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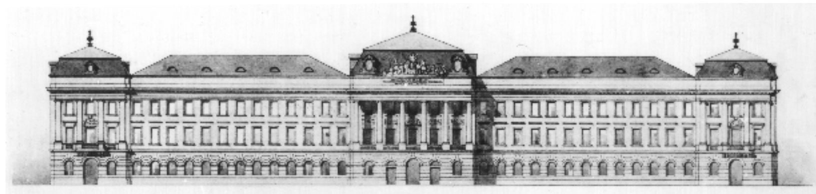

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Revisiting Support in Abstract Argumentation Systems

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Revisiting Support in Abstract Argumentation Systems

Sylwia Polberg [1] **Nir Oren** [2]

Abstract. The original argumentation framework by Dung has been extended in various ways, from adding values and preferences to more advanced relations. A number of such generalizations studied the problem of positive interactions between argument, which has led to the development of evidential, necessary and deductive supports and their respective frameworks. It is thus natural to compare those relations and analyze whether one can be translated into the other. Although a positive answer was given in the necessary and deductive cases, it was claimed that evidential support cannot be expressed by any other type and that it cannot be handled together with them in a single framework. In this paper we show that it is not the case and that there exists a natural translation between argumentation frameworks with necessities (AFNs) and evidential argumentation systems (EASs). We provide a full translation of AFNs into EASs and one the other way around for a subclass of EASs with binary attack. Finally, we introduce the concept of a minimal form of a framework and prove it preserved the behavior of standard semantics.

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1 Introduction

In recent years, argumentation has become an influential field in Artificial Intelligence [2]. One of its subareas is abstract argumentation, in which arguments are considered to be abstract entities and thus only the relations between them are taken into account when evaluating a given argumentation scenario. At the heart of abstract argumentation lies the Dung's framework (AF for short) [7], which focuses on the attack relation between arguments and the concept of defense derived from it. Although they are quite powerful, for many applications Dung's AFs appear too simple in order to conveniently model all aspects of an argumentation problem. This has led to the development of a wide range of their enrichments [4], which introduce concepts such as preferences and strengths into the basic model, as well as a positive interaction between arguments, commonly referred to as *support*.

The version of support introduced in [5] had several drawbacks, leading to the development of several other more specialized frameworks with support, with the most recognized being deductive [3], necessary [8], and evidential [11] supports. A natural question that arises concerns the relation between these different frameworks, as well as whether one can be transformed into another, and consequently if it is possible to express them all types of support within a single structure. [6] provided a positive answer to these questions with regards to deductive and necessary support, but claimed that evidential one cannot be represented via necessary (or deductive) type.

In this paper we show that there *is* an intuitive translation between evidential argumentation systems and argumentation frameworks with necessities. We compare the properties of the different support relations in the two frameworks, and discuss additional properties and similarities of both the frameworks with each other, and with Dung's original abstract argument system. Thus, our results show that it is possible to construct an unified environment capable of handling the available types of support and that due to its advanced structure, the evidential framework appears to be a good candidate for further research in this direction. In this paper we also address some of technical issues present in the framework's original formulation and introduce the concept of a minimal form of a framework.

The next section details Dung's argumentation framework. Sections 3 and 4 then describe two abstract frameworks containing both positive and negative argument interactions, namely evidential argument systems (EASs) and abstract frameworks with necessities (AFNs). Section 5 provides a comparison of these systems, and describes a translation between them. Section 6 concludes the paper with a discussion.

2 Dung's Frameworks

We begin with a brief overview of Dung's abstract argumentation framework [7].

Definition 2.1. *A* ***Dung's abstract argumentation framework*** *(AF for short) is a pair* (A, R)*, where* A *is a set of* ***arguments*** *and* $R \subseteq A \times A$ *represents an* ***attack*** *relation.*

AFs can be simply represented as directed graphs. We will now briefly recall the available semantics, for more details we refer the reader to [1].

Definition 2.2. *Let $AF = (A, R)$ be a Dung's framework. We say that an argument $a \in A$ is **defended** by a set $S \subseteq A$ in AF*[1]*, if for each $b \in A$ s.t. $(b, a) \in R$, there exists $c \in S$ s.t. $(c, b) \in R$. A set $S \subseteq A$ is:*

- ***conflict–free** in AF iff for each $a, b \in S$, $(a, b) \notin R$.*
- ***naive** in AF iff it is maximal w.r.t. set inclusion conflict–free.*
- ***admissible** in AF iff it is conflict–free and defends all of its members.*
- ***preferred** in AF iff it is maximal w.r.t. set inclusion admissible.*
- ***complete** in AF iff it is admissible and all arguments defended by S are in S.*
- ***stable** in AF iff it is conflict–free and for each $a \in A \setminus S$ there exists an argument $b \in S$ s.t. $(b, a) \in R$.*

We close the list with the grounded semantics, formally defined by the means of the characteristic function of AF:

Definition 2.3. *The **characteristic function** $F_{AF} : 2^A \to 2^A$ is defined as: $F_{AF}(S) = \{a \mid a \text{ is defended by } S \text{ in } AF\}$. The **grounded extension** is the least fixed point of F_{AF}.*

Lemma 2.4. *A conflict–free set S is admissible iff $S \subseteq F_{AF}(S)$. A conflict–free set S is complete iff $S = F_{AF}(S)$.*

Finally, we would like to recall several important lemmas and theorems from the original paper on AFs [7].

Lemma 2.5. ***Dung's Fundamental Lemma** Let S be an admissible extension, a and b two arguments defended by S. Then $S' = S \cup \{a\}$ is admissible and b is defended by S'.*

Theorem 2.6. *The following holds:*

1. *Every stable extension is a preferred extension, but not vice versa.*
2. *Every preferred extension is a complete extension, but not vice versa.*
3. *The grounded extension is the least w.r.t. set inclusion complete extension.*
4. *The complete extensions form a complete semilattice w.r.t. set inclusion.* [2]

[1] Defense is often substituted with acceptability: say that a is acceptable w.r.t. S if S is defends a.

[2] A partial order $(A, \leq)$ is a complete semilattice iff each nonempty subset of A has a greatest lower bound and each increasing sequence of S has a least upper bound.

3 Evidential Argumentation Systems

Unattacked arguments serve as the strongest source of defense within AFs. However, in many cases, the lack of an attack is insufficient to consider an argument acceptable. In areas such as legal reasoning and medicine, one is required to support a claim with facts or evidence so as to be convincing. For example, it does suffice to claim that a given person committed a crime in order to sentence them. Instead, the prosecution has to prove guilt, by means of evidence. Similarly, medical diagnoses have to be supported by facts such as symptoms or test results.

We can therefore distinguish between two types of arguments. The special arguments, often referred to as *prima facie* or evidence, act as an indisputable source of truth, while the standard ones need to be supported by them in order to be considered acceptable. In order to handle such reasoning, the evidential argumentation systems were created. Furthermore, since standard arguments must be supported, evidential frameworks address a critical drawback of abstract support in BAFs [5], namely that an argument could be present in an extension regardless of whether it is supported or not (see [10] for details). In this section we introduce the framework and describe some of its properties. In doing so, we provide corrections to the original formulation of these systems, as presented in [11, 12].

