Comparing the Power of Different Semantics for Abstract Argumentation

Stefan Woltran

Joint work with Ringo Baumann, Paul Dunne, Wolfgang Dvořák, Thomas Linsbichler, Emilia Oikarinen and Hannes Strass

Oct 24, 2014

Introduction

- Important research stream in abstract argumentation: systematic comparison of semantics
- Some general questions:
  - Which sets of extensions are possible?
  - What is the role of "auxiliary" arguments (i.e. arguments that do not appear in extensions)?
  - How do explicit and implicit conflicts relate to each other?
- We provide answers to these questions for some of the main semantics (based on our papers at KR 2014, ECAI 2014, COMMA 2014)

Background

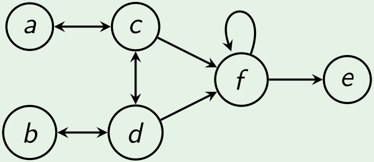
Recap: Some Semantics

Let $F = (A, R)$ be an AF and $S \subseteq A$ such that S is conflict-free in F.

- $S \in \mathit{stb}(F)$ if for each $a \in A \setminus S$, S attacks a.
- $S \in \mathit{adm}(F)$ if each $a \in S$ is defended by S
- $S \in \mathit{pref}(F)$ if $S \in \mathit{adm}(F)$ and for each $T \in \mathit{adm}(F)$, $S \not\subset T$
- $S \in \mathit{sem}(F)$ if $S \in \mathit{adm}(F)$ and for each $T \in \mathit{adm}(F)$, $R^+(S) \not\subset R^+(T)$, where $R^+(U) = U \cup \{b \mid (a, b) \in R, a \in U\}$.
- $S \in \mathit{comp}(F)$ if $S \in \mathit{adm}(F)$ and $\forall a \in A$ defended by S in F, $a \in S$

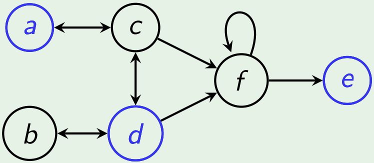
Background (ctd.)

Example



Background (ctd.)

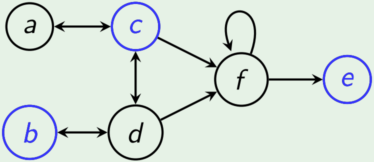
Example



$pref(F) = \{\{a, d, e\},$

Background (ctd.)

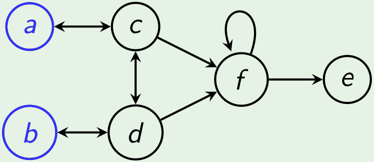
Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\},$

Background (ctd.)

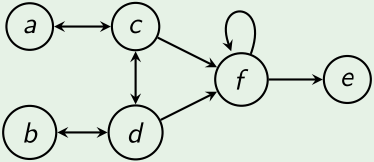
Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$

Background (ctd.)

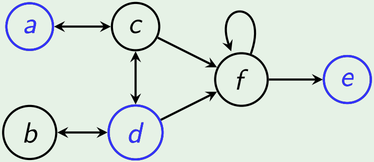
Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$

Background (ctd.)

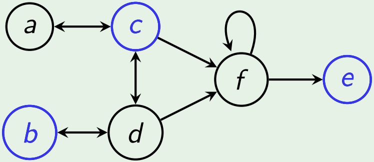
Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$
$sem(F) = stb(F) = \{\{a, d, e\},$

Background (ctd.)

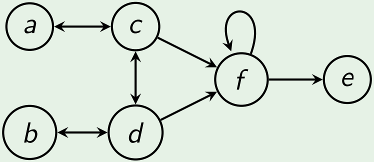
Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$
$sem(F) = stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$

Background (ctd.)

Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$
$sem(F) = stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$

Signatures

Definition

The signature of a semantics σ is defined as

$$\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF }\}.$$

Thus signatures capture all what a semantics can express.

