

From Intractability to Inconceivability ¹

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Workshop on New Trends in Formal Argumentation 2017

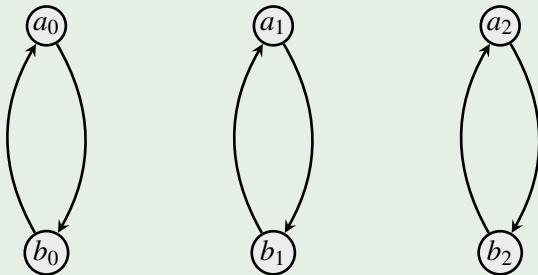
FWF

Der Wissenschaftsfonds.

¹This research has been supported by FWF (project I1102).

Simple Problems...

Example

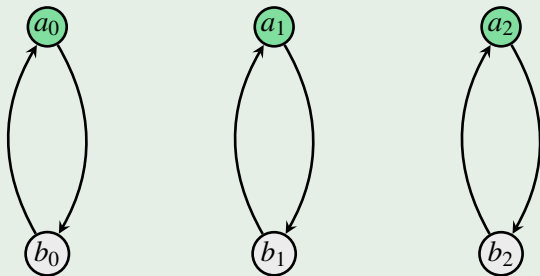


Question

What is some preferred extension?

Simple Problems...

Example



Question

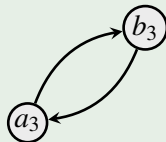
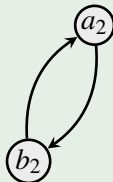
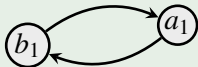
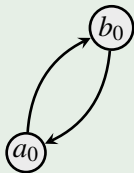
What is some preferred extension?

Answer

The set $\{a_0, a_1, a_2\}$ is a preferred extension.

Simple Problems...

Example

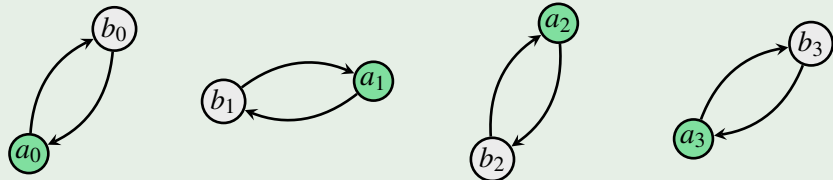


Question

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Simple Problems...

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Question

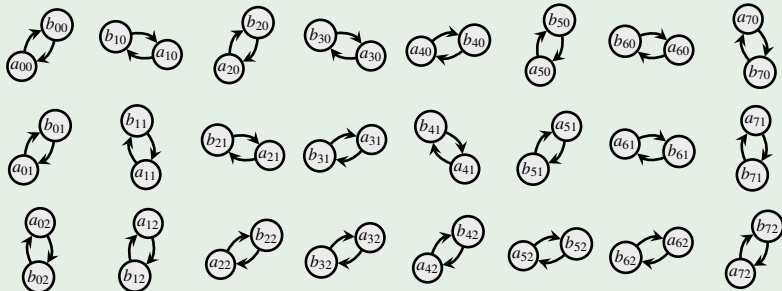
What is some preferred extension?

Answer

The set $\{a_0, a_1, a_2, a_3\}$ is a preferred extension.

Simple Problems..., ...

Example

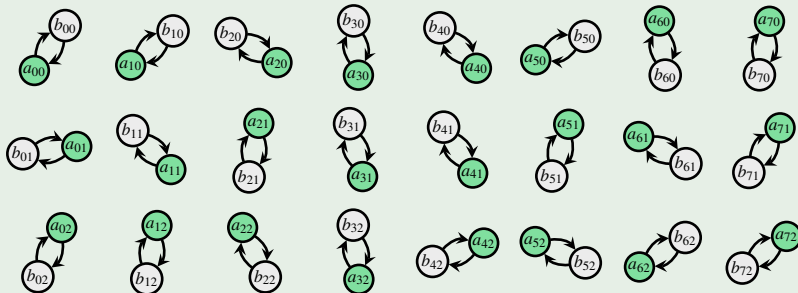


Question

What is some preferred extension?

