Abstract

Claim-augmented argumentation frameworks (CAFs) constitute a generic formalism for conflict resolution of conclusion-oriented problems in argumentation. CAFs extend Dung argumentation frameworks (AFs) by assigning a claim to each argument; so far, semantics for CAFs have been defined by considering the semantics for AFs and interpreting the extensions in terms of the claims of the arguments. However, certain semantics of the originally considered (instantiated) problem which involve maximization of the range on conclusion-level cannot be captured by performing maximization on argument-level. In this paper, we propose therefore an alternative way of defining range-based semantics for CAFs in order to mimic the behavior of the respective semantics of the instantiated problems; we investigate the relation of the newly introduced semantics to their argument-level based counterparts.

1 Introduction

Abstract argumentation frameworks (AFs) as introduced by Dung [6] provide a general schema for analyzing discourses by treating arguments as abstract entities while an attack relation encodes conflicts between them; the acceptance status of arguments is evaluated with respect to different semantics. Moreover, AFs exhibit a close connection to logic programming and other non-monotonic reasoning formalisms by allowing for an alternative way of representing inconsistent and conflicting information. The instantiation of logic programs (LPs) into AFs and generalizations thereof has been frequently discussed in the literature [6, 13, 5] and reveals the close connection of both formalisms in particular by comparing the respective semantics; the correspondence of stable model semantics for LPs with stable semantics in AFs is probably the most fundamental example [6], but also 3-valued stable model semantics or well-founded model semantics admit equivalent argumentation semantics [13].

In a nutshell, an instantiation procedure into AFs includes (1) extraction of arguments and conflicts among them; (2) identification of jointly acceptable arguments (extensions) based on a particular argumentation semantics; (3) inspection of claims of the acceptable arguments in order to draw conclusions about the original system. Instantiation procedures for different formalisms have been established, see e.g. [11, 10, 4, 5]. A generalization of AFs which is ideally suited for instantiation procedures in this spirit are claim-augmented argumentation frameworks (CAFs) [8] which extend AFs by assigning a claim to each argument. In [8], semantics for CAFs are evaluated with respect to the underlying AF, the extensions are then interpreted in terms of the claims of the arguments (inherited semantics). We furthermore mention a particular restriction on the attack relation of CAFs which is satisfied by many instantiation procedures: A CAF is well-formed iff arguments having the same claim attack the same argument. In the following example, we will adapt an instantiation of logic programs to AFs due to [5] by defining an appropriate claim-function for the generated arguments.

Example 1. Consider the logic program $P$ from Figure 1, we will construct a CAF $CF = (A, R, claim)$ by instantiating the AF $(A, R)$ following [5] and extracting the claim-function claim for the constructed arguments.
as follows: Each rule \( r_i : c \leftarrow \text{not } b_1, \ldots, \text{not } b_m \) is interpreted as argument \( A_i \in A \) where the head \( c \) or \( r_i \) corresponds to the claim of \( A_i \) (that is, we define \( \text{claim}(A_i) = c \)). Moreover, the negated atoms determine the potential attackers of \( A_i \) in \( \text{CF} \), that is, an argument \( A_j \) attacks \( A_i \), i.e. \( (A_j, A_i) \in R \), iff \( A_j \) has claim \( b_k \) for some \( k \leq m \). The resulting CAF is depicted in Figure 2. Evaluating \( \text{CF} \) with respect to stable semantics\(^1\) yields no extension; also, \( P \) does not possess a stable model. Observe that the procedure yields a well-formed CAF.

Although the CAF \( \text{CF} \) in Example 1 yields the same results as the original problem with respect to most of the semantics, certain irregularities may arise when it comes to so-called range-based semantics, which take arguments (atoms) into account that are defeated (set to false) in the particular extension (model): Semi-stable semantics \([12, 3]\), which yield admissible sets, thus the evaluation matches the outcome of \( P \) with respect to L-stable model semantics. Observe that the procedure yields a well-formed CAF.

We introduce alternatives based on maximization on claim-level and investigate their relation to inherited semantics in the spirit of [8] which perform maximization on argument-level. The main results of our paper are:

- We introduce alternative definitions for semi-stable and stage semantics for CAFs by shifting maximization of extensions from argument-level to claim-level. A crucial notion therefore is the defeat of claims, where one requires that a claim \( c \) is defeated iff every occurrence of \( c \) is attacked.
- We propose two variants of stable semantics, based on conflict-free, respectively, admissible sets. We show that for well-formed CAFs, both variants of stable semantics as well as inherited stable semantics coincide.
- We compare inherited semantics with cl-semantics. We show that they exhibit similar behaviour concerning incomparability: For general CAFs, incomparability of claim-sets is not guaranteed, whereas for well-formed CAFs, every semantics under consideration yields incomparable claim-sets; moreover, we show that even for well-formed CAFs, both variants of semi-stable and stage semantics potentially yield different claim-sets.

