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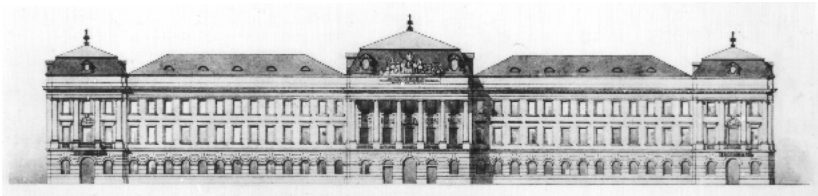

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Characteristics of Multiple Viewpoints in Abstract Argumentation under Complete Semantics

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Characteristics of Multiple Viewpoints in Abstract Argumentation under Complete Semantics

Thomas Linsbichler [1]

Abstract. This report provides a characterization of the signature of complete semantics in abstract argumentation. By that it solves a problem that was left open by recent work on the expressiveness of abstract argumentation semantics.

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1 Introduction

This is an addendum to [1] characterizing the signature of complete semantics. By that we solve what was mentioned to be among the open problems in abstract argumentation [2]. We assume the reader is familiar with the basic definitions and concepts of [1]. Section 2 will show a property of extension-sets under complete semantics which is stricter than the one shown in the original paper and Section 3 will show that this property is sufficient for realizability, giving rise to an exact characterization of the signature of complete semantics given in Section 4.

2 Properties of extension-sets under complete semantics

We recall and extend definitions from [1] concerning complete realizability:

Definition 1. Given an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ and $E \subseteq \mathit{Args}_{\mathbb{S}}$. We define the completion-sets $\mathbb{C}_{\mathbb{S}}(E)$ of E in $\mathbb{S}$ as the set of $\subseteq$-minimal sets $S \in \mathbb{S}$ with $E \subseteq S$. If $|\mathbb{C}_{\mathbb{S}}(E)| = 1$ we denote this single set as $\mathcal{C}_{\mathbb{S}}(E)$.

Definition 2. Let $\mathbb{S} \subseteq 2^{\mathfrak{A}}$. If for a set $\mathbb{T} \subseteq \mathbb{S}$ and a set $P \subseteq (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}})$ it holds that $(a,b) \in P$ for each $a, b \in \mathit{Args}_{\mathbb{T}}$, but $\bigcup \mathbb{T} \notin \mathbb{S}$, then $\bigcup \mathbb{T}$ is a completion-candidate of $\mathbb{S}$ wrt. P. The set of all completion-candidates of $\mathbb{S}$ wrt. P is denoted by $cc_{\mathbb{S}}(P)$. $\mathbb{S}$ is called *com-closed* wrt. P if each completion-candidate t of $\mathbb{S}$ wrt. P has a unique completion-set in $\mathbb{S}$, i.e. $|\mathbb{C}_{\mathbb{S}}(T)| = 1$. Finally, letting T be a completion-candidate of $\mathbb{S}$ wrt. P, we define $X^T_{\mathbb{S},P} = \{x \in \mathit{Args}_{\mathbb{S}} \mid \exists u \in \mathcal{C}_{\mathbb{S}}(T) : (u,x) \notin P, \forall t \in T : (t,x) \in P\}$.

Example 1. Let $\mathbb{S} = \{\emptyset, \{a\}, \{b\}, \{a,b,c\}\}$ and observe that $\{a,b\}$ is a completion-candidate of $\mathbb{S}$ wrt. $\mathit{Pairs}_{\mathbb{S}}$, i.e. $\{a,b\} \in cc_{\mathbb{S}}(\mathit{Pairs}_{\mathbb{S}})$. Moreover, $\{a,b\}$ has a unique completion-set in $\mathbb{S}$, namely $\mathcal{C}_{\mathbb{S}}(\{a,b\}) = \{a,b,c\}$. Since $\{a,b\}$ is the only completion-candidate of $\mathbb{S}$ wrt. $\mathit{Pairs}_{\mathbb{S}}$, $\mathbb{S}$ is com-closed wrt. $\mathit{Pairs}_{\mathbb{S}}$.