Simple Problems..., ...

Example



Question

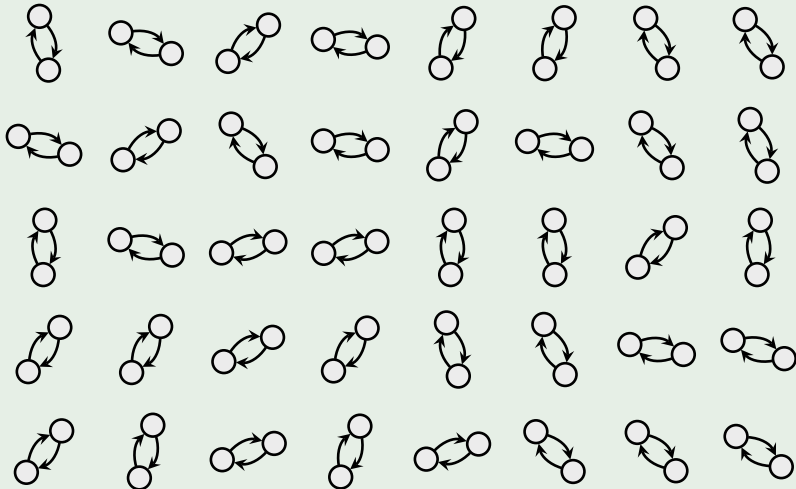
What is some preferred extension?

Answer

The set $\{a_{ij} : i \in \{0, 1, \dots, 7\}, j \in \{0, 1, 2\}\}$ is a preferred extension.

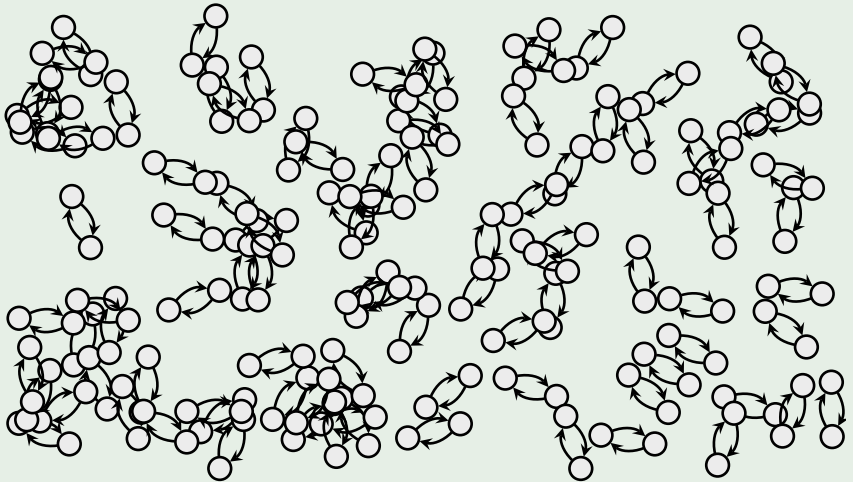
Simple Problems..., ..., ?

Example



Simple Problems..., ..., ?

Example



Tractable vs. Intractable

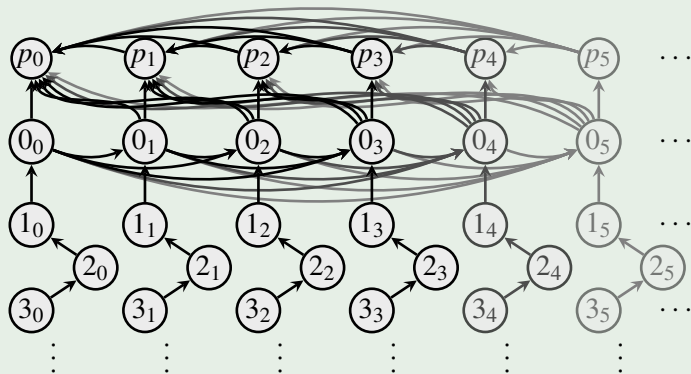
	<i>pr</i>	<i>sm</i>	<i>st</i>	<i>sg</i>	<i>na</i>	<i>c2</i>	<i>s2</i>
Ver_σ	-	-	✓	-	✓	✓	-
Cred_σ	-	-	-	-	✓	-	-
Skept_σ	-	-	-	-	✓	-	-
EX_σ	✓	✓	-	✓	✓	✓	✓
NEX_σ	-	-	-	✓	✓	✓	✓

- ASPIC Variants [Modgil and Prakken, 2014]
- Automata [Baroni et al., 2013]
- Logic Programming [García and Simari, 2004]
- Structured Argumentation . . .