2 Preliminaries

We introduce argumentation frameworks \([6]\) (for a comprehensive introduction, see \([2, 1]\)). We fix \( U \) as countable infinite domain of arguments.

\[\begin{align*}
  r_0 : & \quad a \leftarrow \text{not } d \\
  r_1 : & \quad d \leftarrow \text{not } a \\
  r_2 : & \quad b \leftarrow \text{not } a \\
  r_3 : & \quad c \leftarrow \text{not } a, \text{not } b \\
  r_4 : & \quad f \leftarrow \text{not } f \\
\end{align*}\]

Figure 1: Logic program \( P \).

Figure 2: Resulting CAF \( \text{CF} = (A, R, \text{claim}) \).

\(^1\)A set \( S \) is stable iff it is conflict-free and attacks every argument in \( A \setminus S \).

\(^2\)A set \( S \) is admissible in an AF \( F \) iff it is conflict-free and attacks all attackers of \( S \).
Definition 1. An argumentation framework (AF) is a pair $F = (A, R)$ where $A \subseteq U$ is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. We say that $S \subseteq A$ attacks $b$ if $(a, b) \in R$ for some $a \in S$. Moreover, an argument $a \in A$ is defended (in $F$) by $S \subseteq A$ if each $b$ with $(b, a) \in R$ is attacked by $S$ in $F$.

Furthermore we denote by $S^+_F = \{ b \in A \mid (a, b) \in R \}$ the set of attacked arguments of $S$. If no ambiguity arises, we drop the subscript $F$. We call $S \cup S^+_F$ the range of $S$ in $F$.

Semantics for AFs are defined as functions $\sigma$ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for $\sigma$ the functions $cf$, $adm$, $stb$, $sem$ and $stg$ which stand for conflict-free, admissible, stable, semi-stable and stage extensions, respectively.

Definition 2. Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is conflict-free (in $F$), if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of sets being conflict-free in $F$. For a conflict-free set $S \in cf(F)$, we say $S \in adm(F)$, if each $a \in S$ is defended by $S$ in $F$; $S \in stb(F)$, if each $a \in A \setminus S$ is attacked by $S$ in $F$; $S \in sem(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S \cup S^+_F \subseteq T \cup T^+_F$; $S \in stg(F)$, if there is no $T \in cf(F)$, with $S \cup S^+_F \subseteq T \cup T^+_F$.

We recall that for each AF $F$, $stb(F) \subseteq stg(F) \subseteq cf(F)$ and $stb(F) \subseteq sem(F) \subseteq adm(F)$; also $stb(F) = sem(F) = stg(F)$ in case $stb(F) \neq \emptyset$. Moreover, semantics $\sigma \in \{ stg, stb, sem \}$ deliver incomparable sets, i.e. for all $S, T \in \sigma(F)$, $S \subseteq T$ implies $S = T$; the property is also referred to as I-maximal.

Next we define claim-augmented argumentation frameworks according to [8].

Definition 3. A claim-augmented argumentation framework (CAF) is a triple $(A, R, claim)$ where $(A, R)$ is an AF and $claim : A \rightarrow C$ is a function which assigns a claim to each argument in $A$; $C$ is a set of possible claims. The claim-function is extended to sets in the following way: For a set $E \subseteq A$, $claim(E) = \{ claim(a) \mid a \in E \}$.

The CAF $CF$ is called well-formed if $\{ a \}_{A, R}^F = \{ b \}_{A, R}^F$ for all $a, b \in A$ such that $claim(a) = claim(b)$.

In [8], semantics of CAFs are defined based on the standard semantics of the underlying AF. The extensions are interpreted in terms of the claims of the arguments. We call this variant inherited semantics (i-semantics).

Definition 4. For a CAF $CF = (A, R, claim)$, for a semantics $\sigma$, we define $i$-semantics $\sigma_i(CF) = \{ claim(E) \mid E \in \sigma((A, R)) \}$. We call a set $E \in \sigma((A, R))$ with claim $E = S$ a $\sigma$-realization of $S$ in $CF$.