On the other hand consider $\mathbb{S}' = \mathbb{S} \cup \{\{a,b,d\}\}$. Still $cc_{\mathbb{S}'}(\mathit{Pairs}_{\mathbb{S}'}) = \{\{a,b\}\}$, but now $\{a,b\}$ has two completion-sets in $\mathbb{S}'$, that is $\mathbb{C}_{\mathbb{S}'}(\{a,b\}) = \{\{a,b,c\},\{a,b,d\}\}$. Hence $\mathbb{S}'$ is not com-closed wrt. $\mathit{Pairs}_{\mathbb{S}'}$.

Definition 3. An extension-set $\mathbb{S}$ is com-fortable if it holds that $\bigcap \mathbb{S} \in \mathbb{S}$ and there exists a removal-set $Z \subseteq (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}}$ such that

- $\mathbb{S}$ is com-closed wrt. $\mathit{Pairs}_{\mathbb{S}} \cup Z$,
- for each $T \in cc_{\mathbb{S}}(\mathit{Pairs}_{\mathbb{S}} \cup Z)$ it holds that $U \subseteq \mathit{grd}((U \cup X^T_{\mathbb{S},P}, ((U \cup X^T_{\mathbb{S},P}) \times (U \cup X^T_{\mathbb{S},P})) \setminus P))$ with $U = \mathcal{C}_{\mathbb{S}}(T) \setminus T$ and $P = \mathit{Pairs}_{\mathbb{S}} \cup Z$, and
- for each $S \in \mathbb{S}$ and $a \in S$ it holds that if, for some $b \in \mathit{Args}_{\mathbb{S}}$, $(a,b) \in Z$ and $(b,a) \notin Z$ then there is an $s \in S$ with $(s,b) \notin \mathit{Pairs}_{\mathbb{S}} \cup Z$.

Note that an extension-set $\mathbb{S}$ being com-fortable implies $\mathbb{S} \neq \emptyset$, since otherwise $\bigcap \mathbb{S} = \emptyset \notin \emptyset$.

Example 2. Consider the extension-set $\mathbb{S}$ from Example 1. It can easily be verified that $\mathbb{S}$ is com-fortable. In particular, $\bigcap \mathbb{S} = \emptyset \in \mathbb{S}$ and the empty removal-set $Z = \emptyset$ fulfills all conditions. On the other hand we immediately see that $\mathbb{S}'$ from Example 1 is not com-fortable. As it is not com-closed wrt. $\mathit{Pairs}_{\mathbb{S}'}$ there cannot be a set Z such that it is com-closed wrt. $\mathit{Pairs}_{\mathbb{S}'} \cup Z$.

Example 3. Let $\mathbb{S} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b, c\}, \{a, d, e\}, \{b, d, f\}, \{x, c\}, \{x, d\}\}$. It was discussed in Example 8 of [1] that, despite $\mathbb{S}$ is com-closed wrt. $\mathit{Pairs}_{\mathbb{S}}$, there exists no AF F having $\mathit{com}(F) = \mathbb{S}$. We will argue that $\mathbb{S}$ is also not com-fortable. First assume $Z = \emptyset$. We have $T = \{a, b\} \in \mathit{cc}_{\mathbb{S}}(\mathit{Pairs}_{\mathbb{S}})$ and get $U = \mathcal{C}_{\mathbb{S}}(T) \setminus T = \{c\}$ and $X^{T}_{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \{d\}$. However, $\mathit{grd}(\{c, d\}, \{(c, d), (d, c)\}) = \emptyset$, hence Z violates the second condition of Definition 3. Assuming $Z = \{(d, c)\}$ gives us the same T and U but now $X^{T}_{\mathbb{S},\mathit{Pairs}_{\mathbb{S}} \cup Z} = \emptyset$ and we get $\mathit{grd}(\{c\}, \emptyset) = \{c\}$, fulfilling the second condition. But now we have $\{x, d\} \in \mathbb{S}$ and $(d, c) \in Z$ and $(c, d) \notin Z$ but both $(x, c), (d, c) \in \mathit{Pairs}_{\mathbb{S}} \cup Z$, violating the third condition. Finally, choosing $Z = \{(c, d), (d, c)\}$ fulfills these conditions, but we get $\{x, c, d\} \in \mathit{cc}_{\mathbb{S}}(\mathit{Pairs}_{\mathbb{S}} \cup Z)$ as new completion-candidate of $\mathbb{S}$ wrt. $\mathit{Pairs}_{\mathbb{S}} \cup Z$ which has no completion-set in $\mathbb{S}$. Hence $\mathbb{S}$ is not com-closed wrt. $\mathit{Pairs}_{\mathbb{S}} \cup Z$. It can be verified that there is also no other choice of Z fulfilling the conditions of Definition 3. Therefore $\mathbb{S}$ is not com-fortable.