Definition 3.1. *An* ***evidential argumentation system*** *(EAS) is a tuple (A, R, E) where A is a set of* ***arguments****, $R \subseteq (2^A \setminus \emptyset) \times A$ is the* ***attack*** *relation, and $E \subseteq (2^A \setminus \emptyset) \times A$ is the* ***support*** *relation. We distinguish a special argument $\eta \in A$ s.t. $\nexists(X, y) \in R$ where $\eta \in X$; and $\nexists X$ where $(X, \eta) \in R$ or $(X, \eta) \in E$.*

The special argument η serves as a representation of the *prima facie* arguments and is referred to as evidence or environment. The difference between this and the definition from [12] is the removal of the restriction that there should be no argument x and set X s.t. XRx and XEx.

The core idea of evidential argument systems is that valid arguments (and attackers) need to trace back to the environment. It is captured with the notions of e–support and e–supported attack. The following formulations are the corrected versions of the ones available in [11, 12].

Definition 3.2. *An argument $a \in A$ has* ***evidential support*** *(e–support) from a set $S \subseteq A$ iff $a = \eta$ or there is a non-empty $S' \subseteq S$ such that $S'Ea$ and $\forall x \in S'$, x has evidential support from $S \setminus \{a\}$.*

An argument a has ***minimal e–support*** *from a set S if there is no set $S' \subset S$ such that a has e–support from S'.*

Remark. Note that by this definition η has evidential support from any set.

Example 3.3. In its original version [11], the definition of being e–supported by a set S required that either SEa where $S = \{\eta\}$ or that $\exists T \subset S$ s.t. TEa and $\forall x \in T$, x is e–supported by $S \setminus \{x\}$. Although the provided intuition on what it means to be supported by evidence were correct, this formulation did not fully reflect it. Let us assume a framework $(\{\eta, a, b, c\}, \emptyset, \{(\{a, b\}, c), (\{\eta\}, a), (\{\eta\}, b)\})$. Since both a and b, which are required for c, are supported by η, it should naturally be the case that the set $\{\eta, a, b\}$ e–supports c. If we proceed with the original definition, we see that we need to verify whether $\{\eta, a\}$ e–supports b and $\{\eta, b\}$ e–supports a. This then brings us to verifying whether η is e–supported by $\{a\}$ and $\{b\}$ respectively, which produces a result that c is not e–supported.

Definition 3.4. *A set $S \subseteq A$ carries out an* ***evidence supported attack*** *(e–supported attack) on a iff $(S', a) \in R$ where $S' \subseteq S$, and for all $s \in S'$, s has e–support from S.*

An e–supported attack by S on a is ***minimal*** *iff there is no $S' \subset S$ that carries out an e–supported attack on a.*

Given these notions, we can define semantics for EASs built around the notion of acceptability in a manner similar to those of Dung's. However, in the latter, only the attack relation was considered. For EASs, not only must arguments be defended from attacks, but they must also have sufficient support in order to be acceptable.

Definition 3.5. *An argument a is* ***acceptable*** *with respect to a set of arguments $S \subseteq A$ iff*

- *a is e–supported by S; and*
- *given a minimal e–supported attack by a set $T \subseteq A$ against a, it is the case that S carries out an e–supported attack against a member of T.*

Remark. [11] required that S attacked T on arguments T' s.t. $T \setminus T'$ is no longer an e–supported attack on a. However, it is easy to see that if S attacked T on T' in a way that $T \setminus T'$ is still an e–supported attack on a, then by Definition 3.4 T could not have been a minimal attack in the first place. Thus, the simpler definition above is sufficient.

Definition 3.6. *A set of arguments $S \subseteq A$ is:*

- ***self–supporting*** *iff all arguments in S are e–supported by S.*
- ***conflict–free*** *iff there is no $a \in S$ and $S' \subseteq S$ such that $S'Ra$.*
- ***admissible*** *iff it is conflict–free and all elements of S are acceptable w.r.t. S.*
- ***preferred*** *iff it is maximal w.r.t. set inclusion admissible.*
- ***complete*** *iff it is admissible and all arguments acceptable w.r.t S are in S.*
- ***stable*** *iff it is conflict–free, self–supporting, and for any argument a e–supported by A where $a \notin S$, S e–support attacks either a or every set of arguments minimally e–supporting a.*

Just like in the Dung setting, the grounded semantics is defined via the characteristic function. Also the properties of admissibility and completeness carry on to EASs.

Definition 3.7. *The* ***characteristic function*** *$F_{ES} : 2^A \to 2^A$ is defined as: $F_{ES}(S) = \{a \mid a \text{ is acceptable w.r.t. } S \text{ in } ES\}$. The* ***grounded extension*** *of a finitary framework $ES = (A, R, E)$ is the least fixed point of F_{ES}.*

Lemma 3.8. *A conflict–free set of arguments S is admissible iff $S \subseteq F_{ES}(S)$. A conflict–free set S of arguments is complete iff $S = F_{ES}(S)$.*

Proof. Proof for admissibility can be found in [10]. Completeness follows straightforwardly from it and from the definitions of the semantics and characteristic function. □

While there are many analogies between EASs and AFs, some of the properties have never been proven. Therefore, we will now formally introduce and confirm them and recall the existing ones presented in [10, 11].

Lemma 3.9. *If a set of arguments $S \subseteq A$ is a minimal e–support for some argument $a \in A$, then it is self–supporting.*

Lemma 3.10. *If $S \subseteq A$ is admissible, then it is self–supporting.*

Lemma 3.11. ***EAS Fundamental Lemma** Let S be an admissible set and x, y two arguments acceptable w.r.t. S. Then $S \cup \{x\}$ is admissible and y is acceptable w.r.t. $S \cup \{x\}$.*

Lemma 3.12. *A set S is an e–stable extension iff*
$S = \{a \mid a$ is not e–support attacked by S and is e–supported by $S\}$.

Lemma 3.13. *If a set of arguments $S \subseteq A$ carries out a minimal e–supported attacked on some argument $a \in A$, then it is self–supporting.*

Proof. Results directly from Definition 3.4 and Lemma 3.9. □

Theorem 3.14. *The following holds:*

1. *Every stable extension is a preferred extension, but not vice versa.*
2. *Every preferred extension is a complete extension, but not vice versa.*
3. *The grounded extension is the least complete extension w.r.t. set inclusion.*

Proof.

1. It is easy to see that a stable extension S is also admissible, since it attacks (valid) arguments that are not included in it. Consequently, there can be no admissible extension $S \subset S'$ – it would simply break the conflict–freeness. Example 3.15 illustrates a preferred extension which is not stable.

2. If a preferred extension S was not complete, then by Lemma 3.11 we would be able to include more arguments into it and obtain an admissible extension S' such that $S \subset S'$. Thus, S could not have been preferred in the first place. A case of complete, but not preferred extensions is shown in Example 3.15.

3. Obvious by the definition of the semantics and Lemma 3.8.

□

Example 3.15. Let $(\{\eta, a, b, c, d, e, f\},\ \{(\{b\}, a), (\{b\}, c), (\{c\}, b), (\{c\}, d), (\{d\}, f),\ (\{f\}, f)\},$ $\{(\{\eta\}, b), (\{\eta\}, c), (\{\eta\}, d), (\{\eta\}, f), (\{d\}, e)\})$ be the EAS depicted in Figure 1. The admissible extensions are $\emptyset$, $\{\eta\}$, $\{\eta, b\}$, $\{\eta, c\}$, $\{\eta, b, d\}$ and $\{\eta, b, d, e\}$, with $\{\eta\}$, $\{\eta, c\}$ and $\{\eta, b, d, e\}$ being the complete ones. Obviously, the latter two are preferred. However, only $\{\eta, b, d, e\}$ is stable. Since a is not a valid argument (it is not e–supported in the framework), we do not have to attack it. Although $\{\eta, c\}$ attacks b and d (and by this, also e), it is not in any way in conflict with f. The grounded extension is just $\{\eta\}$.