Signatures

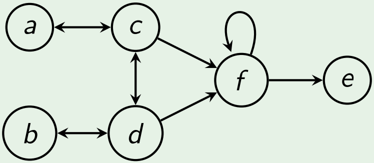
Definition

The signature of a semantics σ is defined as

$$\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF }\}.$$

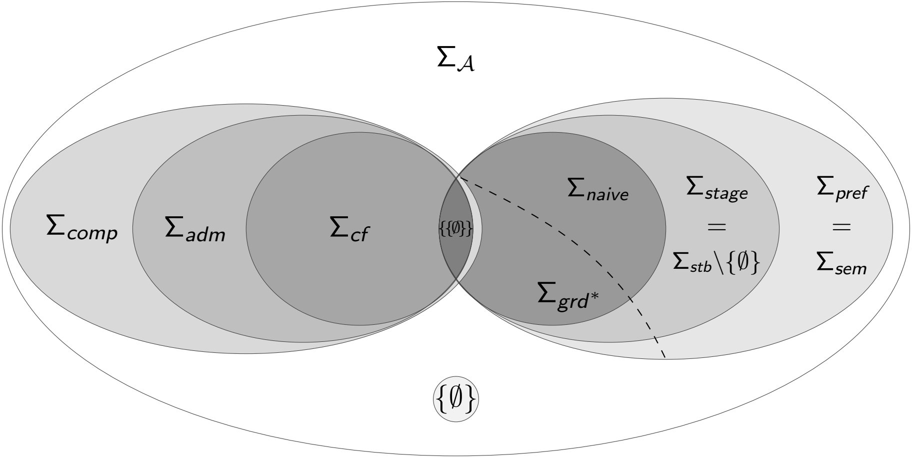
Thus signatures capture all what a semantics can express.

Example



- $\mathcal{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{pref}$
- Natural Questions: $\mathcal{S} \in \Sigma_{sem}$? $\mathcal{S} \in \Sigma_{stb}$?

Signatures (ctd.)



Signatures (ctd.)

Some Notation

Call a set of sets of arguments $\mathcal{S}$ extension-set. Moreover,

- $Args_{\mathcal{S}} = \bigcup_{S \in \mathcal{S}} S$
- $Pairs_{\mathcal{S}} = \{\{a, b\} \mid \exists E \in \mathcal{S} \text{ with } \{a, b\} \subseteq E\}$

Definition

An extension-set $\mathcal{S}$ is called conflict-sensitive if for each $A, B \in \mathcal{S}$ such that $A \cup B \notin \mathcal{S}$ it holds that $\exists a, b \in A \cup B : \{a, b\} \notin Pairs_{\mathcal{S}}$.

Example

Given $\mathcal{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$:
$Args_{\mathcal{S}} = \{a, b, c, d, e\}$
$Pairs_{\mathcal{S}} = \{\{a, b\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, e\}, \{c, e\}, \{d, e\}\}$

Signatures (ctd.)

Some Notation

Call a set of sets of arguments $\mathcal{S}$ extension-set. Moreover,

- $Args_{\mathcal{S}} = \bigcup_{S \in \mathcal{S}} S$
- $Pairs_{\mathcal{S}} = \{\{a, b\} \mid \exists E \in \mathcal{S} \text{ with } \{a, b\} \subseteq E\}$

Definition

An extension-set $\mathcal{S}$ is called conflict-sensitive if for each $A, B \in \mathcal{S}$ such that $A \cup B \notin \mathcal{S}$ it holds that $\exists a, b \in A \cup B : \{a, b\} \notin Pairs_{\mathcal{S}}$.

Example

Given $\mathcal{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$:
$Args_{\mathcal{S}} = \{a, b, c, d, e\}$
$Pairs_{\mathcal{S}} = \{\{a, b\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, e\}, \{c, e\}, \{d, e\}\}$
Observation: $\mathcal{S}$ is conflict-sensitive; $\{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$ is not!

Signatures (ctd.)

Proposition

For any AF F, $\mathit{pref}(F)$ is conflict-sensitive

Signatures (ctd.)