On the other hand the extension-set $\mathbb{S}' = \mathbb{S} \setminus \{\{x, c\}, \{x, d\}\}$ is com-fortable as the removal-set $\{(d, c)\}$ fulfills all properties of Definition 3.

Proposition 1. *For every AF $F \in AF_{\mathfrak{A}}$ it holds that $\mathit{com}(F)$ is com-fortable.*

Proof. Let $F \in AF_{\mathfrak{A}}$ be an arbitrary AF. It is well-known that $\bigcap \mathit{com}(F)$, that is the grounded extension of F, is also a member of $\mathit{com}(F)$. Hence we have to show that there exists a removal-set $Z \subseteq (\mathit{Args}_{\mathit{com}(F)} \times \mathit{Args}_{\mathit{com}(F)}) \setminus \mathit{Pairs}_{\mathit{com}(F)}$ fulfilling the conditions given in Definition 3. Let $Z = ((\mathit{Args}_{\mathit{com}(F)} \times \mathit{Args}_{\mathit{com}(F)}) \setminus \mathit{Pairs}_{\mathit{com}(F)}) \setminus R_F$. In other words, $(a, b) \in Z$ iff $a, b \in \mathit{Args}_{\mathit{com}(F)}$, $(a, b) \notin \mathit{Pairs}_{\mathit{com}(F)}$ and $(a, b) \notin R_F$ (implicit conflicts among $\mathit{Args}_{\mathbb{S}}$ according to hidden power paper). Let $P = \mathit{Pairs}_{\mathit{com}(F)} \cup Z$ which is just the inverse of R_F among arguments $\mathit{Args}_{\mathit{com}(F)}$. (1) Let $T \in \mathit{cc}_{\mathit{com}(F)}(P)$. In other words, T is the union of complete extensions $E_1, \ldots, E_n$ ($n \geq 2$) of F which is conflict-free in F but not a complete extension of F itself. Note that T, being the union of admissible sets, is also admissible in F (cf. Lemma 1 in [1]). Now iteratively adding the defended arguments to T gives a unique complete extension F, hence T has a unique completion-set in $\mathit{com}(F)$, showing that $\mathit{com}(F)$ is com-closed wrt. $\mathit{Pairs}_{\mathit{com}(F)} \cup Z$. (2) Now let E be the unique $\subseteq$-minimal complete extension of F extending T (i.e. $E = \mathcal{C}_{\mathit{com}(F)}(T)$) and let $U = E \setminus T$. As $T \notin \mathit{com}(F)$, T must defend at least one argument of U, which, together with T, defends another argument, and so on. In other words $U = \mathit{grd}(F|_{A_F \setminus T^+})$. Let $F' = (U \cup X^{T}_{\mathit{com}(F),P}, ((U \cup X^{T}_{\mathit{com}(F),P}) \times (U \cup X^{T}_{\mathit{com}(F),P})) \setminus P)$ and note that F' coincides with $F|_{A_F \setminus T^+}$ among arguments in $U \cup X^{T}_{\mathit{com}(F),P}$. Therefore it holds that if an argument $u \in U$ is attacked in F' then it is attacked in $F|_{A_F \setminus T^+}$ and if an argument $u \in U$ attacks an argument of $U \cup X^{T}_{\mathit{com}(F),P}$ in $F|_{A_F \setminus T^+}$ then it also attacks this argument in F'. Hence $\mathit{grd}(F') \supseteq \mathit{grd}(F|_{A_F \setminus T^+})$. Therefore $U \subseteq \mathit{grd}(F')$, which was to show. (3) We have to show that for each $E \in \mathit{com}(F)$ and each $a \in E$, it holds that if for some $b \in \mathit{Args}_{\mathit{com}(F)}$, $(a, b) \in Z$ and $(b, a) \notin Z$, then there is some $c \in E$ with $(c, b) \notin P$. Let $E \in \mathit{com}(F)$,$a \in E$ and assume there is an argument $b \in \mathit{Args}_{\mathit{com}(F)}$, $(a, b) \in Z$

