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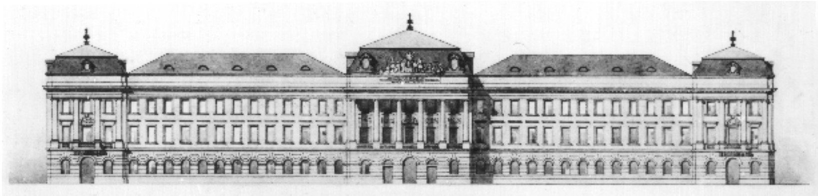

INSTITUT FÜR LOGIC AND COMPUTATION

ABTEILUNG DATENBANKEN UND ARTIFICIAL INTELLIGENCE

Equivalence in Argumentation Frameworks with a Claim-centric View – Classical Results With Novel Ingredients

DBAI-TR-2021-122

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DBAI TECHNICAL REPORT
2021

TECHNISCHE UNIVERSITÄT WIEN
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Ringo Baumann [1] **Anna Rapberger** [2] **Markus Ulbricht** [3]

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Abstract

A common feature of non-monotonic logics is that the classical notion of equivalence does not preserve the intended meaning in light of additional information. Consequently, the term strong equivalence was coined in the literature and thoroughly investigated. In the present paper, the knowledge representation formalism under consideration are claim-augmented argumentation frameworks (CAFs) which provide a formal basis to analyze conclusion-oriented problems in argumentation by adapting a claim-focused perspective. CAFs extend Dung AFs by associating a claim to each argument representing its conclusion. In this paper, we investigate both ordinary and strong equivalence in CAFs. Thereby, we take the fact into account that one might either be interested in the actual arguments or their claims only. The former point of view naturally yields an extension of strong equivalence for AFs to the claim-based setting while the latter gives rise to a novel equivalence notion which is genuine for CAFs. We tailor, examine and compare these notions and obtain a comprehensive study of this matter for CAFs. We conclude by investigating the computational complexity of naturally arising decision problems.

1 Introduction

Equivalence is an important subject of research in knowledge representation and reasoning. Given a knowledge base $\mathcal{K}$, finding an equivalent one, say $\mathcal{K}'$, helps to obtain a better understanding or more concise representation of $\mathcal{K}$. From a computational point of view, equivalence is particularly interesting whenever a certain subset of a collection of information can be replaced without changing the intended meaning. In propositional logics, for example, replacing a subformula ϕ of Φ with an equivalent one, say ϕ', yields a formula $\Phi[\phi/\phi']$ equivalent to Φ. That is, we may view ϕ as an independent module of Φ. Within the KR community it is folklore that this is usually not the case for non-monotonic logics (apart from folklore, we refer the reader to (Baumann and Strass 2016) for a rigorous study of this matter).

Motivated by this observation, the notion of strong equivalence was introduced in the literature. In a nutshell, strong equivalence requires the aforementioned property by design: $\mathcal{K}$ and $\mathcal{K}'$ are strongly equivalent if for any $\mathcal{H}$, the knowledge bases $\mathcal{K} \cup \mathcal{H}$ and $\mathcal{K}' \cup \mathcal{H}$ are equivalent. Although a naive implementation would require to iterate over an infinite number of possible $\mathcal{H}$, researchers discovered techniques to decide strong equivalence of two knowledge bases efficiently, most notably for logic programming (Lifschitz, Pearce, and Valverde 2001) and argumentation frameworks (AFs) (Oikarinen and Woltran 2011). In this paper, we extend this line of research to a recent extension of AFs, called Claim-augmented argumentation frameworks (CAFs).

Abstract argumentation frameworks as proposed by Dung (Dung 1995) in his seminal 1995 paper are by now a major research area in knowledge representation and reasoning. They have been thoroughly investigated since then and various extensions have been proposed in order to extend their expressive power. For example, researchers considered the addition of supports (Cayrol and Lagasquie-Schiex 2005), recursive (Baroni et al. 2011) and collective (Nielsen and Parsons 2006) attacks, or probabilities (Thimm 2012) to mention a few. CAFs as introduced by (Dvořák and Woltran 2020) provide means for conclusion-oriented reasoning in argumentation. While traditional argumentation formalisms focus on the identification of acceptable arguments, the emphasis in claim-augmented argumentation lies instead on the argument's conclusions (*claims*). Building on the basic observation that a claim can be supported by different arguments, it becomes evident that the traditional argument-focused perspective is often insufficient to capture claim-based reasoning. CAFs address this issue by extending AFs with a function which assigns a claim to each argument. They are in particular well-suited to analyze instantiation-based approaches, e.g., instantiations of logic programs (Caminada et al. 2015b), rule-based formalisms like ABA+ (Bondarenko, Toni, and Kowalski 1993; Caminada et al. 2015a), or logic-based instantiations (Besnard and Hunter 2001; Gorogiannis and Hunter 2011), where the focus lies on the claims of the arguments which have been constructed during the process.

The goal of this paper is to investigate equivalence notions for reasoning with a claim-centered point of view. Due to their generality, CAFs form an ideal basis to obtain a comprehensive study of this matter. Our main contributions are:

- We provide characterization results of strong equivalence between CAFs via semantics-dependent kernels for each CAF semantics which has been considered in the literature so far. Moreover, we discuss ordinary equivalence for CAFs and present dependencies between semantics for this weaker equivalence notion.