Proposition

For any AF F, $pref(F)$ is conflict-sensitive

Recall: $\mathcal{S}$ is conflict-sensitive if for each $A, B \in \mathcal{S}$ such that $A \cup B \notin \mathcal{S}$ it holds that $\exists a, b \in A \cup B : (a, b) \notin Pairs_{\mathcal{S}}$.

Proof:

1. Let F be an AF. $adm(F)$ is conflict-sensitive: Suppose $B, C \in adm(F)$ such that $B \cup C \notin adm(F)$, but for all $b, c \in B \cup C$, $(b, c) \in Pairs_{adm(F)}$. $B \cup C$ defends itself in F. Thus, $(b, c) \in R_F$ for some pair $\{b, c\} \subseteq B \cup C$. But then, for all $D \in adm(F)$, $\{b, c\} \not\subseteq D$. Hence, $\{b, c\} \notin Pairs_{adm(F)}$, a contradiction.
2. For any conflict-sensitive $\mathcal{S}$, its subset-maximal elements form a set $\mathcal{S}'$ that is conflict-sensitive, too (follows from $Pairs_{\mathcal{S}} = Pairs_{\mathcal{S}'}$).

Signatures (ctd.)

Proposition

For any non-empty, incomparable conflict-sensitive extension set $\mathcal{S}$, there exists an AF F, such that $pref(F) = \mathcal{S}$.

Theorem

$\Sigma_{pref} = \{\mathcal{S} \mid \mathcal{S} \neq \emptyset$ is incomparable and context-sensitive $\}$.

Signatures (ctd.)

Proposition

For any non-empty, incomparable conflict-sensitive extension set $\mathcal{S}$, there exists an AF F, such that $\mathit{pref}(F) = \mathcal{S}$.

Theorem

$\Sigma_{\mathit{pref}} = \{\mathcal{S} \mid \mathcal{S} \neq \emptyset$ is incomparable and context-sensitive $\}$.

Important Consequence

Call a semantics $\sigma : AF_{\mathcal{A}} \to 2^{2^{\mathcal{A}}}$ reasonable I-maximal if, for any AF F: (i) $\sigma(F) \neq \emptyset$; (ii) $\sigma(F) \subseteq \mathit{cf}(F)$; (iii) $\sigma(F)$ is incomparable; and (iv) for all $S_1, S_2 \in \sigma(F)$ $(S_1 \neq S_2)$ there exist $a, b \in S_1 \cup S_2$ with $(a, b) \in R$.

For any reasonable I-maximal semantics σ, $\Sigma_\sigma \subseteq \Sigma_{\mathit{pref}}$.

Signatures (ctd.)

Proposition (Conjoining)

For any AFs F_1, F_2 such that $\mathcal{S} = \mathit{pref}(F_1) \cap \mathit{pref}(F_2) \neq \emptyset$ there exists an AF F with $\mathit{pref}(F) = \mathcal{S}$.

Signatures (ctd.)

Proposition (Conjoining)

For any AFs F_1, F_2 such that $\mathcal{S} = \mathit{pref}(F_1) \cap \mathit{pref}(F_2) \neq \emptyset$ there exists an AF F with $\mathit{pref}(F) = \mathcal{S}$.

Proof Sketch:
$\mathcal{S} \neq \emptyset$ by assumption and that $\mathcal{S}$ is incomparable, is easy to see. We have to show that $\mathcal{S}$ is conflict-sensitive.
Take any $A, B \in \mathcal{S}$, $A \neq B$. We show that $\exists a, b \in A \cup B : (a, b) \notin \mathit{Pairs}_{\mathcal{S}}$.
Clearly, $A, B \in \mathit{pref}(F_1)$ and we know $\mathcal{S}_1 = \mathit{pref}(F_1)$ is conflict-sensitive.
Thus, $\exists a, b \in A \cup B : (a, b) \notin \mathit{Pairs}_{\mathcal{S}_1}$. Since $\mathcal{S}_1 \supseteq \mathcal{S}$, $\mathit{Pairs}_{\mathcal{S}_1} \supseteq \mathit{Pairs}_{\mathcal{S}}$.