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- Set Theoretic Approach for Arbitrary Infinities
 - Zermelo-Fraenkel Set Theory
 - Axiom of Choice, Zorn's Lemma, Well-Ordering Theorem
 - Transfinite Induction
 - Bourbaki-Witt

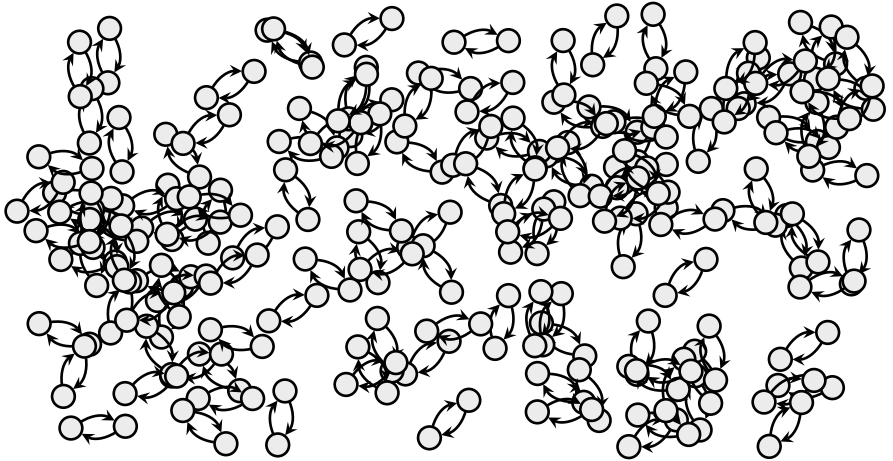
Collapse I

Example



Collapse of stable, semi-stable, stage, cf2, stage2 semantics in ZFC.

Simple Problems?

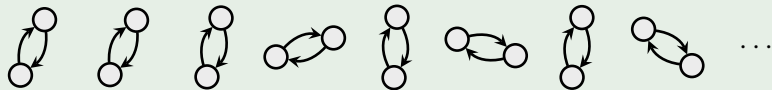


Simple Problems?



Collapse II

Example



Possible collapse of stable, semi-stable, stage, cf2, stage2, preferred, naive semantics in ZF, i.e. models of ZF where AC does not hold.

Collapse and Perfection

	<i>co</i>	<i>na</i>	<i>pr</i>	<i>st</i>	<i>sg</i>	<i>sm</i>	<i>c2</i>	<i>s2</i>	<i>gr</i>	<i>id</i>	<i>eg</i>
well-founded	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
bipartite	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
finite	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	✓
limited controversial	✓	AC	AC	AC	AC	AC	AC	AC	✓	AC	AC
symmetric loop-free	✓	AC	AC	AC	AC	AC	AC	AC	✓	AC	AC
finitary	✓	AC	AC	-	AC	AC	?	?	✓	AC	AC
symmetric	✓	AC	AC	-	-	-	AC	-	✓	AC	AC
planar	✓	AC	AC	-	?	-	?	?	✓	AC	AC
finitely superseded	✓	AC	AC	-	-	-	-	-	✓	AC	AC
finitarily superseded	✓	AC	AC	-	-	-	-	-	✓	AC	AC
arbitrary	✓	AC	AC	-	-	-	-	-	✓	AC	AC

Table: Perfection results.

