AF (A, R). A set $S \subseteq A$ is stage (resp. semi-stable) in F, if

- S is conflict-free (resp admissible) in F
- for each $S' \subseteq A$, if S' conflict-free (admissible) then $S^+ \not\subset S'^+$..

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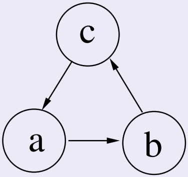

$cf(F) = \{\emptyset, \{a\}, \{b\}, \{c\}\}$
$adm(F) = \{\emptyset\}$

$stage(F) = \{\{a\}, \{b\}, \{c\}\}$
$semi(F) = \{\emptyset\}$

Decision Problems on AFs

Let be σ a semantic for AFs then we are interested in the following problems:

- Credulous Acceptance (Cred_σ): Given AF $F = (A, R)$ and $a \in A$; is a contained in at least one σ-extension of F?
- Skeptical Acceptance (Skept_σ): Given AF $F = (A, R)$ and $a \in A$; is a contained in every σ-extension of F?

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Theorem ([Dunne and Caminada(2008)])

Cred_{semi} *and* Skept_{semi} *are* $\text{P}_{||}^{\text{NP}}$*- hard.*
Cred_{semi} *is* Σ_2^p *- easy.* / Skept_{semi} *is* Π_2^p *- easy.*

Complexity of stage / semi-stable semantics

Theorem ([Dvořák and Woltran(2009)])

Cred *for stage / semi-stable semantics is* Σ_2^p*-complete.*
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Proof membership.

Credulous Acceptance of $a \in A$

- Guess a set S such that $a \in S$.
- Verify that S is conflict-free (admissible)
- Verify that S is $\subseteq^+$-maximal (in co-NP)
  - Guess a set S' such that $S^+ \subset S'^+$
  - Test if S' is conflict-free (admissible)

□

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Proof membership.

co-Skeptical Acceptance of $a \in A$

- Guess a set S such that $a \notin S$.
- Verify that S is conflict-free (admissible)
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Hardness - Skeptical Acceptance

To prove the hardness we reduce the Π_2^p-hard problem $\mathrm{QSAT}_2^\forall$ to Skept.

Definition ($\mathrm{QSAT}_2^\forall$)

Given: A quantified boolean formula in CNF: $\Phi = \forall Y \, \exists Z \, \Psi(Y, Z)$.
Question: Is Φ true?

Example:

$$\forall y_1, y_2 \, \exists z_3, z_4 \, (y_1 \vee y_2 \vee z_3) \wedge (\neg y_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg y_1 \vee \neg y_2 \vee z_4)$$

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$$\forall y_1, y_2 \; \exists z_3, z_4 \; (y_1 \lor y_2 \lor z_3) \land (\neg y_2 \lor \neg z_3 \lor \neg z_4) \land (\neg y_1 \lor \neg y_2 \lor z_4)$$

In our reduction

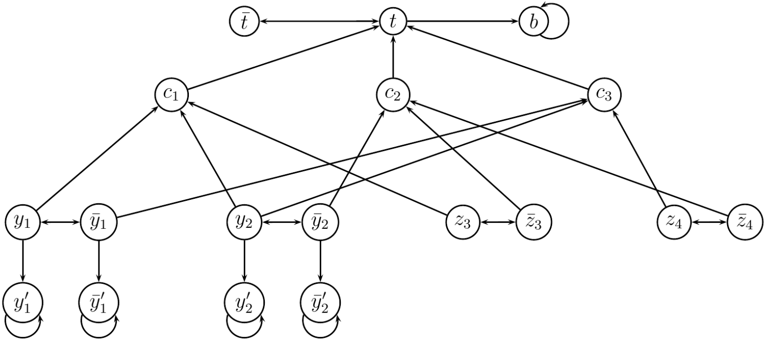
- we map each formula to Φ to an AF F_Φ and an argument $t \in F_\Phi$
- such that Φ is true iff t is skeptically accepted in F_Φ.