and $(b, a) \notin Z$. By the definition of Z this means that b attacks a but is not attacked by a in F. Since E must be admissible it has to attack b in F, which means that there is some $c \in E$ with $(c, b) \in R_F$, i.e. $(c, b) \notin P$. □

3 Realizability

Definition 4. Given a com-fortable (with removal-set Z) extension-set $\mathbb{S}$ and an argument $a \in \mathit{Args}_{\mathbb{S}}$, let $P = \mathit{Pairs}_{\mathbb{S}} \cup Z$. We define the completion-formula $\mathcal{C}_a^{\mathbb{S},P}$ of argument a as $\top$ if $a \in \bigcap \mathbb{S}$ and

$$\bigvee_{S \in cc_{\mathbb{S}}(P) \text{ s.t. } a \in (\mathcal{C}_{\mathbb{S}}(S) \setminus S)} \bigwedge S.$$

otherwise. $\mathcal{C}_a^{\mathbb{S},P}$ converted to CNF is denoted by $\mathcal{CC}_a^{\mathbb{S},P}$.

The extended defense-formula $\mathcal{ECD}_a^{\mathbb{S},P}$ of a is $\mathcal{D}_a^{\mathbb{S}} \vee \mathcal{C}_a^{\mathbb{S},P}$ in CNF.

Example 4. Let $\mathbb{S} = \{\emptyset, \{a\}, \{b\}, \{a, c, d\}, \{a, b, c, d\}\}$. It can be verified that $\mathbb{S}$ is com-fortable with the empty removal set. Moreover observe that we have a single completion-candidate $cc_{\mathbb{S}}(\mathit{Pairs}_{\mathbb{S}}) = \{\{a, b\}\}$ which has completion-set $\mathcal{C}_{\mathbb{S}}(\{a, b\}) = \{a, b, c, d\}$. We get $\mathcal{CC}_a^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \mathcal{CC}_b^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \bot = \{\emptyset\}$ and $\mathcal{CC}_c^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \mathcal{CC}_d^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = a \wedge b = \{\{a\}, \{b\}\}$. Moreover, we have $\mathcal{ECD}_a^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \mathcal{ECD}_b^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = \top = \emptyset$ and $\mathcal{ECD}_c^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = a \wedge (b \vee d) = \{\{a\}, \{b, d\}\}$, $\mathcal{ECD}_d^{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}} = a \wedge (b \vee c) = \{\{a\}, \{b, c\}\}$.