- We introduce novel equivalence concepts based on argument renaming which are genuine for CAFs. We show that ordinary equivalence up to renaming coincides with ordinary equivalence while strong equivalence up to renaming can be characterized via kernel isomorphism.
- We present a rigorous complexity analysis of deciding equivalence between two CAFs for all of the aforementioned equivalence notions. We show that deciding ordinary equivalence can be computationally hard, up to the third level of the polynomial hierarchy while strong equivalence is computationally tractable. Moreover, we show that strong equivalence up to renaming has the same complexity as the graph isomorphism problem.

Full proofs can be found in the appendix.

2 Background

Abstract Argumentation. We fix a non-finite background set $\mathcal{U}$. An argumentation framework (AF) (Dung 1995) is a directed graph $F = (A, R)$ where $A \subseteq \mathcal{U}$ represents a set of arguments and $R \subseteq A \times A$ models *attacks* between them. In this paper we consider finite AFs only.

For two arguments $a, b \in A$, if $(a, b) \in R$ we say that a *attacks* b as well as a *attacks* (the set) E given that $b \in E \subseteq A$. We frequently use the so-called *range* of a set E defined as $E^{\oplus}_F = E \cup E^+_F$ where $E^+_F = \{a \in A \mid E \text{ attacks } a\}$.

A set $E \subseteq A$ is conflict-free in F (for short, $E \in cf(F)$) iff for no $a, b \in E$, $(a, b) \in R$. A set E *defends* an argument a if any attacker of a is attacked by some argument of E. A *semantics* is a function $\sigma : \mathcal{F} \to 2^{2^{\mathcal{U}}}$ with $F \mapsto \sigma(F) \subseteq 2^A$. This means, given an AF $F = (A, R)$ a semantics returns a set of subsets of A. These subsets are called σ-*extensions*.

In this paper we consider so-called *naive*, *admissible*, *complete*, *grounded*, *preferred*, *stable*, *semi-stable* and *stage* semantics (abbr. na, ad, co, gr, pr, stb, ss, stg). Apart from naive, semi-stable and stage semantics (Verheij 1996; Caminada 2006), all mentioned semantics were already introduced in (Dung 1995).

Definition 2.1. Let $F = (A, R)$ be an AF and $E \in cf(F)$.

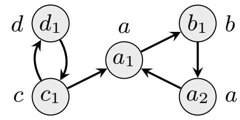
1. $E \in na(F)$ iff E is $\subseteq$-maximal in $cf(F)$,
2. $E \in ad(F)$ iff E defends all its elements,
3. $E \in co(F)$ iff $E \in ad(F)$ and for any a defended by E we have, $a \in E$,
4. $E \in gr(F)$ iff E is $\subseteq$-minimal in $co(F)$, and
5. $E \in pr(F)$ iff E is $\subseteq$-maximal in $ad(F)$,
6. $E \in stb(F)$ iff $E \in cf(A)$ and E attacks any $a \in A \setminus E$,
7. $E \in ss(F)$ iff $E \in ad(F)$ and there is no $D \in ad(F)$ with $E^{\oplus}_F \subsetneq D^{\oplus}_F$,
8. $E \in stg(F)$ iff $E \in cf(F)$ and there is no $D \in cf(F)$ with $E^{\oplus}_F \subsetneq D^{\oplus}_F$.

Claim-based Argumentation. A *claim-augmented argumentation framework (CAF)* (Dvorák and Woltran 2020) is a triple $\mathcal{F} = (A, R, cl)$ where $F = (A, R)$ is an AF and $cl : A \to \mathcal{C}$ is a function which assigns a claim to each argument in A; $\mathcal{C}$ is a set of (countable infinite) possible claims. The claim-function is extended to sets in the natural way, i.e. for a set $E \subseteq A$, we let $cl(E) = \{cl(a) \mid a \in E\}$.

There are several ways in which semantics for AFs extend to CAFs. The most basic one is to choose an appropriate AF semantics and consider the claims of the induced extensions.

Definition 2.2. For a CAF $\mathcal{F} = (A, R, cl)$, $F = (A, R)$, and a semantics σ, we define the inherited variant of σ *(i-σ)* as $\sigma_c(\mathcal{F}) = \{cl(E) \mid E \in \sigma(F)\}$. We call $E \in \sigma(F)$ with $cl(E) = S$ a σ_c-realization of S in $\mathcal{F}$.

Example 2.3. Consider the following CAF $\mathcal{F}$:



Let us focus on stable semantics. For the underlying AF F we have the unique stable extension $E = \{c_1, b_1\}$. It is thus easy to see that $stb_c(\mathcal{F}) = \{\{c, b\}\}$. Moreover, $\{c_1, b_1\}$ is a stb_c-realization of E.

Let us now turn to the semantics which actually operate on the level of the claims instead of focusing on the underlying arguments. For this, we need to generalize the notion of defeat to claims. A set of arguments $E \subseteq A$ *defeats* a claim $c \in cl(A)$ in $\mathcal{F}$ if E attacks every $a \in A$ with $cl(a) = c$ (in F); we write $E^{\ast}_{\mathcal{F}} = \{c \in cl(A) \mid E \text{ defeats } c \text{ in } \mathcal{F}\}$ to denote the set of all claims which are defeated by E in $\mathcal{F}$. The claim-range of a set of claims $S = cl(E)$ is denoted by $E^{\circledast}_{\mathcal{F}} = cl(E) \cup E^{\ast}_{\mathcal{F}}$.

Example 2.4. Consider again the CAF $\mathcal{F}$ from the previous example. Although c_1 defeats a_1, it does not defeat the claim a. However, $E = \{c_1, b_1\}$ defeats a, i.e. $a \in E^{\ast}_{\mathcal{F}}$. The claim-range of E is thus $E^{\circledast}_{\mathcal{F}} = \{a, b, c, d\}$.