Basic relations between different semantics carry over from standard AFs, i.e. for any CAF $CF$, $stb_i(CF) \subseteq sem_i(CF) \subseteq adm_i(CF)$ and $stb_i(CF) \subseteq stg_i(CF) \subseteq cf_i(CF)$; moreover, if $stb_i(CF) \neq \emptyset$ then $stb_i(CF) = sem_i(CF) = stg_i(CF)$. However, the next example shows that we lose fundamental properties of semantics like I-maximality of stable, semi-stable and stage semantics.

Example 2. Let $CF = (A, R, claim)$ with $(A, R) = (\{ x_1, x_2, y \}, (\{ x_1, x_2 \}, (x_2, x_1), (x_2, y)))$ and $claim(x_1) = x, i \leq 2, claim(y) = y$. Then $stb_i(CF) = sem_i(CF) = stg_i(CF) = \{ \{ x \}, \{ x, y \} \}$. Note that $CF$ is not well-formed.

3 Range-based Semantics in CAFs

For standard argumentation frameworks, the range of a set $E$ of arguments is defined as the union of $E$ together with all arguments it attacks; hence a claim-centered variant of range-based semantics requires explicit concepts for the defeat of claims. In the current section, we will discuss defeat on claim-level and the range of a claim-set which both exhibit certain differences to its argument-based counter-parts. In Sections 3.1, 3.2 and 3.3, we will discuss claim-centered variants of stable, semi-stable and stage semantics, respectively.

We will introduce the range of a claim-set $S \subseteq claim(A)$ in a CAF $CF = (A, R, claim)$, that is, we will define, for any claim-set $S$, the set of all claims it defeats. Since each claim-set depends on a particular realization in the underlying AF $(A, R)$, we will first introduce claim-defeat on argument-level.

Definition 5. Let $CF = (A, R, claim)$, $E \subseteq A$ and $c \in claim(A)$. We say that $E$ defeats $c$ iff $E$ attacks every $a \in A$ with $claim(a) = c$. We define $dis_{CF}(E) = \{ c \in claim(A) \mid \forall x \in A, claim(x) = c \ \exists y \in E \ \text{s.t.} \ (y, x) \in R \}$. If no ambiguity arises, we drop the subscript $CF$.

Observe that $dis_{CF} : A \rightarrow claim(A)$ is monotone, i.e. if $E \subseteq E'$ then $dis_{CF}(E) \subseteq dis_{CF}(E')$ for any $E, E' \subseteq A$.

Next we will consider claim-defeat with respect to a claim-set $S$ independently of a particular realization. The general idea is to consider, for each realization $E$ of $S$, the set of defeated claims $dis_{CF}(E)$ as potential candidates to identify the range of $S$. Observe that, in contrast to the range of a set of arguments, the range of a set of claims $S$ is in general not unique since $S$ can possess multiple realizations; moreover, we restrict ourselves to $\sigma$-realizations of $S$ for some semantics $\sigma$ in order to exclude for example conflicting realizations.
Definition 6. Let $CF = (A, R, claim)$, $S \subseteq claim(A)$ and consider a semantics $\sigma$. Then $D_{\sigma, CF}(S) = \{ dis_{CF}(E) \mid E \in \sigma((A, R)), claim(E) = S \}$; moreover, $R_{\sigma, CF}(S) = \{ S \cup S' \mid S' \in D_{\sigma, CF}(S) \}$ represents every possible range of $S$ with respect to $\sigma$. If no ambiguity arises, we drop the subscript $CF$.

Observe that for every claim-set $S$ and two semantics $\sigma, \sigma'$ with $\sigma((A, R)) \subseteq \sigma'((A, R))$ it holds that $D_{\sigma, CF}(S) \subseteq D_{\sigma', CF}(S)$. Indeed, if $dis_{CF}(E) \in D_{\sigma, CF}(S)$ for some $E \subseteq A$, then $E \in \sigma((A, R)) \subseteq \sigma'((A, R))$, and thus $dis_{CF}(E) \in D_{\sigma', CF}(S)$. Moreover notice that, in general, $|R_{\sigma, CF}(S)| \geq 1$, that is, the range of a claim-set potentially consists of multiple alternatives. However, for well-formed CAFs $CF$, it holds that for every two sets $E, E' \subseteq A$ with $claim(E) = claim(E')$, $E^+ = E'^+$, thus $dis_{CF}(E) = dis_{CF}(E')$. It follows that the range of a claim-set $S$ is unique if the CAF is well-formed. This also implies that, for well-formed CAFs, the range is independent of the particular realization with respect to a semantics $\sigma$.

Lemma 1. Let $CF = (A, R, claim)$ be well-formed and let $S \subseteq claim(A)$. Then $|R_{\sigma, CF}(S)| = 1$.