Signatures (ctd.)

Proposition (Conjoining)

For any AFs F_1, F_2 such that $\mathcal{S} = \mathit{pref}(F_1) \cap \mathit{pref}(F_2) \neq \emptyset$ there exists an AF F with $\mathit{pref}(F) = \mathcal{S}$.

Proof Sketch:
$\mathcal{S} \neq \emptyset$ by assumption and that $\mathcal{S}$ is incomparable, is easy to see. We have to show that $\mathcal{S}$ is conflict-sensitive.
Take any $A, B \in \mathcal{S}$, $A \neq B$. We show that $\exists a, b \in A \cup B : (a, b) \notin \mathit{Pairs}_{\mathcal{S}}$.
Clearly, $A, B \in \mathit{pref}(F_1)$ and we know $\mathcal{S}_1 = \mathit{pref}(F_1)$ is conflict-sensitive.
Thus, $\exists a, b \in A \cup B : (a, b) \notin \mathit{Pairs}_{\mathcal{S}_1}$. Since $\mathcal{S}_1 \supseteq \mathcal{S}$, $\mathit{Pairs}_{\mathcal{S}_1} \supseteq \mathit{Pairs}_{\mathcal{S}}$.

Remark: Conjoining holds for all major semantics except complete.

Compact Argumentation Frameworks

Definition

Given a semantics σ, the set of compact argumentation frameworks under σ is defined as $CAF_\sigma = \{F \mid Args_{\sigma(F)} = A_F\}$. We call an AF $F \in CAF_\sigma$ σ-compact.

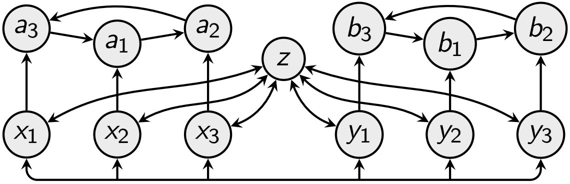
In words, σ-compact AFs guarantee that each of its arguments appears in at least one σ-extension.

σ-compact AFs are a candidate for a normal form, since they cannot be simplified (wrt. to the number of arguments).

Compact Argumentation Frameworks (ctd.)

Observation

$CAF_{sem} \subset CAF_{pref}$.



$pref(F) = \{\{z\}, \{x_1, a_1\}, \{x_2, a_2\}, \{x_3, a_3\}, \{y_1, b_1\}, \{y_2, b_2\}, \{y_3, b_3\}\}$.
$sem(F) = (pref(F) \setminus \{\{z\}\})$.

Compact Argumentation Frameworks (ctd.)

What about complexity?

- Credulous acceptance holds by definition
- Skeptical acceptance holds only for isolated arguments

Compact Argumentation Frameworks (ctd.)

What about complexity?

- Credulous acceptance holds by definition
- Skeptical acceptance holds only for isolated arguments

Theorem

For $\sigma \in \{pref, sem\}$, given $F \in CAF_\sigma$ and S, deciding whether $S \in \sigma(F)$ remains coNP-complete.

Compact Argumentation Frameworks (ctd.)

What about complexity?

- Credulous acceptance holds by definition
- Skeptical acceptance holds only for isolated arguments

Theorem

For $\sigma \in \{pref, sem\}$, given $F \in CAF_\sigma$ and S, deciding whether $S \in \sigma(F)$ remains coNP-complete.

Theorem

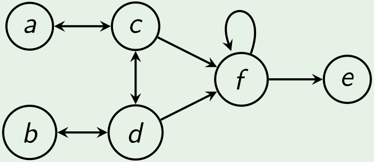
Given AF F, deciding whether $F \in CAF_{pref}$ (resp. $F \in CAF_{sem}$) is NP-complete (resp. Σ_2^P-complete).

Compact Argumentation Frameworks (ctd.)