Observe that the range of a set of claims is not a well-defined concept: In our example CAF $\mathcal{F}$, the claim-range of $\{a\}$ could either be $\{a, b\}$ induced by the realization $\{a_1\}$ or it could be $\{a\}$, which is induced by the realization $\{a_2\}$. Nonetheless, we can define semantics based on the claim-range by focusing on the underlying set E of arguments. We consider *cl-preferred*, *cl-naive*, *cl-cf-stable*, *cl-ad-stable*, *cl-semi-stable* and *cl-stage* semantics (abbr. $cl\text{-}pr$, $cl\text{-}na$, $cl\text{-}stb_{cf}$, $cl\text{-}stb_{ad}$, $cl\text{-}ss$, $cl\text{-}stg$) as introduced in (Rapberger 2020; Dvorák, Rapberger, and Woltran 2020a).

Definition 2.5. Let $\mathcal{F} = (A, R, cl)$ be a CAF with underlying AF $F = (A, R)$. For a set of claims $S \subseteq cl(A)$,

- $S \in cl\text{-}pr(\mathcal{F})$ if S is $\subseteq$-maximal in $ad_c(\mathcal{F})$;
- $S \in cl\text{-}na(\mathcal{F})$ if S is $\subseteq$-maximal in $cf_c(\mathcal{F})$;
- $S \in cl\text{-}stb_\tau(\mathcal{F})$, $\tau \in \{cf, ad\}$, if there is a τ_c-realization E of S which defeats any $c \in cl(A) \setminus S$ (i.e., $E^{\circledast}_{\mathcal{F}} = cl(A)$);
- $S \in cl\text{-}ss(\mathcal{F})$ if there is an ad_c-realization E of S in $\mathcal{F}$ such that there is no $D \in ad(F)$ with $E^{\circledast}_{\mathcal{F}} \subsetneq D^{\circledast}_{\mathcal{F}}$;
- $S \in cl\text{-}stg(\mathcal{F})$ if there is an cf_c-realization E of S in $\mathcal{F}$ such that there is no $D \in cf(F)$ with $E^{\circledast}_{\mathcal{F}} \subsetneq D^{\circledast}_{\mathcal{F}}$.

A set $E \subseteq A$ *cl-σ-realizes* the claim-set S in $\mathcal{F}$ if $cl(E) = S$ and E satisfies the respective requirements; e.g., $E \in cf(F)$ and $E^{\circledast}_{\mathcal{F}} = cl(A)$ for cl-cf-stable semantics. We call E a cl-σ-realization of S in $\mathcal{F}$.

Example 2.6. Consider the semantics $cl\text{-}stb_{cf}$. We have that $S = \{c, b\} \in cl\text{-}stb_{cf}(\mathcal{F})$ since the realization $E = \{c_1, b_1\}$ for S has full claim-range as we already observed before. Moreover, $S' = \{d, a\} \in cl\text{-}stb_{cf}(\mathcal{F})$ as well: We consider the realization $E' = \{d_1, a_1\}$. The claims c and b are defeated by E' and hence, $E'^{\oplus}_{\mathcal{F}} = \{a, b, c, d\}$. Note that E' is not a stable extension of the underlying AF.

Basic relations between i-semantics carry over from AF semantics, e.g., $stb_c(\mathcal{F}) \subseteq ss_c(\mathcal{F}) \subseteq pr_c(CF) \subseteq co_c(CF) \subseteq ad_c(\mathcal{F}) \subseteq cf_c(\mathcal{F})$ and $stb_c(\mathcal{F}) \subseteq stg_c(\mathcal{F}) \subseteq na_c(\mathcal{F}) \subseteq cf_c(\mathcal{F})$. As shown in (Dvorák, Rapberger, and Woltran 2020a), we have $stb_c(\mathcal{F}) \subseteq cl\text{-}stb_{ad}(\mathcal{F}) \subseteq cl\text{-}stb_{cf}(\mathcal{F}) \subseteq cl\text{-}stg(\mathcal{F}) \subseteq na_c(\mathcal{F})$ and $cl\text{-}stb_{ad}(\mathcal{F}) \subseteq cl\text{-}ss(\mathcal{F}) \subseteq pr_c(\mathcal{F})$. Moreover, each $cl\text{-}\sigma$-claim-set of $\mathcal{F}$ is $\subseteq$-maximal in $\sigma_c(\mathcal{F})$ for $\sigma \in \{pr, na\}$.

Notation. We write $\mathcal{F} = (F, cl)$ as an abbreviation for $\mathcal{F} = (A, R, cl)$ with AF $F = (A, R)$ (similar for CAFs $\mathcal{G}$ or $\mathcal{H}$ for which we denote the corresponding AFs by G and H, respectively). Also, we use the subscript-notation $A_{\mathcal{F}}$, $R_{\mathcal{F}}$, $cl_{\mathcal{F}}$, and $F_{\mathcal{F}}$ to indicate the affiliations.

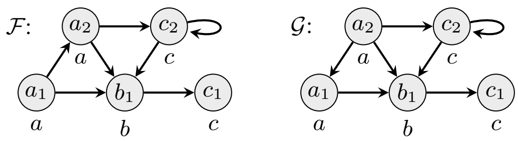
3 Equivalence in CAFs

In this section, we discuss ordinary and strong equivalence for CAFs. We introduce a novel kernel which characterizes strong equivalence for cl-cf-stable and cl-stage semantics; moreover, we show that the remaining semantics can be characterized via known kernels for AFs.