Definition 5. Given a com-fortable (with removal-set Z) extension-set $\mathbb{S}$, let $P = \mathit{Pairs}_{\mathbb{S}} \cup Z$. We define the canonical completion-argumentation-framework as

$$F_{\mathbb{S},P}^{com} = (\mathit{Args}_{\mathbb{S}} \cup D_{\mathbb{S},P} \cup C_{\mathbb{S},P}, R_{\mathbb{S},P}^{cf} \cup R_{\mathbb{S},P}^{def} \cup R_{\mathbb{S},P}^{com})$$

where

$$\begin{aligned}
D_{\mathbb{S},P} &= \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{\alpha_{a,\gamma} \mid \gamma \in \mathcal{ECD}_a^{\mathbb{S},P}\}, \\
C_{\mathbb{S},P} &= \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{\beta_{a,\gamma} \mid \gamma \in \mathcal{CC}_a^{\mathbb{S},P}\}, \\
R_{\mathbb{S},P}^{cf} &= (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus P, \\
R_{\mathbb{S},P}^{def} &= \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{(b, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, a) \mid \gamma \in \mathcal{ECD}_a^{\mathbb{S},P}, b \in \gamma\}, \\
R_{\mathbb{S},P}^{com} &= \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{(b, \beta_{a,\gamma}), (\beta_{a,\gamma}, \beta_{a,\gamma}), (\beta_{a,\gamma}, a), (a, \beta_{a,\gamma}) \mid \gamma \in \mathcal{CC}_a^{\mathbb{S},P}, b \in \gamma\}.
\end{aligned}$$

Example 5. The canonical completion-argumentation-framework of extension-set $\mathbb{S}$ from Example 4 with $\mathit{Pairs}_{\mathbb{S}}$ is depicted in Figure 1. It is easy to verify that $com(F_{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}}^{com}) = \mathbb{S}$. In particular, note that $\{a, b\} \notin \mathbb{S}$ is admissible in $F_{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}}^{com}$, but as it defends both c and d it is, as expected, not a complete extension of $F_{\mathbb{S},\mathit{Pairs}_{\mathbb{S}}}^{com}$.

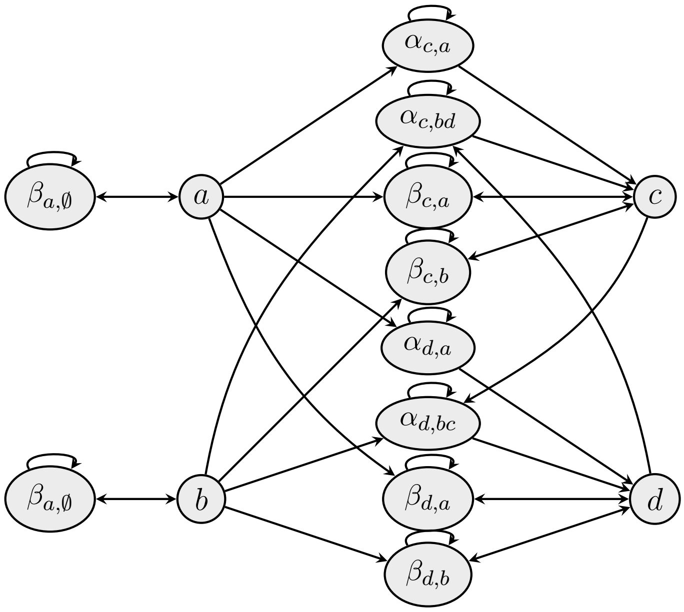

Figure 1: $F^{com}_{\mathbb{S},Pairs_{\mathbb{S}}}$ for $\mathbb{S}$ as given in Example 4

Lemma 1. *Given a com-fortable (with removal-set Z) extension-set $\mathbb{S}$, let $P = Pairs_{\mathbb{S}} \cup Z$. It holds that*

1. *If $S \in \mathbb{S}$ then S defends itself from $D_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$.*
2. *If S defends itself from $D_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$ then $\forall a \in S \exists T \in \mathbb{S}$ with $a \in T$ and $T \subseteq S$.*
3. *If $S \subseteq Args_{\mathbb{S}}$ defends $a \in Args_{\mathbb{S}} \setminus (S \cup \bigcap \mathbb{S})$ from $C_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$ then there is some $T \in cc_{\mathbb{S}}(P)$ with $T \subseteq S$ and $a \in \mathcal{C}_{\mathbb{S}}(T)$.*

Proof. (1) Let $S \in \mathbb{S}$ and $a \in S$. By definition of $\mathcal{D}^{\mathbb{S}}_a$ (cf. Definition 13 in [1]) it holds that $a \models \mathcal{D}^{\mathbb{S}}_a$, hence also $a \models \mathcal{CD}^{\mathbb{S}}_a$ and $a \models \mathcal{ECD}^{\mathbb{S}}_a$ meaning that for each argument $\alpha \in D_{\mathbb{S},P}$ is attacked by $\mathbb{S}$, hence S defends itself from $D_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$.