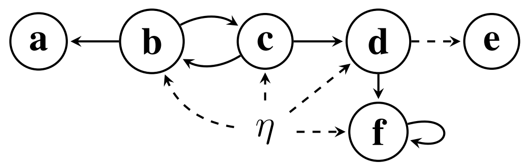

Figure 1: Sample EAS

Finally, in order to make certain proofs more feasible, we would like to propose an alternative definition of e–support, more in line with the sequence style found in other frameworks with support [8, 13], and in the argument chains of [12]. We will also show that a given EAS is equivalent to its minimal form w.r.t. the standard semantics.

Definition 3.16. *Given a set of arguments $X \subseteq A$, an* ***evidential sequence*** *for an argument $a \in X$ is a sequence of distinct elements of X $(a_0, .., a_n)$ s.t. $a_n = a$, $a_0 = \eta$, and if $n > 0$, then $\forall_{i=1}^{n}$ there exists a nonempty $T \subseteq \{a_0, ..., a_{i-1}\}$ s.t. TEa_i.*

Theorem 3.17. *Let $X \subseteq A$ be a set of arguments and $a \in A$. a is e–supported by X iff there exists an evidential sequence for a on $X \cup \{a\}$.*

Proof. Without the loss of generality, let us focus on the minimal case. Let a be e–supported by X. If $a = \eta$, then we trivially obtain an evidential sequence. Therefore, assume $a \neq \eta$. Since a is e–supported, then it is also supported and there exists a set of arguments $T \subseteq X$ s.t. TEa. If $a \in T$, then since all elements of T (and thus a) are e–supported by $X \setminus \{a\}$, then there has to exist a suitable $T' \subseteq X \setminus \{a\}$ s.t. $T'Ea$ and T' can be used for e–support. Thus, from now on we assume that $a \notin T$.

We now have a pair $(X \setminus \{a\}, (a))$, with each argument $x \in X \setminus \{a\}$ e–supported by $X \setminus \{a, x\}$ (see reasoning above). We now have to show that there exists an argument $a' \in X \setminus \{a\}$ that can be safely "removed", i.e. every $x \in X \setminus \{a, a'\}$ is e–supported by $x \in X \setminus \{a, a', x\}$. if $X \setminus \{a\}$ consists out of η only, then we are trivially done. Thus, let us assume there are more elements.

Assume there is no such a' that can be safely removed. This means that for every $a' \in X \setminus \{a\}$, there exists a different argument $a'' \in X \setminus \{a\}$ s.t. it is not e–supported by $X \setminus \{a, a'\}$. We can now "trace" this removal. Let us start with an arbitrary argument and denote it x_1, the argument it removes by x_2 and so on. We then have that x_1 removes (at least) x_2, x_2 removes x_3,..., x_n removes a certain x_i. Simply speaking, we obtain a cycle within this removal. It is easy to see that the set could not have possibly contained only e–supported arguments and we reach a contradiction with our assumptions: we have that x_n is e–supported by $X \setminus \{a\}$. By definition, at least x_{n-1} has to be e–supported by $X \setminus \{a, x_n\}$, and then x_{n-2} by $X \setminus \{a, x_n, x_{n-1}\}$. We can continue this way until we reach that x_i has to be e–supported by $X \setminus \{a, x_n, x_{n-1}, ..., x_{i+1}\}$ and thus obtain a contradiction. We can repeat the process starting with any argument from the set to show that this holds for the whole set $X \setminus \{a\}$. Hence, we can conclude that if the set is e–supported, there needs to exist an argument "safe for removal". We can now then retrieve this argument, say b, and obtain a pair $(X \setminus \{a, b\}, (b, a))$. We can repeat retrieving the "safe" elements and expanding our pair until we have only one element left. It is easy to see, that it has to be η and our evidential sequence can be easily obtained through the pair.

The other way around is straightforward. □

Theorem 3.18. *Let $ES = (A, R, E)$ be an EAS. The* ***minimal form*** *of ES is a framework $ES^{min} = (A, R', E')$, where $R' \subseteq R$ (respectively $E' \subseteq E$) consists of those elements (T, a) in R (E) s.t. $\nexists T' \subseteq T, (T', a) \in R$ (E). Then a set S is a σ–extension in ES where $\sigma \in \{$admissible, preferred, complete, grounded,stable$\}$ iff it is a σ–extension in ES^{min}.*

Proof. First of all, we prove that a set of arguments S attacks an argument a in ES iff it attacks it in ES^{min}. Based on the way R' is obtained, it holds that $\forall(X, a) \in R$, $\exists(X', a) \in R'$ s.t. $X' \subseteq X$. This means that if there exists $S' \subseteq S$ s.t. SRa, then there exists $S'' \subseteq S$ s.t. $S''R'a$. Thus, an argument attacked by S in ES also has to be attacked by S in ES^{min}. Since $R' \subseteq R$, it trivially follows that an argument attacked by S in ES^{min} is also attacked by S in ES. In a similar fashion we can show that a is supported by S in ES iff it is supported by S in ES^{min}. From this correspondence of e–support follows; if set S supports a by S' and S' is e–supported by $S \setminus \{a\}$ in ES, then naturally S supports a by $S'' \subseteq S'$ in ES and since S' is e–supported by $S \setminus \{a\}$, so is S''. The other way around is trivial since $E' \subseteq E$. Based on this and the correspondence between the attacks, we can show the equivalence between e–supported attacks. Finally, it also holds that S is a minimal e–support/ e-supported attack on a in ES iff it is one in ES^{min}. Should S be a minimal e–support for a in ES but not in ES^{min}, then it means there exists $S' \subset S$ e–supporting a in ES^{min}. However, this also means that $S' \subset S$ e–supports a in ES and we reach a contradiction. We can prove that it cannot be the case that S is minimal in ES^{min} but not in ES in the same way. From this and the correspondence between the attacks in both frameworks, equivalence of minimal e–supported attacks follows.

Based on this, it follows straightforwardly that a set S is conflict–free in ES iff it is conflict–free in ES^{min}. Using the correspondence between e–support and e–supported attacks, it is easy to prove that a set S is stable in ES iff it is such in ES^{min} and that an argument a is acceptable w.r.t. S in ES iff it is acceptable in ES^{min}. Consequently, admissible, complete and preferred semantics produce the same extensions in both frameworks. Based on completeness and Theorem 3.14, the same can be shown for the grounded semantics. □

4 Abstract Frameworks with Necessities

The necessary support introduced in [9] aimed at exposing the intuition that if an argument a necessarily supports b, then acceptance of a is required for the acceptance of b. Although initially defined in a binary manner, in a more recent work [8] it was dropped in favor of the set form. Moreover, some of the problems with the previous semantics were fixed and additional labeling–based formulations were introduced. The new definition is thus as follows.