Let us start with ordinary equivalence of CAFs.

Definition 3.1. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *ordinary equivalent* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$, if $\rho(\mathcal{F}) = \rho(\mathcal{G})$.

Example 3.2. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:



Although $\mathcal{F}$ and $\mathcal{G}$ disagree only on the direction of the attack between the arguments a_1 and a_2, we observe that $\mathcal{F}$ and $\mathcal{G}$ are not ordinary equivalent under i-stable semantics: $stb_c(\mathcal{F}) = \emptyset$ while $\mathcal{G}$ has the unique i-stable claim-set $\{a, c\}$ witnessed by the stable extension $\{a_2, c_1\}$ of G.

If we consider instead cl-stable semantics, we observe that the two CAFs agree on their outcome: First notice that $\{a, c\}$ is also cl-ad-stable (cl-cf-stable) in $\mathcal{G}$ (every stb_c-realization is admissible and has full claim-range). Moreover, we have that $\{a, c\}$ is also cl-ad-stable (cl-cf-stable) in $\mathcal{F}$ since the set $\{a_1, c_1\}$ is admissible and defeats every remaining claim. As a side remark, we mention that the claim-set $\{a, c\}$ has two realizations in $\mathcal{F}$ and $\mathcal{G}$ since both of the sets $\{a_1, c_1\}$, $\{a_2, c_1\}$ are conflict-free and have full claim-range. We obtain that the CAFs $\mathcal{F}$ and $\mathcal{G}$ are ordinary equivalent with respect to $cl\text{-}stb_{ad}$ and $cl\text{-}stb_{cf}$ semantics.

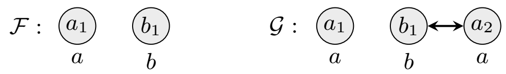
There are only few relations between the semantics for ordinary equivalence. We summarize them as follows:

Proposition 3.3. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$,*

- $\mathcal{F} \equiv^{\rho}_{o} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{cl\text{-}pr}_{o} \mathcal{G}$, $\rho \in \{ad_c, pr_c\}$;
- $\mathcal{F} \equiv^{co_c}_{o} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$, $\rho \in \{gr_c, cl\text{-}pr\}$;
- $\mathcal{F} \equiv^{cf_c}_{o} \mathcal{G} \Leftrightarrow \mathcal{F} \equiv^{cl\text{-}na}_{o} \mathcal{G}$;
- $\mathcal{F} \equiv^{na_c}_{o} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$, $\rho \in \{cf_c, cl\text{-}na\}$.

Interestingly, we observe that the relations for AF semantics presented in (Oikarinen and Woltran 2011) do not carry over to inherited semantics. This is due to the fact that i-preferred (i-naive) semantics are not necessarily $\subseteq$-maximal i-admissible (i-conflict-free) claim-sets; for CAFs, this role is instead taken over by cl-preferred (cl-naive) semantics.

Example 3.4. Assume we are given two CAFs as follows:

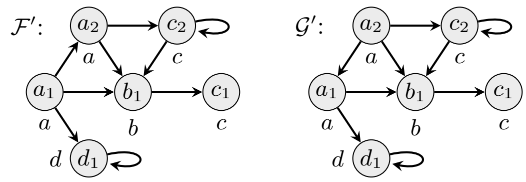


Clearly, $ad_c(\mathcal{F}) = ad_c(\mathcal{G}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. On the other hand, $\{a, b\}$ is the unique i-preferred claim-set of $\mathcal{F}$ while $pr_c(\mathcal{G}) = \{\{a\}, \{a, b\}\}$ witnessed by the extensions $\{a_1, a_2\}$ and $\{a_1, b_1\}$. Thus $\mathcal{F} \equiv^{ad_c}_{o} \mathcal{G} \not\Rightarrow \mathcal{F} \equiv^{pr_c}_{o} \mathcal{G}$. The example furthermore shows $\mathcal{F} \equiv^{cf_c}_{o} \mathcal{G} \not\Rightarrow \mathcal{F} \equiv^{na_c}_{o} \mathcal{G}$ since cf_c and ad_c as well as the respective variants of naive and preferred semantics coincide in $\mathcal{F}$ and $\mathcal{G}$.

The relations presented in Proposition 3.3 follow since cl-preferred claim-sets are $\subseteq$-maximal in $ad_c(\mathcal{F})$, $co_c(\mathcal{F})$ and $pr_c(\mathcal{F})$ for any CAF $\mathcal{F}$; moreover, the i-grounded claim-set is the $\subseteq$-minimal i-complete extension. Similar observations hold for conflict-free and naive semantics; additionally, we observe that $\mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$, $\rho \in \{cl\text{-}na, na_c\}$, implies $\mathcal{F} \equiv^{cf_c}_{o} \mathcal{G}$ since cf_c semantics satisfies downward closure (every subset of a conflict-free set is conflict-free). We can construct counter-examples for the remaining cases.

A crucial observation is that ordinary equivalence is not robust when it comes to expansion of the frameworks, e.g., if an update in the knowledge base induces new arguments or attacks. Let us illustrate this at the following example:

Example 3.5. Assume we are given an updated version of $\mathcal{F}$ and $\mathcal{G}$ from Example 3.2 where an additional argument has been introduced. Let $\mathcal{F}'$ and $\mathcal{G}'$ be given as follows:



$\mathcal{F}'$ and $\mathcal{G}'$ no longer agree on their cl-ad-stable claim-sets: In $\mathcal{G}'$, the set $\{a_2, c_1\}$ does not defeat claim d, thus $cl\text{-}stb_{ad}(\mathcal{G}') = \emptyset$ while $\{a, c\}$ remains cl-ad-stable in $\mathcal{F}'$.