Definition

Strict Signatures for semantics σ: $\Sigma^c_\sigma = \{\sigma(F) \mid F \in CAF_\sigma\}$

Example



Recall $pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{pref}$.
However, $pref(F)$ cannot be expressed without f: e occurs together with a, b, c, d in $pref(F)$, thus in an AF over arguments $\{a, b, c, d, e\}$, e would be isolated and thus contained in any (or none) preferred extension.
It follows that $\Sigma^c_{pref} \subset \Sigma_{pref}$.

Implicit Conflicts

Definition

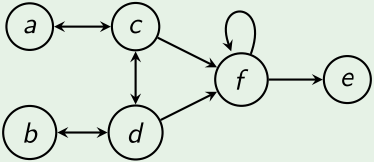
Given AF $F = (A, R)$, $a, b \in A$, σ a semantics. $\{a, b\}$ is a σ-implicit conflict if $\{a, b\} \notin Pairs_{\sigma(F)}$, $(a, b) \notin R$ and $(b, a) \notin R$.

Implicit Conflicts

Definition

Given AF $F = (A, R)$, $a, b \in A$, σ a semantics. $\{a, b\}$ is a σ-implicit conflict if $\{a, b\} \notin Pairs_{\sigma(F)}$, $(a, b) \notin R$ and $(b, a) \notin R$.

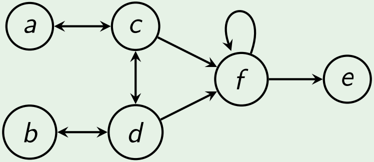
Example



- Recall $pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ and $stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$.
- Implicit conflicts for *pref*: $\{a, f\}$, $\{b, f\}$
- Implicit conflicts for *stb*: $\{a, b\}$, $\{a, f\}$, $\{b, f\}$

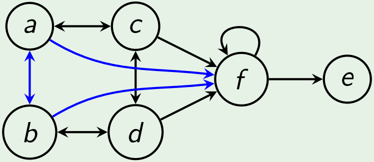
Implicit Conflicts (ctd.)

Making Implicit Conflicts Explicit



Implicit Conflicts (ctd.)

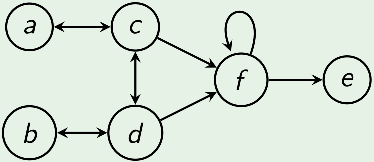
Making Implicit Conflicts Explicit



We have $stb(F) = stb(F') = \{\{a, d, e\}, \{b, c, e\}\}$.

Implicit Conflicts (ctd.)

Making Implicit Conflicts Explicit



But: there exists no AF $F' = (A, R')$ such that $pref(F') = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ and F' is free of *pref*-implicit conflicts.

Implicit Conflicts (ctd.)

Conflict-Explicit Conjecture (Stable Case)

For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ such that $stb(F) = stb(F')$ and F' is free of *stb*-implicit conflicts.

Implicit Conflicts (ctd.)

Conflict-Explicit Conjecture (Stable Case)

For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ such that $stb(F) = stb(F')$ and F' is free of *stb*-implicit conflicts.

Some remarks:

- There exist examples where just adding attacks to R is not sufficient
- Given the above conjecture holds, we have an exact characterization for Σ^c_{stb}

Summary

We have analysed argumentation semantics with respect to their capabilities in terms of

- realizing sets of extensions
- making use of "auxiliary" arguments
- eliminating implicit conflicts

Summary

We have analysed argumentation semantics with respect to their capabilities in terms of

- realizing sets of extensions
- making use of “auxiliary” arguments
- eliminating implicit conflicts

Why important?

- Theory: comparison of semantics
- Practice: insights can be exploited to tune systems

Future Research Directions

- So far, we have not compared semantics in terms of their "space-efficiency"
- Exact characterizations for complete semantics and for compact signatures
- Labellings would provide a closer link between signatures and compact signatures, but results are still missing
- Normalforms: minimize arguments (compact AFs in the best case) and implicit conflicts (for stable, there is hope that all implicit conflicts can be removed)