Definition 4.1. *An **abstract argumentation framework with necessities** (AFN) is a tuple (A, R, N) where A is a set of **arguments**, $R \subseteq A \times A$ represents the **attack** relation and $N \subseteq (2^A \setminus \emptyset) \times A$ represents the **necessity** relation.*

We say that a attacks b iff aRb. Abusing notation, we will write SRC to denote that there exists an argument $a \in S$ and $b \in C$ such that aRb.

Note that although the support relations of EASs and AFNs are structurally the same, the intuition behind what it means to be supported by a set is different. In EASs, we say that a set of arguments $S \subseteq A$ supports an argument $a \in A$ if $\exists X \subseteq S$ s.t. XEa. In AFNs, we are presented with a dual situation. S supports a if $\forall X \subseteq S$ s.t. XNa, $X \cap S \neq \emptyset$. This will be especially visible when we present the definition of a powerful sequence (Defn 4.2) and a translation between the two frameworks.

While EAS semantics imply acyclicity of the support relation among the accepted arguments through the requirement for evidence η, AFNs make this requirement explicit. One of the possible formulations for doing so is by the means of the powerful sequence:

Definition 4.2. *An argument a is **powerful** in $S \subseteq A$ iff $a \in S$ and there is a sequence $a_0, ..., a_k$ of elements of S such that:*

- $a_k = a$
- *there is no $E \subseteq A$ s.t. ENa_0*
- *for $1 \leq i \leq k$: for each $E \subseteq A$, if ENa_i then $E \cap \{a_0, ..., a_{i-1}\} \neq \emptyset$.*

*A set of arguments $S \subseteq A$ is **coherent** iff each $a \in S$ is powerful in S. A set of arguments is **strongly coherent** iff it is coherent and conflict–free w.r.t. R.*

Remark. There is a subtle difference between the sequences in AFNs (Defn. 4.2) and EASs (Defn. 3.16). The former states that if a supporter set exists, then it has an element in the sequence. The evidential sequence requires that a supporter exists and is contained in the sequence. This results from the fact that every valid argument (apart from η) needs to be supported by some set in the first place to even have a chance of tracing back to evidence. Thus, unsupported arguments are ”filtered out” immediately.

Just like in EASs, the definition of defense (acceptability) in AFNs extends Dung's definition by introducing support requirements. Semantics are then defined as usual.

Definition 4.3. *Let $S \subseteq A$ and $a \in A$. We say that S **defends** a iff $S \cup \{a\}$ is coherent and for each $b \in A$, if bRa then for each coherent $C \subseteq A$ that contains b, $S\,R\,C$.*

A set of arguments $S \subseteq A$ is :

- ***admissible** iff it is strongly coherent and defends all of its arguments.*

- ***preferred*** *iff it is maximal w.r.t. set inclusion admissible.*
- ***complete*** *iff it is admissible and all arguments defended by S are in S.*

The grounded semantics is again defined via the characteristic function.

Definition 4.4. *The **characteristic function** $F_{FN} : 2^A \to 2^A$ is defined as $F(S) = \{a \mid S$ defends a in $FN\}$. The **grounded extension** is the least fixed point of F_{FN}.*

We can now define the stable semantics, and show some properties of AFNs. Just like we did in the case of EASs, we will show that a given AFN is equivalent to its minimal form w.r.t. the standard semantics.

Definition 4.5. *The set of arguments **deactivated** by S is defined by $S^+ = \{a \mid SRa$ or there exists $E \subseteq A$ s.t. ENa and $S \cap E = \emptyset\}$. Then a complete extension S is **stable** iff $S^+ = A \setminus S$.*

Theorem 4.6. *The following properties holds:*

1. *Every stable extension is a preferred extension, but not vice versa.*
2. *Every preferred extension is a complete extension, but not vice versa.*
3. *The grounded extension is the least w.r.t. set inclusion complete extension.*

Example 4.7. Let $(\{a, b, c, d, e\}, \{(b, a), (e, a), (c, d)\}, \{(\{b\}, b), (\{b, d\}, e), (\{a\}, c)\})$ be an AFN depicted in Figure 2. The admissible (and at the same time complete) extensions of this framework would be $\{d, e\}$, $\{a, c\}$ and $\emptyset$. The first two are the preferred and stable ones. $\emptyset$ is the grounded extension.

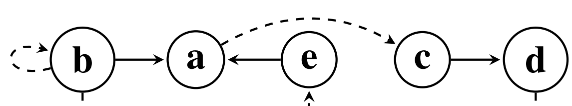

Figure 2: Sample AFN

Theorem 4.8. *Let $FN = (A, R, N)$ be an AFN. The **minimal form** of FN is a framework $FN^{min} = (A, R, N')$, where $N' \subseteq N$ consists of those elements (T, a) in N s.t. $\nexists T' \subseteq T, (T', a) \in N$. Then a set S is a σ–extension in FN where $\sigma \in \{$admissible, preferred, complete, grounded,stable$\}$ iff it is a σ–extension in FN^{min}.*

Proof. Let us first show that a has a powerful sequence on S in FN iff it has one in FN^{min}. Let $(a_0, ..., a_n)$ be a sequence for a in FN. Since no arguments are removed and there is no support in N' that would not come from N, it is easy to see that the conditions that $a_n = a$ and $\nexists T \subseteq A$ s.t. $TN'a_0$ are easily satisfied. Given a nonzero a_i, it holds that for all $T \subseteq A$ s.t.

$TNa, T \cap \{a_0, ..., a_{i-1}\} \neq \emptyset$. Since it holds for all such $T's$, then it also holds only for those that were included in N'. Thus, the sequence is also powerful in FN^{min}. Now, assume a is powerful in S in FN^{min}. The sequence from FN^{min} trivially meets the endpoint requirements also in FN. By the definition of N', it is easy to see that $\forall (X, a) \in N$, $\exists (X', a) \in N'$ s.t. $X' \subseteq X$. Therefore, if given a nonzero a_i it holds that $\forall T \subseteq A$ s.t. $TN'a, T \cap \{a_0, .., a_{i-1}\} \neq \emptyset$, then it also has to be the case that $\forall T \subseteq A$ s.t. $TNa, T \cap \{a_0, ..., a_{i-1}\} \neq \emptyset$. Thus, the sequence is also powerful in FN.

Consequently, it holds that every coherent in FN is coherent in FN^{min} and vice versa. Since the attack relation is unchanged, strong coherence follows. Based on the equivalence of coherence and attack, defense can be proved easily. Consequently, the admissible, complete and preferred semantics produce the same extensions on FN and FN^{min}. By completeness and Theorem 4.6 it also holds that grounded extensions coincide. Let us not show that the same follows for stability. Since complete extensions coincide, what remains to be shown is that the deactivated sets are the same. Let S be a stable extension in FN and $a \in S^+$. Since the complete extensions coincide, what remains to be shown is that the deactivated sets are the same. If a is in the deactivated set due to attack, then naturally it also is in the deactivated set in FN^{min}. If there exists $T \subseteq A$ s.t. TNa and $S \cap T = \emptyset$, then naturally for every $T' \subseteq T$, $S \cap T' = \emptyset$. Thus, there has to exist such a T' s.t. $T'N'a$ and $S \cap T' = \emptyset$ and a qualifies for the deactivated set in FN^{min}. Since an argument deactivated by attack in FN^{min} is also deactivated in FN and $N' \subseteq N$, it is easy to see that an extension stable in FN^{min} is also stable in FN. □