Let us introduce a stronger notion of equivalence which addresses such situations. We say that two CAFs are *strongly equivalent* to each other if they possess the same extensions independently of any such (simultaneous) expansions of the frameworks. Before we can define this notion formally, we require an additional concept which ensures that the expansion of the frameworks is well-defined.

Definition 3.6. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *compatible* to each other if $cl_{\mathcal{F}}(a) = cl_{\mathcal{G}}(a)$ for all $a \in A_{\mathcal{F}} \cap A_{\mathcal{G}}$. The union $\mathcal{F} \cup \mathcal{G}$ of two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ is defined componentwise, i.e., $\mathcal{F} \cup \mathcal{G} = (A_{\mathcal{F}} \cup A_{\mathcal{G}}, R_{\mathcal{F}} \cup R_{\mathcal{G}}, cl_{\mathcal{F}} \cup cl_{\mathcal{G}})$.

We are ready to introduce strong equivalence for CAFs.

Definition 3.7. Two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ are *strongly equivalent* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv_s^\rho \mathcal{G}$, iff $\rho(\mathcal{F} \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ for each CAF $\mathcal{H}$ which is compatible with $\mathcal{F}$ and $\mathcal{G}$.

The definition extends strong equivalence for AFs. We write $F \equiv_s^\sigma G$ to denote strong equivalence of two AFs F and G w.r.t. the semantics σ.

Strong equivalence for AFs has been characterized via syntactic equivalence of so-called (semantics-dependent) kernels. Let us recall the definitions of the stable and the naive kernel (Oikarinen and Woltran 2011; Baumann, Linsbichler, and Woltran 2016) as they exhibit interesting overlaps with our novel kernel for cl-cf-stable semantics.

Definition 3.8. For an AF $F = (A, R)$, we define the *stable kernel* $F^{sk} = (A, R^{sk})$ with

$$R^{sk} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\};$$

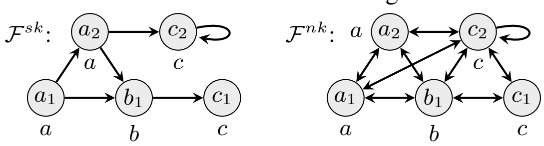
and the *naive kernel* $F^{nk} = (A, R^{nk})$ with

$$R^{nk} = R \cup \{(a, b) \mid a \neq b, \{(a, a), (b, b), (b, a)\} \cap R \neq \emptyset\}.$$

For a CAF $\mathcal{F} = (F, cl)$, we write $\mathcal{F}^{sk}$ ($\mathcal{F}^{nk}$) to denote (F^{sk}, cl) ((F^{nk}, cl), respectively).

The stable kernel characterizes strong equivalence for stable and stage semantics, i.e., $F \equiv_s^\sigma G$ iff $F^{sk} = G^{sk}$ for $\sigma \in \{stb, stg\}$ (Oikarinen and Woltran 2011); similarly, $F \equiv_s^\sigma G$ iff $F^{nk} = G^{nk}$ for $\sigma \in \{cf, na\}$ (Baumann, Linsbichler, and Woltran 2016).

Example 3.9. For the CAF $\mathcal{F}$ from Example 3.2, the stable kernel $\mathcal{F}^{sk}$ and the naive kernel $\mathcal{F}^{nk}$ are given as follows:



In the remaining part of this section, we characterize strong equivalence for all semantics under consideration by identifying appropriate kernels. Let us start with cl-cf-stable semantics. An interesting observation is that the CAFs $\mathcal{F}'$ and $\mathcal{G}'$ from Example 3.5 yield the same cl-cf-stable claim-sets even after the argument d_1 has been added. In fact, it can be shown that $\mathcal{F}$ and $\mathcal{G}$ yield the same cl-cf-stable claim-sets under any possible expansion. The reason is that the direction of the attack between a_1 and a_2 is irrelevant since both arguments possess the same claim a. Thus it suffices to include one of them in a cl-cf-stable claim-set in case not both of them are attacked.

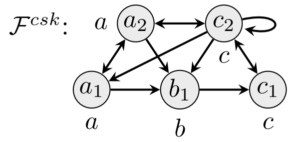
Let us now introduce the cf-stable kernel for CAFs.

Definition 3.10. For a CAF $\mathcal{F} = (A, R, cl)$, we define the cf-stable kernel as $\mathcal{F}^{csk} = (A, R^{csk}, cl)$ with

$$R^{csk} = R \cup \{(a, b) \mid a \neq b,$$
$$(a, a) \in R \vee (cl(a) = cl(b) \wedge \{(b, a), (b, b)\} \cap R \neq \emptyset)\}.$$

We denote the underlying AF (A, R^{csk}) by F^{csk}.

Example 3.11. Consider again our previous CAF $\mathcal{F}$. We construct the cf-stable kernel $\mathcal{F}^{csk}$ of $\mathcal{F}$ as follows:



Remark 3.12. The cf-stable kernel consists of a combination of the stable and the naive kernel for AFs, where the claim-independent part stems from the stable kernel while the case where two arguments have the same claim relates to the naive kernel. In a nutshell, it is save to introduce attacks (a, b), $a \neq b$ where a is self-attacking without changing stable semantics because attacks of this form neither interfere with the conflict-free extensions of an AF nor change the range of a conflict-free set. In case two arguments have the same claim, it is irrelevant which of these arguments is included in an extension. It is thus save to introduce attacks between two arguments in case their union is conflicting.