3.1 Stable Semantics

We will introduce two variants of stable semantics based on maximization on claim-level. The first variant requires the underlying realization of a claim-set $S$ to be conflict-free, while the second variant requires admissibility. We clarify the relation between both variants as well as the relation to $i$-stable semantics and compare them also with regard to I-maximality of their extensions.

Definition 7. Let $CF = (A, R, claim)$ and $S \subseteq claim(A)$. $S$ is a cf-cl-stable claim-set, in symbols $S \in cl-stb_{cf}(CF)$, iff there exists $S' \in D_{\sigma, CF}(S)$ such that $S \cup S' = claim(A)$.

The proposed variant of claim-based stable semantics relaxes the definition of inherited stable semantics in the way that it is no longer required that a stab-realization of a cf-cl-stable claim-set exists. Consider the CAF $CF = (A, R, claim)$ from Figure 3 with claim($a_1$) = claim($a_2$) = $a$, claim($b$) = $b$. Here, stb$_h(CF) = \emptyset$ but cl-stb$_{cf}(CF) = \{ \{a\} \}$: The cf-realization $E = \{a_1\}$ satisfies $dis_{CF}(E) = \{b\}$ and therefore, $claim(E) \cup dis_{CF}(E) = claim(A)$. Observe that $CF$ is not well-formed. Furthermore notice that the cf-cl-stable claim-set $\{a\}$ is in fact not adm-realizable in $(A, R)$. Thus in contrast to standard AF semantics where each stable extension satisfies admissibility, a cl-stb$_{cf}$-realization in the underlying AF is not necessarily admissible. Thus we consider also a stronger notion of stable semantics which requires adm-realizability in the underlying AF.

Definition 8. Let $CF = (A, R, claim)$ and $S \subseteq claim(A)$. $S$ is an adm-cl-stable set, in symbols $S \in cl-stb_{adm}(CF)$, if there exists $S' \in D_{adm, CF}(S)$ such that $S \cup S' = claim(A)$.

Proposition 1. For any CF = (A, R, claim), stb$_h(CF) \subseteq cl-stb_{adm}(CF) \subseteq cl-stb_{cf}(CF)$.

Proof. Let $S \in stb_h(CF)$ and consider a stab-realization $E \subseteq A$. Observe that $E \in adm((A, R))$. Let $c \in claim(A) \setminus S$, then for all $x \in A$ with claim($x$) = $c$, $x \in A \setminus E$. Since $E$ is stable in $(A, R)$ we have that $E$ attacks each argument $x \in A \setminus E$, therefore $c \in dis_{CF}(E)$. Thus $dis_{CF}(E) = claim(A) \setminus S$ and therefore we have found a set $T = dis_{CF}(E) \in D_{adm, CF}(S)$ with $S \cup T = claim(A)$, i.e. $S \in cl-stb_{adm}(CF)$. Moreover, cl-stb$_{adm}(CF) \subseteq cl-stb_{cf}(CF)$ follows from the fact that each admissible set is also conflict-free.

In the CAF $CF = (A, R, claim)$ from Figure 3 we have cl-stb$_{adm}(CF) \neq cl-stb_{cf}(CF)$ since cl-stb$_{adm}(CF) = \emptyset$ but cl-stb$_{cf}(CF) = \{ \{a\} \}$. A small modification of the CAF $CF$ also shows that cl-stb$_{adm}(CF) \neq stb_h(CF)$: Let $CF_1 = (A, R \setminus \{(a_2, a_1)\}, claim)$, then cl-stb$_{adm}(CF_1) = \{ \{a\} \}$ (witnessed by the adm-realization $\{a_1\}$ in $(A, R)$) but $stb_h(CF_1) = \emptyset$. Observe that both $CF$ and $CF_1$ are not well-formed. We will show next that for well-formed CAFs, all considered variants of stable semantics are in fact equal.

Proposition 2. For any well-formed CAF CF = (A, R, claim), cl-stb$_{adm}(CF) = cl-stb_{cf}(CF) = stb_h(CF)$.

Proof. We will show that cl-stb$_{cf}(CF) \subseteq stb_h(CF)$, the other direction is due to Proposition 1.

Let $S \in cl-stb_{cf}(CF)$, then there is some set $S' \in D_{cf, CF}(S)$ such that $S \cup S' = claim(A)$ (recall that $|D_{cf, CF}(S)| = 1$ by Lemma 1). We consider a maximal cf-realization $E \subseteq A$ of $S$, that is, $E \in cf((A, R))$ with $E = claim(S)$ and for every set $E' \in cf((A, R))$ with $E' = claim(S)$, $E' \subseteq E$. We show that $E_R = A \setminus E$. Let
Figure 4: CAF $CF = (A, R, \text{claim})$ with $\text{claim}(b_1) = \text{claim}(b_2) = b$ and $\text{claim}(x) = x$ for $x \in A \setminus \{b_1, b_2\}$. 