Reduction (informal)

We first demonstrate our reduction on an example QBF:

$$\forall y_1, y_2 \; \exists z_3, z_4 \; (y_1 \vee y_2 \vee z_3) \wedge (\neg y_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg y_1 \vee \neg y_2 \vee z_4)$$

The resulting framework F_Φ:



Reduction (formal)

Reduction

Given a $QBF_{\forall}^{2}$ formula $\Phi = \forall Y \exists Z \bigwedge_{c \in C} c$, we define $F_{\Phi} = (A, R)$, where

$$
\begin{aligned}
A &= \{t, \bar{t}, b\} \cup C \cup Y \cup \bar{Y} \cup Y' \cup \bar{Y}' \cup Z \cup \bar{Z} \\
R &= \{\langle c, t \rangle \mid c \in C\} \cup \\
&\quad \{\langle x, \bar{x} \rangle, \langle \bar{x}, x \rangle \mid x \in Y \cup Z\} \cup \\
&\quad \{\langle y, y' \rangle, \langle \bar{y}, \bar{y}' \rangle, \langle y', y' \rangle, \langle \bar{y}', \bar{y}' \rangle \mid y \in Y\} \cup \\
&\quad \{\langle l, c \rangle \mid \text{literal } l \text{ occurs in } c \in C\} \cup \\
&\quad \{\langle t, \bar{t} \rangle, \langle \bar{t}, t \rangle, \langle t, b \rangle, \langle b, b \rangle\}.
\end{aligned}
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Lemma

For every stage (resp. semi-stable) extension S of an AF $F_\Phi = (A, R)$:

1. $b \notin S$, *as well as* $y' \notin S$ *and* $\bar{y}' \notin S$ *for each* $y \in Y$.
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If $x \in Y \cup Z$ then we define $T = (S \setminus \{c \in C \mid \langle \bar{x}, c \rangle \in R\}) \cup \{\bar{x}\}$. Once more we have that T is conflict-free and that T is admissible if S is. For the removed arguments $c \in C$, we have $c \in T^+$.
The only argument attacked by such c is t, but $t \in T^+$, since we can already assume $\{t, \bar{t}\} \cap S \neq \emptyset$. Therefore we have $S^+ \subset T^+$. ↯ □

Lemma

Let $Y^* = Y \cup \bar{Y} \cup Y' \cup \bar{Y}'$ *and* S, T *be conflict-free sets then:*

1. $S \cap Y^* \subseteq T \cap Y^*$ *iff* $(S \cap Y^*)^+ \subseteq (T \cap Y^*)^+$
2. $S \cap Y^* = T \cap Y^*$ *iff* $(S \cap Y^*)^+ = (T \cap Y^*)^+$

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$\Leftarrow$: Assume now $(S \cap Y^*)^+ \subseteq (T \cap Y^*)^+$ and let $l \in S \cap Y^*$. (l is either of form y or $\bar{y}$)
As $l \in S \cap Y^*$ we have $l, \bar{l}, l' \in (S \cap Y^*)^+$ and thus $l, \bar{l}, l' \in (T \cap Y^*)^+$.
But then, $l \in T \cap Y^*$ follows from $l' \in (T \cap Y^*)^+$. ✓

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But then, $l \in T \cap Y^*$ follows from $l' \in (T \cap Y^*)^+$. ✓
By symmetry (2) follows. □

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$$\big(I_Y \cup I_Z \cup \{\bar{x} \mid x \in (Y \cup Z) \setminus (I_Y \cup I_Z)\}\big) \cap c \neq \emptyset.$$

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Consider now the set

$$T = I_Y \cup I_Z \cup \{\bar{x} \mid x \in (Y \cup Z) \setminus (I_Y \cup I_Z)\} \cup \{t\}.$$

T is admissible and $T^+ = A \setminus \bar{I}'_Y$.
As $S \cap \bar{I}'_Y = \emptyset$ and $b \notin S^+$ this implies $S^+ \subset T^+$ ↯ □

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We have to show that t is contained in all semi-stable extensions of F_Φ iff Φ is true. (The if direction is already captured by the last lemma)

Hardness - Skeptical Acceptance Semi-Stable

Theorem

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Proof.

We prove the only-if direction by showing that if Φ is false, then there exists a semi-stable extension S of F_Φ such that $t \notin S$.
In case Φ is false, there exists an $I_Y \subseteq Y$, such that for each $I_Z \subseteq Z$, there exists a $c \in C$, such that

$$\left(I_Y \cup I_Z \cup \{\bar{x} \mid x \in (Y \cup Z) \setminus (I_Y \cup I_Z)\}\right) \cap c = \emptyset. \tag{1}$$

Consider now a maximal (wrt. $\leq^+$) admissible (in F_Φ) set S, such that $I_Y \subseteq S$. S then has to be a semi-stable extension. □

proof (ctd).

Consider now a maximal (wrt. $\leq^+$) admissible (in F_Φ) set S, such that $I_Y \subseteq S$. S then has to be a semi-stable extension.

It remains to show $t \notin S$. We prove this by contradiction and assume $t \in S$.

As S is admissible, S defends t and therefore it defeats all $c \in C$.

Further as all attacks against C come from $Y \cup \bar{Y} \cup Z \cup \bar{Z}$, the set $U = \left(I_Y \cup (S \cap (Z \cup \bar{Z})) \cup \{\bar{y} \mid y \in Y \setminus I_Y\}\right)$ defeats all $c \in C$.