In what follows, we will prove that the cf-kernel characterizes strong equivalence for claim-level cf-stable and stage semantics. To this end we will first discuss some general observations. The following lemma states that two CAFs having different arguments are not strongly equivalent.

Lemma 3.13. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $A_{\mathcal{F}} \neq A_{\mathcal{G}}$ implies $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$ for any considered semantics ρ.*

Proof. W.l.o.g., we may assume that there is $a \in A_{\mathcal{F}}$ with $a \notin A_{\mathcal{G}}$. To prove the statement, we distinguish the following cases: (a) $(a, a) \notin R_{\mathcal{F}}$ and (b) $(a, a) \in R_{\mathcal{F}}$. We present the construction for case (a): For a fresh argument x and a fresh claim c, let $\mathcal{H} = (A_{\mathcal{H}}, R_{\mathcal{H}}, cl_{\mathcal{H}})$ with

$$A_{\mathcal{H}} = (A_{\mathcal{F}} \cup A_{\mathcal{G}} \cup \{x\}) \setminus \{a\};$$
$$R_{\mathcal{H}} = \{(x, b) \mid b \in (A_{\mathcal{F}} \cup A_{\mathcal{G}}) \setminus \{a\}\};$$

and $cl_{\mathcal{H}}(b) = cl_{\mathcal{F}}(b)$ for $b \in A_{\mathcal{F}} \cup A_{\mathcal{G}}$ and $cl_{\mathcal{H}}(x) = c$; that is, we introduce a new argument having a fresh claim c which attacks every argument except a. It can be checked that $\{cl_{\mathcal{H}}(a), c\} \in \rho(\mathcal{F} \cup \mathcal{H})$ for every semantics under consideration. Observe that $\{cl_{\mathcal{H}}(a), c\}$ is not a claim-extension under any semantics in $\mathcal{G} \cup \mathcal{H}$ since a is not present in $\mathcal{G} \cup \mathcal{H}$ and x does attack every remaining argument. $\square$

The following lemma implies that two strongly equivalent CAFs $\mathcal{F}$ and $\mathcal{G}$ possess the same self-attacking arguments.

Lemma 3.14. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $(a, a) \in R_{\mathcal{F}} \Delta R_{\mathcal{G}}$ implies $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$ for any semantics ρ under consideration.*

The following lemma states that a CAF admits the same cl-cf-stable (cl-stage) claim-sets as its cf-stable kernel.

Lemma 3.15. *For any CAF $\mathcal{F}$, $\rho(\mathcal{F}) = \rho(\mathcal{F}^{csk})$ for the semantics $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stg\}$.*

Moreover, it can be shown that syntactic equivalence of cf-stable kernels of two CAFs $\mathcal{F}$ and $\mathcal{G}$ implies that the kernels coincide under any possible expansion.

Lemma 3.16. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ implies $(\mathcal{F} \cup \mathcal{H})^{csk} = (\mathcal{G} \cup \mathcal{H})^{csk}$ for any CAF $\mathcal{H}$ compatible with $\mathcal{F}$ and $\mathcal{G}$.*

We are now ready to prove our first main result stating that two CAFs $\mathcal{F}$ and $\mathcal{G}$ are strongly equivalent to each other w.r.t. cl-cf-stable and cl-stage semantics if and only if their cl-stable kernels coincide.

Theorem 3.17. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ iff $\mathcal{F} \equiv_s^\rho \mathcal{G}$ for $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stg\}$.*

Proof. First suppose we have $\mathcal{F}^{csk} = \mathcal{G}^{csk}$. In this case, $(\mathcal{F} \cup \mathcal{H})^{csk} = (\mathcal{G} \cup \mathcal{H})^{csk}$ for any compatible CAF $\mathcal{H}$ by Lemma 3.16. We infer $\rho(\mathcal{F} \cup \mathcal{H}) = \rho((\mathcal{F} \cup \mathcal{H})^{csk})$ as well as $\rho((\mathcal{G} \cup \mathcal{H})^{csk}) = \rho(\mathcal{G} \cup \mathcal{H})$ from Lemma 3.15. Hence $\mathcal{F} \equiv_s^\rho \mathcal{G}$ follows.

Now suppose $\mathcal{F}^{csk} \neq \mathcal{G}^{csk}$. Due to Lemma 3.15 we may assume $\rho(\mathcal{F}^{csk}) = \rho(\mathcal{G}^{csk})$; moreover, $A_\mathcal{F} = A_\mathcal{G} (= A)$ by Lemma 3.13. We thus have that $R_{\mathcal{F}^{csk}} \neq R_{\mathcal{G}^{csk}}$. W.l.o.g., let $(a, b) \in R_{\mathcal{F}^{csk}} \setminus R_{\mathcal{G}^{csk}}$; we apply Lemma 3.14 to assume $a \neq b$. Moreover, observe that $(a, a) \notin R_\mathcal{G}^{csk}$ (and thus, $(a, a) \notin R_\mathcal{F}^{csk}$) since otherwise $(a, b) \in R_{\mathcal{G}^{csk}}$ by definition of the cf-stable kernel. We distinguish the following cases: (a) $cl(a) \neq cl(b)$, and (b) $cl(a) = cl(b)$.