$x \in A \setminus E$ and let $\text{claim}(x) = c$. If $c \notin S$, then $c \in S'$ by definition of cf-cl-stable semantics, thus $E$ attacks $x$. Consider now the case $c \in S$, i.e., there is an argument $y \in E$ such that $\text{claim}(y) = c$ and observe that $E \cup \{x\}$ is not conflict-free by maximality of $E$; thus either (a) $(x, y) \in R$ or there is $z \in E$ such that either (b) $(z, x) \in R$ or (c) $(x, z) \in R$. In case (a) then also $(y, x) \in R$ by well-formedness; in case (b) we are done; in case (c) we have $(y, z) \in R$ by well-formedness and therefore $E$ is not conflict-free, contradiction. 

Recall that i-stable claim-sets are not necessarily I-maximal (c.f. Example 2). As a consequence of Proposition 1 we deduce that cf-cl-stable claim-sets are not I-maximal for arbitrary CAFs. In [7] it has been shown that i-stable semantics yield I-maximal claim-sets for well-formed CAFs. By Proposition 2, we conclude that cl-stable claim-sets satisfy I-maximality if well-formedness is guaranteed.

**Proposition 3.** For any well-formed CAF $CF$, both cl-stb$_{cf}(CF)$ and cl-stb$_{adm}(CF)$ are I-maximal.

### 3.2 Semi-stable Semantics

We consider the following claim-based variant of semi-stable semantics which relaxes $adm$-cl-stable semantics by dropping the requirement that the range of a claim-set must consist of all claims in the framework. Instead, we consider claim-sets with maximal range.

**Definition 9.** Let $CF = (A, R, \text{claim})$, $S \subseteq \text{claim}(A)$ is a cl-semi-stable claim-set, in symbols $S \in \text{cl-sem}(CF)$, iff there exists $S' \in \mathcal{D}_{adm,CF}(S)$ such that there is no $T \in \mathcal{D}_{adm}(CF)$, $T' \in \mathcal{D}_{adm,CF}(T)$ with $S \cup S' \subset T \cup T'$.

As an example, consider the CAF $CF = (A, R, \text{claim})$ from Figure 4 with $\text{claim}(b_1) = \text{claim}(b_2) = b$ and $\text{claim}(x) = x$ for $x \in A \setminus \{b_1, b_2\}$. First notice that $\text{stb}_3(CF) = \text{cl-stb}_3(CF) = \text{cl-stb}_{adm}(CF) = \emptyset$ since $b_1$ and $c$ are mutually attacking, thus either $a$ or $d$ are not attacked. Admissible claim-sets are $S_1 = \{b\}$, $S_2 = \{c\}$ and $S_3 = \{b, c\}$: then $\mathcal{D}_{adm}(S_1) = \{\emptyset, \{a, c\}\}$ and $\mathcal{D}_{adm}(S_2) = \mathcal{D}_{adm}(S_3) = \{\{d\}\}$. Observe that $S_2$ is not cl-semi-stable, since $S_2 \cup \{d\} \subseteq S_3 \cup \{d\}$; moreover, $S_1$ is cl-semi-stable, since $S_1 \cup \{a, c\} = \{a, b, c\} \not\subseteq S_3 \cup \{d\}$ $S_3$ is cl-semi-stable, since $S_3 \cup \{d\} = \{b, c, d\} \not\subseteq S_1 \cup \{a, c\}$. It follows that cl-semi-stable claim-sets are not necessarily I-maximal. Notice that $CF$ is not well-formed.

Since for well-formed CAFs, the range is unique and moreover, the function $\text{dis}_{CF}$ is monotone, we conclude that cl-semi-stable semantics yields I-maximal claim-sets if well-formedness is satisfied.

**Proposition 4.** For any well-formed CAF $CF$, $\text{cl-sem}(CF)$ is I-maximal.

This observation accords with the analysis of i-semi-stable claim-sets: I-maximality of i-semi-stable claim-sets is not guaranteed in the general case but for well-formed CAFs, as we show next.