5 Comparison of evidential and necessary support

We now consider how necessary support can be expressed as evidential support. We show how to translate a given AFN into an EAS and prove that the process preserves the semantics. Clearly, moving attack from binary to set–form is trivial. The support, however, requires some comments. First of all, recall our explanation concerning the differences between support in AFNs and EASs, i.e. the N vs E relation. Let $A_1, ..., A_n$ be sets supporting an argument a in N. We say that a set of arguments S supports a iff for every such A_i, $S \cap A_i \neq \emptyset$. Verifying whether S supports a corresponds to checking whether S satisfies a propositional formula $\bigvee A_1 \wedge ... \wedge \bigvee A_n$, where $\bigvee A_i$ should be understood as a disjunction of elements of A_i. Should $A_1, ..., A_n$ be supporting a by E, we would produce a formula $\bigwedge A_1 \vee ... \vee \bigwedge A_n$, where $\bigwedge A_i$ stands for the conjunction of elements of A_i. Therefore, a translation between these relations can be seen as a conversion between CNF and DNF. The important question is, on how arguments should be tied to evidence. In EASs, evidence is the sole confirmation of validity and arguments need to be able to trace back to it (as the evidential sequence makes explicit). In AFNs, validity is obtained through acyclicity — we must be able to trace back from a valid argument to arguments that require no support (as per the powerful sequence). Therefore, if we want unsupported arguments to be able to provide validity in the EAS setting, it is easy to see that they (and only they) should be backed up by η. This observation allows us to define a translation as follows.

Translation 1. Let $FN = (A, R, N)$ be an AFN. The corresponding EAS $ES^{FN} = (A', R', E)$ is created as follows:

- $A' = A \cup \{\eta\}$.
- For every two arguments a, b s.t. $(a, b) \in R$, put $(\{a\}, b)$ in R'.
- Let a be an argument in A and $Z = \{Z_1, .., Z_n\}$ be a collection of all sets Z_i s.t. $Z_i Na$. If Z is empty, add $(\{\eta\}, a)$ to E. Otherwise, for every subset Z' of $\bigcup_{i=1}^{n} Z_i$ s.t. $\forall_{i=1}^{n} Z' \cap Z_i \neq \emptyset$, add (Z_i, a) to E.

Although the translation of support presented above is correct and the semantics return the desired extensions in the obtained framework, it is not the most optimal one. By this we mean that it can create redundant elements in E. For example, given argument a s.t. $\{a, b\}Na$ and $\{c\}Na$, our intent would be to receive $\{a, c\}Ea$ and $\{b, c\}Ea$. However, the translation would also give us $\{a, b, c\}Ea$. Although the framework behaves in the desired way (see Theorems 3.18), a cleaner transformation would be more desirable. Please note that it cannot be fixed by assuming that we take into account only minimal sets Z', since the elements of N might not be incomparable in the first place. Even though again the produced extensions of a framework produced by such a minimal transformation would be satisfactory (see Theorems 3.18 and 4.8 on minimal forms), we would "lose" some of the relations. Therefore, we would like to propose another way of translating the support:

Translation 1 (Continued)**.** Let a be an argument in A and $Z = \{Z_1, .., Z_n\}$ be a collection of all sets Z_i s.t. $Z_i Na$. If Z is empty, add $(\{\eta\}, a)$ to E. Otherwise, for all Z' in $Z_1 \times ... \times Z_n$, add (Z'_S, a) to E, where Z'_S is the set of all elements in Z'.

Theorem 5.1. *An argument a is powerful in $S \cup \{a\} \subseteq A$ in FN iff it is e–supported by $S \cup \{\eta\}$ in ES^{FN}.*

Proof. Let $(a_0, ..., a_n)$, where $a_n = a$, be a powerful sequence for a on $S \cup \{a\}$. Then $(\eta, a_0, .., a_n)$ is an evidential sequence for a on $S \cup \{a, \eta\}$. Since a_0 requires no support in FN, then by Translation 1 it is supported by η in ES^{FN} and the evidential condition is satisfied. Let a_i be an arbitrary, nonzero element of the powerful sequence. For any set X s.t. XNa_i, we know that $X \cap \{a_0, .., a_{i-1}\} \neq \emptyset$. Thus, if at least one such supporting set X exists, it is easy to see by Translation 1 (and its continuation) that there is $X' \subseteq \{a_0, .., a_{i-1}\}$ s.t. $X'Ea_i$ and that the evidential condition is satisfied. If no supporting set X exists, then we have that a_i is supported by η, and the condition is again satisfied. Thus, we have a valid evidential sequence on $S \cup \{a, \eta\}$ and by Theorem 3.17, e–support from $S \cup \{\eta\}$. The other way around is straightforward, though we note the requirement on a_0: if an argument is supported by η in ES^{FN}, then it requires no support in FN and thus the powerful condition on a_0 is met. □

Theorem 5.2. *A set of arguments $S \subseteq A$ is coherent in FN iff $S \cup \{\eta\}$ is self–supporting in ES^{FN}.*

Proof. Follows straightforwardly from Theorem 5.1 and the fact that η is e–supported by an arbitrary set. Although here we consider only sets that contain at least η, please note that $\emptyset$ is also self–supporting in ES^{FN} and obviously coherent in FN. $\square$

There is an important difference between the definitions of defense (acceptability) in EASs and AFNs concerning support. In EASs, an argument a has to be e–supported by the set S. Consequently, it does not have to be the case that $S \cup \{a\}$ is self–supporting. In AFNs it is required that $S \cup \{a\}$ is coherent, which by Theorem 5.2 is visibly a stronger restriction. However, in order to have a chance to be an extension, a set has to be coherent (self–supporting) in the first place. Therefore, we focus on such sets in our analysis.

Theorem 5.3. *Let $S \subseteq A$ ($S \cup \{\eta\}$ once translated into an EAS) be a coherent (self–supporting) set in FN (ES^{FN}). An argument $a \in A$ is defended by S in FN iff it is acceptable w.r.t $S \cup \{\eta\}$ in ES^{FN}.*

Proof. We begin with the support part of defense/acceptability. If a is defended by S, then $S \cup \{a\}$ is coherent, and by Theorem 5.2 $S \cup \{a, \eta\}$ is self–supporting in ES^{FN}. Thus, a has an evidential sequence on $S \cup \{a, \eta\}$, and by Theorem 3.17 e–support from $S \cup \{\eta\}$. The other way around, it is easy to see that if $S \cup \{\eta\}$ is a self–supporting set and e–supports a, then $S \cup \{a, \eta\}$ is also self–supporting.Thus, by Theorem 5.2 $S \cup \{a\}$ coherent.

Let us now continue with the attack. Let $a \in A$ and b be an argument in A s.t. bRa. Since a is defended by S in FN, then for each coherent set $C \subseteq A$ s.t. $b \in C$, SRC. Thus, after Translation 1, we have that for $\{b\}R'a$ and for every (and thus also minimal) self–supporting set $C \cup \{\eta\}$ containing b, $S \cup \{\eta\}$ attacks $C \cup \{\eta\}$. It is easy to see that $C \cup \{\eta\}$ is an e–supported attack against a and since $S \cup \{\eta\}$ is assumed to be self–supporting, the attack it carries out against $C \cup \{\eta\}$ is also e–supported. As all attacks, and only those attacks, in ES_{FN} come from FN and η cannot attack or be attacked in the framework, acceptability follows straightforwardly.