(a) In case $cl(a) \neq cl(b)$, consider two newly introduced arguments x, y and fresh claims c, d. We consider the AF $\mathcal{H}_1 = (A \cup \{x, y\}, R_1, cl_1)$ where
$$R_1 = \{(x, y)\} \cup \{(y, h) \mid h \in A \cup \{x\}\} \cup \{(x, h) \mid h \in A \setminus \{a, b\}\},$$
and the function cl_1 is given as follows: $cl_1(x) = c$, $cl_1(y) = d$, and the other claims coincide with the given ones, i.e. $cl_1(h) = cl_\mathcal{F}(h)$ if $h \in A$. First observe that $\{d\}$ is i-stable in both $\mathcal{F}^{csk} \cup \mathcal{H}_1$ and $\mathcal{G}^{csk} \cup \mathcal{H}_1$ and thus guarantees that $\rho(\mathcal{F}^{csk} \cup \mathcal{H}_1)$ and $\rho(\mathcal{G}^{csk} \cup \mathcal{H}_1)$ are non-empty. It can be checked that $S = \{cl(a), c\}$ is cl-cf-stable and cl-stage in $\mathcal{F}^{csk} \cup \mathcal{H}_1$ (since $\{a, x\}$ is stable); on the other hand, $S \notin \rho(\mathcal{G}^{csk} \cup \mathcal{H}_1)$ since b is not defeated by $\{a, x\}$. However, this is our only candidate since S has no other cf-realization in $\mathcal{G}^{csk} \cup \mathcal{H}_1$.

(b) Now consider the case $cl(a) = cl(b)$ and observe that $(a, a), (b, b), (b, a) \notin R_\mathcal{G}^{csk}$ (otherwise $(a, b) \in R_{\mathcal{G}^{csk}}$). Since $\mathcal{F}$ and $\mathcal{G}$ contain the same self-attacks, we furthermore have $(a, a), (b, b) \notin R_{\mathcal{F}^{csk}}$. Having established this situation let us construct $\mathcal{H}_2$ as follows: For fresh arguments x, y, z and fresh claims c, d, e, we consider $\mathcal{H}_2 = (A \cup \{x, y, z\}, R_2, cl_2)$ where
$$R_2 = \{(a, h) \mid h \in (A \cup \{x\}) \setminus \{a, b\}\} \cup \{(a, x), (x, x), (b, y), (y, y), (z, b), (b, z), (z, y)\}$$
and as before we let $cl_2(h) = cl_\mathcal{F}(h)$ for $h \in A$; for the fresh arguments let $cl_2(x) = c$, $cl_2(y) = d$, as well as $cl_2(z) = e$. It can be checked that each CAF admits a stable extension; thus it suffices to show that the cl-cf-stable claim-sets disagree. First observe that we now have $\{cl_2(a)\} \in \rho(\mathcal{G}^{csk} \cup \mathcal{H}_2)$ since $\{a, b\}$ is a stable extension in $G^{csk} \cup H_2$. On the other hand, we have that $\{cl_2(a)\}$ is neither $cl\text{-}stb_{cf}$-realizable nor $cl\text{-}stg$-realizable in $\mathcal{F}^{csk} \cup \mathcal{H}_2$.

In every case, we have found some $\mathcal{H}$ enforcing inequality, i.e. $\rho(\mathcal{F}^{csk} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{csk} \cup \mathcal{H})$. By Lemma 3.15, we get $\rho(\mathcal{F} \cup \mathcal{H}) = \rho((\mathcal{F} \cup \mathcal{H})^{csk}) = \rho(\mathcal{F}^{csk} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{csk} \cup \mathcal{H}) = \rho((\mathcal{G} \cup \mathcal{H})^{csk}) = \rho(\mathcal{G} \cup \mathcal{H})$. It follows that $\mathcal{F} \not\equiv_\rho^s \mathcal{G}$. □

The remaining semantics under consideration can be characterized via known AF kernels. We recall the AF kernels from the literature (Oikarinen and Woltran 2011).

Definition 3.18. For an AF $F = (A, R)$, we define the *admissible kernel* $F^{ak} = (A, R^{ak})$ with
$$R^{ak} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R, \{(b, a), (b, b)\} \cap R \neq \emptyset\};$$
the *complete kernel* $F^{gk} = (A, R^{gk})$ with
$$R^{ck} = R \setminus \{(a, b) \mid a \neq b, (a, a), (b, b) \in R\};$$
and the *grounded kernel* $F^{gk} = (A, R^{gk})$ with
$$R^{gk} = R \setminus \{(a, b) \mid a \neq b, (b, b) \in R, \{(b, a), (a, a)\} \cap R \neq \emptyset\}.$$

It has been shown that the grounded (complete) kernel characterizes strong equivalence for grounded (complete) semantics; moreover, for any two AFs F and G we have $F \equiv_s^\sigma G$ iff $F^{ak} = G^{ak}$ for $\sigma \in \{ad, pr, ss\}$ (Oikarinen and Woltran 2011). We write $F^{k(\rho)}$ to denote the kernel which characterizes strong equivalence for the semantics ρ.

To prove that strong equivalence for the remaining semantics can be characterized using known AF kernels, we make use of the following lemma which states that each CAF $\mathcal{F}$ has the same σ_c-claim-sets as its kernel $\mathcal{F}^{k(\sigma)}$ for any AF semantics σ under consideration; moreover, the cl-ad-stable and cl-semi-stable claim-sets of $\mathcal{F}$ and $\mathcal{F}^{ak}$ coincide.

Lemma 3.19. *For any CAF $\mathcal{F}$, (a) $\sigma_c(\mathcal{F}^{k(\sigma)}) = \sigma_c(\mathcal{F})$ for any considered AF semantics σ; and (b) $\rho(\mathcal{F}) = \rho(\mathcal{F}^{ak})$ for $\rho \in \{cl\text{-}stb_{ad}, cl\text{-}ss\}$.*

For inherited semantics, the result is immediate by known results for AFs; for cl-ad-stable and cl-semi-stable semantics, the statement follows by the additional observation that the range of every admissible set of F remains unchanged.