As we know that for each $z \in Z$, either z or $\bar{z}$ is contained in S. We get an equivalent characterization for U by

$U = \left(I_Y \cup I_Z \cup \{\bar{x} \mid x \in (Y \cup Z) \setminus (I_Y \cap I_Z)\}\right)$ with $I_Z = S \cap Z$.

Thus, for all $c \in C$,

$$\left(I_Y \cup I_Z \cup \{\bar{x} \mid x \in (Y \cup Z) \setminus (I_Y \cup I_Z)\}\right) \cap c \neq \emptyset,$$

which contradicts assumption (1). □

Hardness - Skeptical Acceptance under Stage Semantics

Theorem

Skept_{stage} *is* Π_2^p*-hard.*

Hardness - Skeptical Acceptance under Stage Semantics

Theorem

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Proof.

Similar to the proof of the previous theorem.
For details see [Dvořák and Woltran(2009)] □

Hardness - Credulous Acceptance

Theorem

Credulous acceptance for stage or semi-stable semantics is Σ_2^p-hard.

Hardness - Credulous Acceptance

Theorem

Credulous acceptance for stage or semi-stable semantics is Σ_2^p-hard.

Proof.

We have shown that a $QBF_{\forall}^2$ formula Φ is true iff t is contained in each semi-stable extension of F_Φ. This is equivalent to $\bar{t}$ is not contained in any semi-stable extension of F_Φ. Thus the co-credulous acceptance is also Π_2^p-hard. □

Fixed-Parameter-Tractability

Stage and Semi-stable Extensions can be specified in MSOL:

$$
\begin{aligned}
U \subseteq^+_R V &= \forall x\Big(\big(x \in U \vee \exists y(y \in U \wedge \langle y, x\rangle \in R)\big) \rightarrow \\
&\qquad \big(x \in V \vee \exists y(y \in V \wedge \langle y, x\rangle \in R)\big)\Big) \\
U \subset^+_R V &= U \subseteq^+_R V \wedge \neg(V \subseteq^+_R U) \\
\mathrm{cf}_R(U) &= \forall x, y\big(\langle x, y\rangle \in R \rightarrow (\neg x \in U \vee \neg y \in U)\big) \\
\mathrm{adm}_R(U) &= \mathrm{cf}_R(U) \wedge \forall x, y\Big(\big(\langle x, y\rangle \in R \wedge y \in U\big) \rightarrow \\
&\qquad \exists z(z \in U \wedge \langle z, x\rangle \in R)\Big) \\
\mathrm{semi}_{(A,R)}(U) &= \mathrm{adm}_R(U) \wedge \neg\exists V(V \subseteq A \wedge \mathrm{adm}_R(V) \wedge U \subset^+_R V) \\
\mathrm{stage}_{(A,R)}(U) &= \mathrm{cf}_R(U) \wedge \neg\exists V(V \subseteq A \wedge \mathrm{cf}_R(V) \wedge U \subset^+_R V)
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\end{aligned}
$$

By Courcelles theorem the problems Cred_{semi}, Skept_{semi}, Cred_{stage}, Skept_{stage} are fixed parameter tractable wrt tree-width of AF.

Fixed-Parameter-Tractability

Definition (cycle rank)

An acyclic graph has $cr(G) = 0$.
If G is strongly connected then $cr(G) = 1 + min_{v \in V_G} cr(G \setminus v)$.
Otherwise, $cr(G)$ is the maximum cycle rank among all strongly connected components of G.

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The problems $\text{Skept}_{semi}, \text{Skept}_{stage}$ *(resp.* $\text{Cred}_{semi}, \text{Cred}_{stage}$*) remain* Π_2^p*-hard (resp.* Σ_2^p*-hard), even if restricted to AFs which have a cycle-rank of* 1*.*

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Proof.

Every framework of the form F_Φ has cycle-rank 1 and therefore we have an reduction from $QBF_\forall^2$ formulas to an AF with cycle-rank 1. □

Conclusion

Main Results:

- We answered two questions about the complexity of semi-stable semantics raised by Dunne and Caminada (2008).
  Cred_{semi} is Σ_2^p-complete / Skept_{semi} is Π_2^p-complete
- We extended this results to stage semantics:
  Cred_{stage} is Σ_2^p-complete / Skept_{stage} is Π_2^p-complete
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- But these problems are tractable on AFs of bounded tree-width.

Future Work:

- Finding tractable algorithms for AFs of bounded tree-width.
- Identify further tractable fragments.

Paul E. Dunne and Martin Caminada.
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