**Proposition 5.** For any well-formed CAF $CF$, $\text{sem}_i(CF)$ is I-maximal.

Proof. Towards a contradiction, assume that there are two semi-stable claim-sets $S, S' \in \text{sem}_i(CF)$ such that $S \subseteq S'$. We consider $\text{sem}$-realizations $E, E'$ for $S, S'$ respectively and recall that semi-stable extensions are I-maximal on argument level, i.e. there is $E \Delta E' \neq \emptyset$. Observe that $E^+ \subseteq E'^+$ holds by well-formedness: Let $x \in E^+$, then there is $y \in E$ such that $(y, x) \in R$. By assumption $S \subseteq S'$, there exists $z \in E'$ such that $\text{claim}(y) = \text{claim}(z)$, thus $(z, x) \in R$ by well-formedness. It follows that every argument $x \in E \setminus E'$ is defended by $E'$ and thus $E' \cup \{x\} \cup (E' \cup \{x\})^+ \supseteq E' \cup E'^+$, contradiction to $E'$ being semi-stable.

However, a closer comparison of cl-semi-stable and i-semi-stable semantics reveals the inherent difference between maximization on claim- vs. argument-level. As already discussed in the introduction, the well-formed CAF $CF$ from Example 1 yields $\text{sem}_i(CF) = \{\{a\}, \{d, b\}\}$, while $\text{cl-sem}(CF) = \{\{d, b\}\}$, thus $\text{sem}_i(CF) \not\subseteq \text{cl-sem}(CF)$. The following example extends Example 1 in order to show $\text{cl-sem}(CF) \not\subseteq \text{sem}_i(CF)$.

**Example 3.** We extend the CAF $CF = (A, R, \text{claim})$ from Example 1: Let $CF = (A \cup \{b, e\}, R', \text{claim}')$ with $R' = R \cup \{(e, c), (e, b), (A_3, e), (A_3, b), (b, A_3)\}$ and $\text{claim}(x) = x$ for $x \in \{b, e\}$. Then $\{a\}$ is the only i-semi-stable claim-set. For cl-semi-stable claim-sets, consider $\text{adm}(CF) = \{\{d\}, \{b, d\}, \{a\}\}$; inspecting the range yields $\{d, a\}$, $\{b, d, a, c\}$ and $\{a, c, d\}$ and thus $\text{cl-sem}(CF) = \{\{d, b\}\}$. Observe that $CF$ is indeed well-formed.
and, furthermore, (Lemma 1. However, for well-formed CAFs, I-maximality is guaranteed for cl-stage semantics. The following proposition is an immediate consequence of from Lemma 1.

**Proposition 6.** For any well-formed CAF $\text{CF}$, cl-stage ($\text{CF}$) is I-maximal.

We will show that also for i-stage semantics, I-maximality is satisfied if the CAF is well-formed.

**Proposition 7.** For any well-formed CAF $\text{CF}$, stg$_i$($\text{CF}$) is I-maximal.

**Proof.** Towards a contradiction, assume that there are $S_1, S_2 \in$ stg$_i$($\text{CF}$) such that $S_1 \subset S_2$. Consider stg-realizations $E_1, E_2$ of $S_1$ and $S_2$. So $E_1 \cup E_2^+, E_2 \cup E_2^+$ are incomparable and both subset-maximal. By well-formedness, $E_1^+ \subset E_2^+$. Indeed, let $x \in A$ be attacked by $E_1$, i.e. there is $a \in E_1$ such that $(a, x) \in R$. Since $\text{claim}(E_1) \subset \text{claim}(E_2)$, there is $b \in E_2$ such that $\text{claim}(b) = \text{claim}(a)$. By definition of well-formedness, $(b, x) \in R$. Since $E_1^+ \subset E_2 \cup E_2^+$, it must be the case that $E_1 \not\subset E_2 \cup E_2^+$, i.e. there exists $a \in E_1$ such that $a \notin E_2$ and $a \notin E_2^+$. Let $E = E_2 \cup \{a\}$, then (i) $E$ is conflict-free since $a \notin E_2^+$ and $a$ does not attack $E_2$ (assume otherwise, then there is some $b \in E_2$ such that $b \in E_1^+$, but then also $b \in E_2^+$ since $E_1^+ \subset E_2^+$, contradiction) and, furthermore, $(a, a) \notin R$ since $a \in E_1$; and (ii) $E_2^+ \not\subset E^+$ by definition of $E$ (actually, $E^+ = E_2^+$ since $\text{claim}(a) \in \text{claim}(E_2)$). Therefore there is a conflict-free set $E \subseteq A$ such that $E \cup E^+ \supset E_2 \cup E_2^+$, contradiction to the subset-maximality of $E_2 \cup E_2^+$.  