Now let $a \in A$ be an argument acceptable w.r.t. $S \cup \{\eta\}$. Thus, given any set $C \subseteq A$ that carries out a minimal e–supported attack on a, $S \cup \{\eta\}$ support attacks a member of C. Since $S \cup \{\eta\}$ is self–supporting, any attack carried out by it will be e–supported. By Lemma 3.13, C is self–supporting, and thus $C \setminus \{\eta\}$ is coherent in FN. Since C attacks a, there exists $C' \subseteq C$ s.t. $C'R'a$. By Translation 1 all such sets C' consist of exactly one element and if $\{c\}R'a$ in ES_{FN}, then cRa in FN. The attack by $S \cup \{\eta\}$ against C follows a similar analysis. Please note that although technically we attack only minimal e–supported attacks on a, it is easy to see that it cannot be the case that there exists an unattacked e–supported attack on a. Every such attack either contains a minimal one, or is one – either case, it still remains attacked. Consequently, it holds that for every coherent set $C \setminus \{\eta\}$ s.t. CRa, SRC in FN. Defense follows straightforwardly.

$\square$

Theorem 5.4. *Let $FN = (A, R, N)$ be an AFN and $ES_{FN} = (A', R', E)$ its corresponding EAS. Then a set S is a σ–extension in FN where $\sigma \in \{$admissible, preferred, complete, grounded,stable$\}$ iff $S \cup \{\eta\}$ is a σ–extension in ES_{FN}.*

Proof. It it is easy to see by Translation 1 that a given set is conflict–free in FN iff it is conflict–free in ES_{FN}. Then admissible, preferred and complete semantics follow straightforwardly from Lemma 3.10, Theorems 5.2 and 5.3. Please note that although we consider sets that have at least η in ES^{FN}, $\emptyset$ is trivially conflict–free and admissible as well. Since complete extensions coincide and the grounded extensions are the least w.r.t. set inclusion complete both in FN and ES_{FN} by Theorems 3.14 and 4.6, then grounded extensions coincide as well. Thus, we focus on the stable semantics. First, note that by Theorem 3.14 every stable extension in ES_{FN} is also complete. Since complete extensions coincide in FN and ES_{FN}, what remains to be done is the analysis of the elements not present in the extension.

Let S^+ be the deactivated set and $a \in S^+$. If a is in the set because SRa, then naturally $S \cup \{\eta\}$ carries out an e–supported attack on a, whether a is valid or not. Let us focus on the case when a is in the set due to lack of support. If a is powerful in A, this means that for every powerful sequence, part of the sequence is not accepted in the set. If a is not powerful, then by Theorem 5.1 it is not e–supported in $A \cup \{\eta\}$ in ES_{FN} and thus does not affect the stable extension of the EAS. Therefore, let us assume there exists at least one powerful sequence $(a_0, ..., a_n, a)$ for A. Without the loss of generality, we can assume this sequence is minimal. Let $0 \leq i \leq n$ be the position of the first argument in the sequence that does not belong to S. If it is $i = 0$, then since a_0 requires no support and is in S^+, it has to be the case that SRa_0. For other $t \neq 0$, since all the required support for a_i is in S but $a_i \in S^+$, then again it has to be the case that SRa_i. This minimal powerful sequence for a in FN gives rise to a minimal evidential sequence in ES_{FN} (see proof of Theorem 5.1), from which by Theorem 3.17 we can obtain a minimal set e–supporting a. Since it is the case that for any sequence S carries out an attack, then by Translation 1 so does $S \cup \{\eta\}$ and as it is a self–supporting set, the attack is e–supported. Consequently, stability conditions in ES_{FN} are satisfied.

Assume now that $S \cup \{\eta\}$ is stable in ES_{FN}, but S is not stable in FN. This means there exists an argument $a \in A \setminus S$ that is not in the deactivated set. Consequently, it has to be the case that a is not attacked by S (and thus not e–support attacked by $S \cup \{\eta\}$) and either requires no support or is supported by S (which means it has to be supported by $S \cup \{\eta\}$ in ES_{FN}). Since a either does not need support or is sufficiently supported by St, it has a powerful on $S \cup \{a\}$ in FN. Consequently, it is e–supported by $S \cup \{\eta\}$ in ES_{FN} by Theorem 5.1. Since a is not in the stable extension in ES_{FN}, either it or all of the sets minimally e–supporting it are attacked. In the first case, a is also attacked in FN and we reach a contradiction. If a required no support in FN, then it is supported by η in ES_{FN}, and as η cannot be attacked, it has to be the case that a is, again yielding a contradiction. If $S \cup \{\eta\}$ attacks all sets minimally e–supporting a, then naturally all coherent sets containing a are also attacked in FN. Since $S \cup \{a\}$ is a coherent set and S is conflict–free, it cannot be the case that at the same time all powerful sequences of a are attacked (through an element different than a) and S supports a. Thus, S has to be stable in FN.

□

Example 4.7 (Continued). Recall the AFN $(\{a, b, c, d, e\}, \{(b, a), (e, a), (c, d)\}, \{(\{b\}, b), (\{b, d\}, e), (\{a\}, c)\})$. By Translation 1 we obtain its EAS $((\{\eta, a, b, c, d, e\}, \{(\{b\}, a), (\{e\}, a), (\{c\}, d)\}, \{(\{b\}, b), (\{b\}, e), (\{d\}, e), (\{a\}, c)\}), (\{\eta\}, a), (\{\eta\}, d)\})$. The maximal self–supporting sets are $\{\eta, a, c\}$ and $\{\eta, d, e\}$, thus b is correctly recognized as invalid. Our admissible extensions are $\emptyset$, $\{\eta\}$, $\{\eta, a, c\}$ and $\{\eta, d, e\}$. It is easy to see they satisfy the completeness criterion as well. The latter two are also preferred. Since $\{\eta, a, c\}$ attacks d and thus cuts off support of e, it is stable, and similarly for $\{\eta, d, e\}$. Our grounded extension is just η. Thus, it is easy to see that our results agree for both the AFN and translated EAS.

Let us now consider a possible translation from EASs to AFNs. Since AFNs employ only binary attacks, a full translation procedure would require the introduction of additional, virtual, arguments and would further blur our analysis of support. Instead, we focus on translating support between the two frameworks and illustrate the process through an example.

The differences between the E and N sets require a shift between relations similar to the one in Translation 1. The biggest difficulty here is the handling of η. We note that η is the sole confirmation of "truth" in an EAS — its presence at the start of an evidential sequence is required for an argument to be valid. Any argument not supported by η effectively attempts to justify its own truth, thus acting as a self–supporter. Due to acyclicity requirements in AFNs, such an argument would (correctly) not be treated as valid. Note that an argument supported by at least one set (argument) will always trace back to some elements of the framework. If it can only trace to such a self–supporter or to itself, it would again be discarded in AFNs as intended.

This intuition can also be considered from a more structural point of view. Since any powerful sequence originates at an argument that requires no support, the translation from EASs to AFNs should simply ensure that η is the only argument meeting this requirement. Consequently, any other argument that requires no support in the EAS should be disqualified in its corresponding AFN, which is achieved easily by using a support cycle.