(2) Assume S defends itself from $D_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$ and let $a \in S$. Each attacker $\alpha \in D_{\mathbb{S},P}$ of a has to be attacked by S, meaning that $S \models \mathcal{ECD}^{\mathbb{S},P}_a$. By definition, this means that (a) $S \models \mathcal{D}^{\mathbb{S}}_a$ or (b) $S \models \mathcal{C}^{\mathbb{S},P}_a$. In case of (a) we immediately get that there is some $T \in \mathbb{S}$ with $T \subseteq S$ and $a \in T$ (see also Lemma 6 of [1]). In case of (b) we know that there is some $T \in cc_{\mathbb{S}}(P)$ wit $T \subseteq S$ and $a \in T$. As T must be the union of elements of $\mathbb{S}$ the result follows.

(3) Let $S \subseteq Args_{\mathbb{S}}$ and $a \in Args_{\mathbb{S}} \setminus (S \cup \bigcap \mathbb{S})$ and assume S defends a from $C_{\mathbb{S},P}$ in $F^{com}_{\mathbb{S},P}$. Each attacker $\beta \in C_{\mathbb{S},P}$ of a has to be attacked by S, meaning that $S \models \mathcal{CC}^{\mathbb{S},P}_a$, hence also $S \models \mathcal{C}^{\mathbb{S},P}_a$. Therefore there must be some $T \subseteq S$ with $T \in cc_{\mathbb{S}}(P)$ and $a \in \mathcal{C}_{\mathbb{S}}(T)$. □

Proposition 2. *Given a com-fortable (with removal-set Z) extension-set $\mathbb{S}$, it holds that $\mathbb{S} = F^{com}_{\mathbb{S},P}$ with $P = Pairs_{\mathbb{S}} \cup Z$.*

Proof. ($\subseteq$) Let $S = \bigcap \mathbb{S}$. For each $a \in S$ it holds that $\mathcal{C}_a^{\mathbb{S},P}$ is $\top$, hence both $\mathcal{CC}_a^{\mathbb{S},P}$ and $\mathcal{ECD}_a^{\mathbb{S},P}$ contain no clauses, therefore a is not attacked by arguments in $C_{\mathbb{S},P}$ and $D_{\mathbb{S},P}$. Moreover, $(s,a) \in \mathit{Pairs}_{\mathbb{S}}$ for each $s \in \mathit{Args}_{\mathbb{S}}$, hence a has no attackers in $F^{com}_{\mathbb{S},P}$. This means S is admissible in $F^{com}_{\mathbb{S},P}$. For each other argument $b \in \mathit{Args}_{\mathbb{S}} \setminus S$ it holds that $\mathcal{CC}_a^{\mathbb{S},P}$ has at least one (empty) clause γ, hence in order for S to defend b from $C_{\mathbb{S},P}$ there must be a completion-candidate $T \in \mathit{cc}_{\mathbb{S}}(P)$ with $T \subseteq S$ and $b \in \mathcal{C}_{\mathbb{S}}(T)$ (cf. Lemma 1.3). But this cannot be the case since $S \subseteq S'$ for each $S' \in \mathbb{S}$. Therefore b is not defended by S from $C_{\mathbb{S},P}$, hence S is complete in $F^{com}_{\mathbb{S},P}$.