The following examples show that even for well-formed CAFs, i-stage and cl-stage semantics potentially yield different claim-sets.

**Example 4.** Let $\text{CF}_1 = (A, R, \text{claim})$ with $(A, R)$ given in Figure 5a, claim($c_1$) = claim($c_2$) = $c$, claim(a) = $a$ and claim(b) = $b$. Then $\{b\}$ is the only i-stage claim-set. Observe that $\text{CF}_1$ is indeed well-formed. Consider now the cl-stage claim-sets. The conflict-free sets are $\{a\}$ and $\{b\}$. Inspecting the range yields $\{a, b\}$ in both cases and therefore cl-stage($\text{CF}_1$) = $\{\{a\}, \{b\}\}$, i.e. cl-stage($\text{CF}_1$) $\not\subset$ stg$_i$($\text{CF}_1$).

**Example 5.** Let $\text{CF}_2 = (A, R, \text{claim})$ as in Figure 5b, claim($d_1$) = $d$, claim($c_1$) = $c$, claim(a) = $a$, claim(b) = $b$. Then stg$_i$($\text{CF}_2$) = $\{\{a, d\}, \{b\}\}$ but cl-stage($\text{CF}_1$) = $\{\{a\}, \{d\}\}$, that is, stg$_i$($\text{CF}_2$) $\not\subset$ cl-stage($\text{CF}_2$).

### 3.4 Relations between Semantics

We start with a general observation which clarifies the relation between inherited and claim-level semantics for CAFs where every argument poses a unique claim. In that case, both variants coincide with the standard AF semantics interpreted in terms of the claims since the claims in the CAF can be identified with the arguments in the underlying AF. It follows that negative results concerning the relations between the semantics carry over from standard AFs, i.e. counter-examples showing that two AF semantics $\sigma$, $\tau$ are not in a subset-relation can be adapted to CAFs.

**Proposition 8.** Let $\text{CF} = (A, R, \text{claim})$, let $\sigma$, $\tau$ be semantics. If there exists an AF $F$ such that $\sigma(F) \not\subset \tau(F)$ then there exists a (well-formed) CAF $\text{CF}$ such that $\alpha(\text{CF}) \not\subset \beta(\text{CF})$ for $\alpha \in \{\text{cl-$\sigma$,$\alpha_c$}, \beta \in \{\text{cl-$\tau$,$\tau_c$}\}$. 

![Figure 5: Examples of CAFs $\text{CF}_1$, $\text{CF}_2$ with cl-stg($\text{CF}_1$) $\not\subset$ stg($\text{CF}_1$) and stg$_i$($\text{CF}_2$) $\not\subset$ cl-stg($\text{CF}_2$).]
Example 3 shows that well-formed CAFs can be faithfully translated (with respect to standard argumentation semantics) to SETAFs, i.e. AFs which allow for collective attacks of arguments; of particular interest is the behavior of the findings underline the inherent difference of argument-based vs. claim-based maximization of the range: While \(-cl\)-stable semantics in fact corresponds to \(L\)-stable model semantics. We furthermore stud-

-cl-stable semantics correspond to stable semantics on argument-level for well-formed CAFs, this is not the case for semi-stable and stage semantics; we have shown that both i-semi-stable and cl-semi-stable semantics as well as i-stage and cl-stage semantics are incomparable, even for well-formed CAFs.

Figure 6: Relations between semantics. An arrow from \(\sigma\) to \(\tau\) indicates that \(\sigma(CF) \subseteq \tau(CF)\) for each CAF \(CF\).

**Proposition 9.** The relations between the semantics depicted in Figure 6 hold.

\[ \text{Proof.}\]