Question

In the infinite case:

- *Computational Complexity*
- *Intertranslatability*
- *Signatures*

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- ~~*Computational Complexity*~~
- *Intertranslatability*
- *Signatures*

Question

In the infinite case:

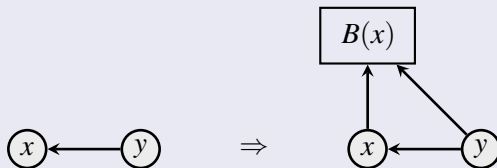
- *Computational Complexity*
- *Intertranslatability*
- *Signatures*

Example

- Admissibility based semantics widely yield the same comparability, regardless of ZF or ZFC;
- In ZF, given extension set $\{0, 1\}^\omega$ we can give an AF with matching semantic evaluation;
- In ZF, a collection of pairs of arguments with symmetric conflicts might not provide maximal extensions;
- How do cf-based semantics compare?

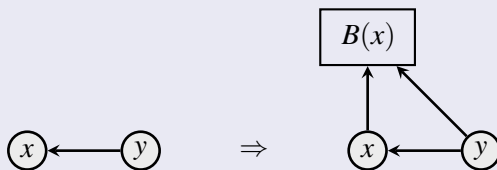
Facilitating Collapse for Translations

Definition

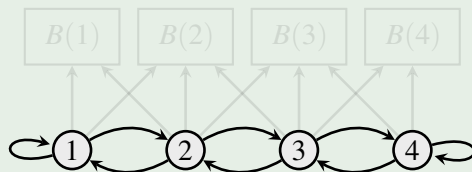


Facilitating Collapse for Translations

Definition

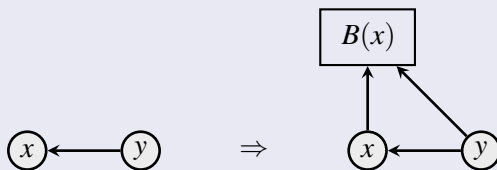


Example

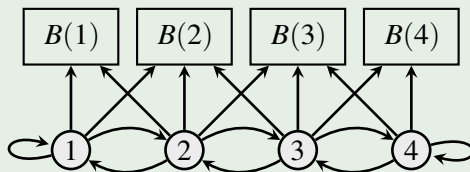


Facilitating Collapse for Translations

Definition

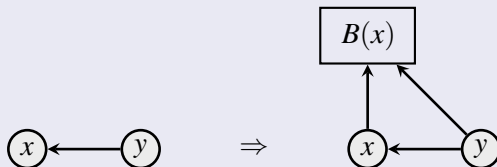


Example



Facilitating Collapse for Translations

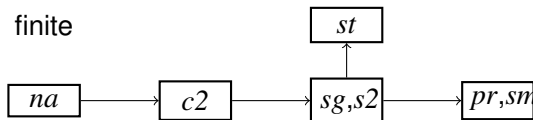
Definition



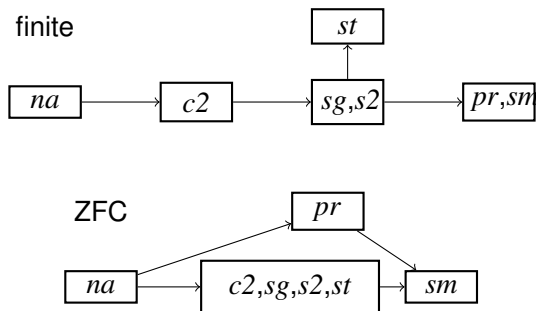
Theorem

- *In ZFC stable, stage, cf2 and stage2 semantics provide the same expressiveness.*
- *In ZF without AC even naive, stable, stage, cf2 and stage2 are comparable.*

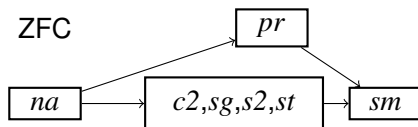
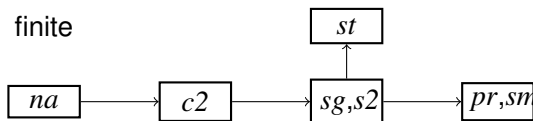
Expressiveness



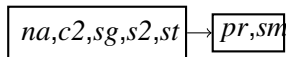
Expressiveness



Expressiveness



ZF



- The possibility of collapse can be considered a valuable tool.
- Inconceivable (i.e. collapsing) subframeworks can enforce other extensions.
- Similarly, can we make use of intractability for expressiveness in terms of tractable extensions?
- For general (finite) AFs, can we tractably detect intractability of subframeworks?

References



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