Now let $S \in \mathbb{S}$ but $S \neq \bigcap \mathbb{S}$. By Lemma 1.1, S defends itself from arguments $C_{\mathbb{S},P}$. Moreover it defends itself form arguments $\mathit{Args}_{\mathbb{S}}$ by the third condition of the removal-set Z which makes $\mathbb{S}$ com-fortable and by construction of $F^{com}_{\mathbb{S},P}$. Finally it defends itself from arguments $D_{\mathbb{S},P}$ by construction of $F^{com}_{\mathbb{S},P}$. Therefore S is admissible in $F^{com}_{\mathbb{S},P}$. In order to show that S is complete assume, towards a contradiction, there is an $a \in \mathit{Args}_{\mathbb{S}} \setminus S$ which is defended by S. As $a \notin \bigcap \mathbb{S}$, there must be a $T \in \mathit{cc}_{\mathbb{S}}(P)$ with $T \subseteq S$ and $a \in \mathcal{C}_{\mathbb{S}}(T)$ by Lemma 1.3. But as $a \notin S$ this is already a contradiction to $\mathbb{S}$ being com-closed wrt. P, as on the one hand a is in the unique completion-set of T and on the other hand S extends T but does not contain a.

($\supseteq$) Let $S = \mathit{grd}(F^{com}_{\mathbb{S},P})$. By the definition of $F^{com}_{\mathbb{S},P}$ an argument a is unattacked iff $a \in \bigcap \mathbb{S}$. Hence $S \supseteq \bigcap \mathbb{S}$. Since we know from before that $\bigcap \mathbb{S} \in \mathit{com}(F^{com}_{\mathbb{S},P})$ it follows that $S = \bigcap \mathbb{S}$. Since $\mathbb{S}$ is assumed to be com-fortable, the result follows.

Now let $E \in \mathit{com}(F^{com}_{\mathbb{S},P})$ but $E \neq \mathit{grd}(F^{com}_{\mathbb{S},P})$. As E defends itself in $F^{com}_{\mathbb{S},P}$, in particular from arguments $D_{\mathbb{S},P}$, it follows by Lemma 1.2 that $\forall a \in E \exists S \in \mathbb{S}$ with $a \in S$ and $S \subseteq E$. If for one such $a \in E$ this $S \in \mathbb{S}$ with $a \in S$ is $S = E$ we are done. So assume that $E \notin \mathbb{S}$. Observe that as E is conflict-free in $F^{com}_{\mathbb{S},P}$ it must hold that $\forall a, b \in E : (a,b) \in P$. Hence, by $\mathbb{S}$ being com-closed wrt. P, $E = \bigcup_{S \in \mathbb{S}, S \subset E} S$ (remember that for each $a \in E$ there is such an $S \in \mathbb{S}$ with $S \subset E$) is a completion-candidate of $\mathbb{S}$ wrt. P, i.e. $E \in \mathit{cc}_{\mathbb{S}}(P)$. By $\mathbb{S}$ being com-closed wrt. P there is a unique completion-set $\mathcal{C}_{\mathbb{S}}(E)$ of E. Let $T = (\mathcal{C}_{\mathbb{S}}(E) \setminus E)$. Since E is complete it must hold that for each $t \in T$, E does not defend T. By the fact that $E \models \mathcal{ECD}_t^{\mathbb{S},P}$ and $E \models \mathcal{CC}_t^{\mathbb{S},P}$ it follows that E defends t from arguments $D_{\mathbb{S},P}$ and $C_{\mathbb{S},P}$. Hence E does not defend t from some argument $a \in \mathit{Args}_{\mathbb{S}}$, that is, by construction of $F^{com}_{\mathbb{S},P}$, $(a,t) \notin P$ and $(e,a) \in P$ for all $e \in E$. But this means $a \in X^T_{\mathbb{S},P}$. We end up with a contradiction to the second property of Z making $\mathbb{S}$ com-fortable. Hence $E \in \mathbb{S}$. $\square$

4 Signature

We can now give an exact characterization of the signature of the complete semantics.

Theorem 1. *The signature of the complete semantics is given by the following collection of extension-sets:*

$$\Sigma_{com} = \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \textit{ is com-fortable}\}.$$

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