The relations between inherited semantics have been already discussed in Section 2; moreover, \(\text{sth}_k(CF) \subseteq \text{cl-stb}_{\text{adm}}(CF) \subseteq \text{cl-stb}_{\text{cf}}(CF)\) for arbitrary CAFs by Proposition 1 and \(\text{sth}_k(CF) = \text{cl-stb}_{\text{adm}}(CF) = \text{cl-stb}_{\text{cf}}(CF)\) for each well-formed CAF \(CF\) by Proposition 2. Moreover, for any CAF \(CF\), for every \(S \in \text{cl-stb}_{\text{adm}}(CF)\) exists \(S' \in \text{adm}_{\text{cf}}(S)\) such that \(S \cup S' = A\) and thus \(S \in \text{cl-sem}(CF)\); furthermore, since each \(S \in \text{cl-sem}(CF)\) is i-admissible by definition, it follows that \(\text{cl-stb}_{\text{adm}}(CF) \subseteq \text{cl-sem}(CF) \subseteq \text{adm}_{\text{cf}}(CF)\). A similar reasoning applies for the \(cf\)-based counter-parts, i.e. for every \(S \in \text{cl-stb}_{\text{cf}}(CF)\) exists \(S' \in \text{adm}_{\text{cf}}(S)\) such that \(S \cup S' = A\) and thus \(S \in \text{cl-stg}(CF)\); moreover, every \(S \in \text{cl-stg}(CF)\) is conflict-free, thus \(\text{cl-stb}_{\text{cf}}(CF) \subseteq \text{cl-stg}(CF) \subseteq \text{cf}_c(CF)\). We present counter-examples for the remaining cases: By Corollary 8, there is a well-formed CAF \(CF\) such that \(\alpha(CF) \nsubseteq \beta(CF)\) for (a) \(\alpha = \text{cf}_c, \beta = \{\text{adm}_c, \text{cl-sem}, \text{sem}_c, \text{cl-stg}, \text{stg}_c, \text{cl-stb}_{\text{cf}}, \text{cl-stb}_{\text{adm}}, \text{sth}_k\}\); (b) \(\alpha = \text{adm}_c, \beta = \{\text{cl-sem}, \text{sem}_c, \text{cl-stg}, \text{stg}_c, \text{cl-stb}_{\text{cf}}, \text{cl-stb}_{\text{adm}}, \text{sth}_k\}\); (c) \(\alpha = \{\text{cl-sem}, \text{sem}_c\}, \beta \in \{\text{cl-stg}, \text{stg}_c, \text{cl-stb}_{\text{cf}}, \text{cl-stb}_{\text{adm}}, \text{sth}_k\}\) and (d) \(\alpha \in \{\text{cl-stg}, \text{stg}_c\}, \beta \in \{\text{adm}_c, \text{cl-sem}, \text{sem}_c, \text{cl-stb}_{\text{cf}}, \text{cl-stb}_{\text{adm}}, \text{sth}_k\}\). Example 3 shows that \(\text{cl-sem}(CF) \neq \text{sem}_{\text{cf}}(CF)\) where \(CF\) is well-formed; moreover, \(\text{cl-stg}(CF) \neq \text{stg}_c(CF)\) using (well-formed) CAFs from Example 4 and Example 5. Counter-examples for general CAFs and stable semantics have been discussed in Section 3.1.

Recall that for inherited semantics, \(\text{sth}_k(CF) = \text{sem}_c(CF) = \text{stg}_c(CF)\) in case \(\text{sth}_k(CF) \neq \emptyset\). One can show that this does not extend to \(cl\)-stable semantics. However, we can obtain the following weaker version.

**Lemma 2.** For any CAF \(CF = (A, R, \text{claim})\), (a) \(\text{cl-stb}_{\text{cf}}(CF) \neq \emptyset\) implies \(\text{cl-stb}_{\text{cf}}(CF) = \text{cl-stg}(CF)\) and (b) \(\text{cl-stb}_{\text{adm}}(CF) \neq \emptyset\) implies \(\text{cl-stb}_{\text{adm}}(CF) = \text{cl-sem}(CF)\).

4 Discussion

In this work, we investigated range-based semantics for claim-augmented argumentation frameworks. We introduced inherited semi-stable and stage semantics in the spirit of \([8]\) which perform maximization on argument-level and developed claim-based alternatives which perform maximization on claim-level. In doing so, we were able to provide a variant of semi-stable semantics which mimics the behavior of L-stable model semantics of LPs; observe that cl-semi-stable semantics in fact corresponds to L-stable model semantics. We further studied two variants of claim-level stable semantics based on conflict-free respectively admissible semantics. Our findings underline the inherent difference of argument-based vs. claim-based maximization of the range: While \(cf\)-cl-stable semantics correspond to stable semantics on argument-level for well-formed CAFs, this is not the case for semi-stable and stage semantics; we have shown that both i-semi-stable and cl-semi-stable semantics as well as i-stage and cl-stage semantics are incomparable, even for well-formed CAFs.

For future work, we plan to extend our investigations to other semantics involving maximization, in particular to preferred and naive semantics. Moreover, we want to connect our findings with studies in \([7]\) where it has been shown that well-formed CAFs can be faithfully translated (with respect to standard argumentation semantics) to SETAFs, i.e. AFs which allow for collective attacks of arguments; of particular interest is the behavior of the variants of range-based semantics we have considered in this work. Another direction of future research is to extend our studies to further classes of CAFs, e.g. attacker-unitary CAFs as introduced in \([7]\).
References