In order to omit the attack issues between EASs and AFNs, let us focus on a subclass of EASs that uses only binary conflicts, i.e. where every set of arguments S s.t. SRa for some $a \in A$, consists of a single element. We denote it by EAS^{bin}.

Translation 2. Let $ES = (A, R, E)$ be an EAS^{bin}. The corresponding AFN $FN^{ES} = (A, R', N)$ is created as follows:

- The set of arguments remains the same.
- For every two arguments a, b s.t. $(\{a\}, b) \in R$, put (a, b) in R'.
- Let $a \neq \eta$ be an argument in A and $Z = \{Z_1, .., Z_n\}$ be a collection of all sets Z_i s.t. $Z_i E a$. If Z is empty, add $(\{a\}, a)$ to N. Otherwise, for every subset Z' of $\bigcup_{i=1}^{n} Z_i$ s.t. $\forall_{i=1}^{n} Z' \cap Z_i \neq \emptyset$, add (Z_i, a) to N.

Although the translation is correct in the sense that extensions produced by the frameworks coincide, just like in the case of Translation 1 we can obtain redundant information. Again, assuming minimality of Z' sets would "drop" some relations in case the original framework was not in minimal form, though still the provided results would be correct (see Theorems 4.8 and 3.18). Unfortunately, the optimization in this case is still a task for future work.

Theorem 5.5. *An argument a e–supported by $S \subseteq A$ in ES iff it is powerful in $S \cup \{a\}$ in FN^{ES}.*

Proof. Since a is e–supported by S, it has an evidential sequence on $S \cup \{a\}$ by Theorem 3.17. Let $(\eta, a_0, ..., a_n)$, where $a_n = a$, be an evidential sequence for a. Then it is also a powerful sequence for a on $S \cup \{a\}$ in FN^{ES}. Since by the definition of EAS $\nexists T \subseteq A$ s.t. $TE\eta$ and Translation 2 adds no additional support relation to η, it holds that $\nexists T \subseteq A$ s.t. $TN\eta$ in FN^{ES}. Therefore, the first two requirements of the powerful sequence are satisfied. By the evidential sequence, we know that $\{\eta\}Ea_0$. Hence, by Translation 2, every set $Z \subseteq A$ s.t. ZNa_0 contains η, and thus a_0 satisfies the powerful requirement. Let a_i, where $1 \leq i \leq n$, be an element of the evidential sequence. We know that for every such a_i, there exists a nonempty $T \subseteq \{\eta, a_0, .., a_{i-1}\}$s.t. TEa_i. Since by Translation 2 for every $Z \subseteq A$ s.t. ZNa_0, $T \cap Z \neq \emptyset$, it holds that for every Z, $Z \cap \{\eta, a_0, .., a_{i-1}\} \neq \emptyset$ and thus the powerful requirements are satisfied.

Let now $(a_0, ..., a_n)$, where $a_n = a$, be a powerful sequence for a on $S \cup \{a\}$ in FN^{ES}. Then it is also an evidential sequence for a on $S \cup \{a\}$ in ES. By Translation 2, it is easy to see that η is the only argument that requires no support in FN^{ES} and thus it is the only candidate for a_0. Thus, the first two requirements of an evidential sequence are satisfied. Let a_i be an arbitrary, nonzero argument. This means that for every $Z \subseteq A$ s.t. ZNa_i, $Z \cap \{a_0, ..., a_{i-1}\} \neq \emptyset$. Please note that Translation 2 guarantees the existence of at least one supporting set Z. Let us assume that a_i does not satisfy the evidential requirements, i.e. $\forall T \subseteq A$ s.t. TEa_i, $T \nsubseteq \{a_0, ..., a_{i-1}\}$. This means that for every such T, there is some argument $t \in T$ s.t. $t \notin \{a_0, ..., a_{i-1}\}$. However, by Translation 2, from such $t's$ we can construct a set Z s.t. ZNa_i. For this set it holds that $Z \cap \{a_0, ..., a_{i-1}\} = \emptyset$, which breaks the powerful requirement and we reach a contradiction. Therefore, the evidential conditions are satisfied and $(a_0, ..., a_n)$ is an evidential sequence. Thus, by Theorem 3.17, a is e–supported by S. □

Theorem 5.6. *A set of arguments $S \subseteq A$ is self–supporting in ES iff it is coherent FN^{ES}.*

Proof. Follows straightforwardly from Theorem 5.5. □

Theorem 5.7. *Let $S \subseteq A$ be a self–supporting (coherent) set in ES (FN^{ES}). An argument $a \in A$ is acceptable w.r.t. S in ES iff it is defended by S in FN^{ES}.*

Proof. We begin with the support part of defense/acceptability. If S is a self–supporting set and e–supports a, then $S \cup \{a\}$ is also self–supporting and by Theorem 5.6 coherent. The other way around is also simple; if $S \cup \{a\}$ is coherent, then it is also self–supporting. This means that a has an evidential sequence on $S \cup \{a\}$, and by Theorem 3.17 is e–supported by S.

Let us now continue with the attack. Please note that by Translation 2, an argument a attacks b in FN^{ES} iff $\{a\}$ attacks b in ES.

Let $a \in A$ be an argument acceptable w.r.t. S. This means that given any set $C \subseteq A$ that carries out a minimal e–supported attack on a, S support attacks a member of C. Since S is self–supporting, any attack carried out by it will be e–supported. Please note that although technically we attack only minimal e–supported attacks on a, it is easy to see that it cannot be the case that there exists an unattacked e–supported attack on a. Every such attack either contains a minimal one, or is one – either case, it still remains attacked. By Lemma 3.13, C is self–supporting, and thus coherent in FN^{ES}. Since C attacks a, then by the fact that ES in an EAS^{bin}, $\exists c \in C$ s.t.

$\{c\}Ra$. By Translation 2, it follows that $cR'a$ in FN^{ES}. The attack by S against C follows a similar analysis. Thus, we have that for every coherent set C s.t. $CR'a$, $SR'C$ in FN^{ES}. Defense follows straightforwardly.

Now let $a \in A$ be defended by S in FN^{ES}. This means that for any argument $b \in A$ s.t. $bR'a$, every coherent set $C \subseteq A$ containing b is attacked by S. By Translation 2, if $bR'a$ in FN^{ES}, then $\{b\}Ra$ in ES. Similar follows for attack of S on C. Since every C is coherent, then by Theorem 5.6 it is also self–supporting. Consequently, it is an e–supported attack against a in ES and as S is assumed to be self–supporting, the attack it carries out against C is also e–supported. Therefore, S can respond to every (and thus also minimal) e–supported attack on a and thus, a is acceptable w.r.t. S.

□

Theorem 5.8. *Let $ES = (A, R, E)$ be an EAS^{bin} and $FN^{ES} = (A, R', N)$ its corresponding AFN. Then a set S is a σ–extension in ES where $\sigma \in \{$admissible, preferred, complete, grounded,stable$\}$ iff it is a σ–extension in FN^{ES}..*

Proof. It it is easy to see by Translation 2 that a given set is conflict–free in ES iff it is conflict–free in FN^{ES}. Then admissible, preferred and complete semantics follow straightforwardly from Lemma 3.10, Theorems 5.6 and 5.7. Since complete extensions coincide and the grounded extensions are the least w.r.t. set inclusion complete both in ES and FN^{ES} by Theorems 3.14 and 4.6, then grounded extensions coincide as well. Thus, we focus on the stable semantics. First, note that by Theorem 3.14 every stable extension in FN^{ES} is also complete. Since complete extensions coincide in ES and FN^{ES}, what remains to be done is the analysis of the elements not present in the extension.

Assume that S is stable in ES, but not in FN^{ES}. This means there exists an argument $a \in A \backslash S$ that is not in the deactivated set. Consequently, a is not attacked by S in FN^{ES} and either requires no support at all or sufficient support is provided by S. If a requires no support, by Translation 2 it has to be the case that $a = \eta$. However, since S is self–supporting/coherent, η has to be in S and we reach a contradiction. If a is supported by S, it is easy to see that we can construct a powerful sequence for it on $S \cup \{a\}$ and by Theorem 5.5 a is e–supported by S in ES. However, by definition of stability in ES, it means that there is $s \in S$ s.t. $\{s\}Ra$ in ES or that every set minimally supporting a is attacked. Consequently, either $sR'a$ or every coherent set containing a is attacked in FN. If it is the first case, then a has to be in the deactivated set and we reach a contradiction. If it is the latter, it means that since $S \cup \{a\}$ is coherent and S does not attack a, S has to attack itself. This breaches the conflict–freeness assumption and we reach a contradiction. Hence, there is no $a \in A \setminus S$ that is not in the deactivated set and S is AFN stable.

Now assume that S is stable in FN^{ES}, but not in ES. This means there exists an argument $a \in A \backslash S$ that is e–supported by A and neither it nor every set of arguments minimally e–supporting it is attacked by S. Since a is in the deactivated set, then either $SR'a$ or $\exists E \subseteq A$ s.t. ENa and $S \cap E - \emptyset$. If it is the first case, then obviously SRa in ES and we reach a contradiction. Let us focus on the case when a is in the set due to lack of support. Since a is e–supported by A, by Theorem 5.5 it is powerful in A. Lack of support means that for every powerful sequence, part of the sequence is not in S. Without the loss of generality, we can assume this sequence is minimal.

Let $1 \leq i \leq n$ be the position of the first argument in the sequence that does not belong to S. Since $a_0 = \eta$, it cannot be the case that it does not belong to S. As all the required support for a_i is in S but $a_i \in S^+$, then it has to be the case that $SR'a_i$ and thus SRa_i and we reach a contradiction. This minimal powerful sequence for a in FN^{ES} gives rise to a minimal evidential sequence in ES (see proof of Theorem 5.5), from which by Theorem 3.17 we can obtain a minimal set e–supporting a. Since it is the case that for any sequence S carries out an attack in FN^{ES}, then by Translation 2 it also carries out an attack in ES and as it is a self–supporting set, the attack is e–supported. Consequently, we reach a contradiction and S is stable in ES.

□

Example 3.15 (Continued)**.** Recall the framework $(\{\eta, a, b, c, d, e, f\}, \{(\{b\}, a), (\{b\}, c), (\{c\}, b), (\{c\}, d), (\{d\}, f), (\{f\}, f)\}, \{(\{\eta\}, b), (\{\eta\}, c), (\{\eta\}, d), (\{\eta\}, f), (\{d\}, e)\})$. We can now construct its corresponding AFN (A, R, N). The set of arguments remains the same and A is simply $\{\eta, a, b, c, d, e, f\}$. Since the EAS has only binary attacks, we can copy them across from the EAS to R. In this example, the necessity relation is $N = E \cup \{(a, a)\}$. a is the only argument that is not supported by anything at all in the EAS. Our admissible extensions are now $\emptyset, \{\eta\}, \{\eta, b\}, \{\eta, b, d\}, \{\eta, b, d, e\}$ and $\{\eta, c\}$, which are exactly the same as in for the EAS. The AFN's complete extensions are $\{\eta\}, \{\eta, b, d, e\}$ and $\{\eta, c\}$ and again we obtain correspondence. The same trivially follows for the preferred semantics. It is easy to see that the stable set is also $\{\eta, b, d, e\}$ and the grounded $\{\eta\}$.

Example 4.7 (Continued)**.** Recall the AFN $FN_1 = (\{a, b, c, d, e\}, \{(b, a), (e, a), (c, d)\}, \{(\{b\}, b), (\{b, d\}, e), (\{a\}, c)\})$ and its EAS $((\{\eta, a, b, c, d, e\}, \{(\{b\}, a), (\{e\}, a), (\{c\}, d)\}, \{(\{b\}, b), (\{b, \}, e), (\{d\}, e), (\{a\}, c)\}), (\{\eta\}, a), (\{\eta\}, d)\})$. Let us now shift it back to AFN form via Translation 2. The produced framework is $FN_2 = (\{\eta, a, b, c, d, e\}, \{(b, a), (e, a), (c, d)\}, \{(\{b\}, b), (\{b, d\}, e), (\{a\}, c)\}), (\{\eta\}, a), (\{\eta\}, d)\})$. Therefore, we retrieve the original AFN extended with evidence and the resulting relations. We obtain four admissible extensions – $\emptyset, \{\eta\}, \{\eta, a, c\}$ and $\{\eta, d, e\}$, out of which only $\emptyset$ is not complete. $\{\eta, a, c\}$ and $\{\eta, d, e\}$ are the preferred and stable extensions. It is easy to see that by removing η from the sets we retrieve the extensions of FN_1.

Please note that a translation removing η from the framework can exist, however, it would require more attention than the one we presented. This comes from the fact that removing support from η to some argument a does not mean that a becomes an argument that requires no support. If it were the case that there is also an argument b supporting a s.t. $\{\eta\}Eb$, in the EAS approach a would have two sequences. Should we translate this example to AFN without evidence, we would have that a now has only one sequence (through b), as $\emptyset$ is not permitted in N. Even if this restriction was relaxed, we would still obtain the same answer due to the definition of the powerful sequence. Although it can of course be adapted to treat "requires no support" and "requires no support/is supported by $\emptyset$" in the same manner, it is easy to see that an translation that requires changing the formal definitions of the target structure is by no means desirable. Moreover, should an implementation of AFNs exist, it would no longer be usable as a solver for EASs.

6 Discussion and Conclusions

This paper's examined the differences and similarities between support as used within Evidential Argument Systems and Argumentation Frameworks with Necessities.

We provided a translation between AFNs and EASs and analyzed a possible translation going in the other direction, thus completing and correcting the analysis carried out in [6]. Additionally, we identified correspondences between the properties of both of these systems to the properties obtained in Dung's argumentation system. We have also introduced the notion of a minimal form of a framework, which although not required in binary frameworks like AFs, can be interesting when faced with structures using set–form relations. Finally, we corrected some important errors in the definitions of EASs.

We are pursuing several avenues of future work. First, as suggested above, we intend to fully formalize the translation from EASs to AFNs. Second, we wish to provide a mapping between the remaining types of support (deductive and abstract) to the systems discussed here (c.f. [6, 12]). Finally, we intend to investigate a more pragmatic issue: given the different types of support in the literature, we will examine which framework (if any) is able to facilitate support between arguments as found in real domains, and if needed, construct a system to provide this type of support to a knowledge engineer.

Support is an important, if somewhat controversial concept in argumentation. This paper serves to unify some of the most popular approaches to its representation. Doing so not only provides important theoretical contributions, but also helps in the representation of real world domains.

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