It can be shown that two CAFs are strongly equivalent under cl-ad-stable and cl-semi-stable semantics iff their admissible kernels coincide.

Theorem 3.20. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^\rho \mathcal{G}$ iff $F^{ak} = G^{ak}$ for $\rho \in \{cl\text{-}stb_{ad}, cl\text{-}ss\}$.*

Moreover, each each inherited semantics σ_c can be characterized by the respective kernel for σ.

Theorem 3.21. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^{\sigma_c} \mathcal{G}$ iff $F \equiv_s^\sigma G$ for any considered AF semantics σ.*

Due to space limits, we shall omit the proofs of the above theorems. The proofs proceed in the same way as the proof of Theorem 3.17; first, we use Lemma 3.19 to show $F^{k(\rho)} = G^{k(\rho)}$ implies strong equivalence of two CAFs $\mathcal{F}$ and $\mathcal{G}$ w.r.t. ρ for the respective kernels $F^{k(\rho)}$ and $G^{k(\rho)}$. For the other direction, we assume that the kernels of $\mathcal{F}$ and $\mathcal{G}$ differ. Depending on the semantics, we consider different cases for which we construct a CAF $\mathcal{H}$ which serves as a witness to show $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$.

For cl-naive and cl-preferred semantics, it can be shown that strong equivalence w.r.t. cl-naive and cl-preferred semantics coincides with strong equivalence w.r.t. their inherited counterparts. This implies that two CAFs are strongly equivalent w.r.t. cl-preferred semantics iff their admissible kernels coincide; likewise, two CAFs are strongly equivalent w.r.t. cl-naive semantics iff their naive kernels coincide.

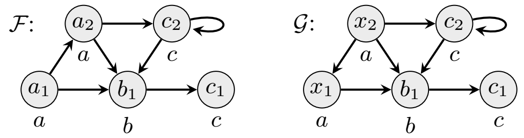
Theorem 3.22. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^{cl\text{-}\sigma} \mathcal{G}$ iff $\mathcal{F} \equiv_s^{\sigma_c} \mathcal{G}$ for $\sigma \in \{na, pr\}$.*

The proof proceeds in a slightly different way: To show $\mathcal{F} \not\equiv_s^{\sigma_c} \mathcal{G}$ implies $\mathcal{F} \not\equiv_s^{cl\text{-}\sigma} \mathcal{G}$, it can be assumed that $\mathcal{F}$ and $\mathcal{G}$ disagree on their σ_c claim-sets. We construct counter-examples $\mathcal{H}$ satisfying $cl\text{-}\sigma(\mathcal{F} \cup \mathcal{H}) \neq cl\text{-}\sigma(\mathcal{G} \cup \mathcal{H})$ in such a way that the claim-set which does not appear in either one of the frameworks becomes a $\subseteq$-maximal σ_c-claim-extension.

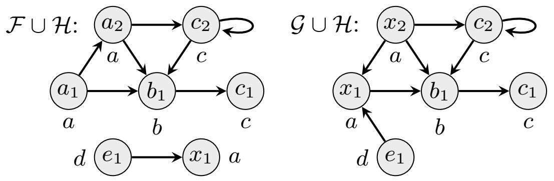
4 Renaming and Equivalence

In the previous section we were assuming that we are interested in the actual arguments and not just the claims and their interactions. In this section, we will also provide another point of view which entirely abstracts from the underlying arguments and thus viewing a CAF as a collection of claims and their relationships. To illustrate this, let us consider the following example.

Example 4.1. Assume we are given again our CAF $\mathcal{F}$ from Example 3.2 together with a CAF $\mathcal{G}$ as follows:



We observe that both CAFs are equivalent w.r.t. cl-cf-stable semantics although the arguments a_1 and a_2 are not even present in $\mathcal{G}$ while the same is true for x_1 and x_2 in $\mathcal{F}$. Moreover, recalling the kernel for $cl\text{-}stb_{cf}$ from Theorem 3.17 we observe that $\mathcal{F}$ and $\mathcal{G}$ would be even strongly equivalent if this mismatch in argument names were not present. This suggests that the usual notion of strong equivalence does not handle situations where we are interested in claims only very well. To illustrate this with a hands-on situation let us suppose we are given $\mathcal{H}$ in a way that a novel argument e_1 with claim e is given which attacks x_1:



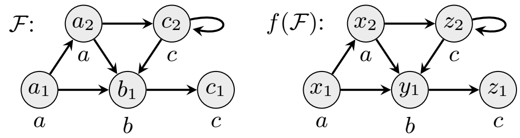
This is fine when insisting on the arguments, but on a claim-level one could of course argue that $\mathcal{H}$ did not yield the same modification on both sides and thus disrupts the similarity between $\mathcal{F}$ and $\mathcal{G}$ in an unintended way.

Our goal is hence to develop notions of equivalence which handle situations like the aforementioned one in a more intuitive way. The first step to formalize the underlying idea is the following notion of a renaming.

Definition 4.2. For a CAF $\mathcal{F}$ and an arbitrary set A' of arguments we call a bijection $f : A_{\mathcal{F}} \to A'$ s.t. for each $a \in A_{\mathcal{F}}$ we have $cl_{\mathcal{F}}(a) = cl_{\mathcal{F}}(f(a))$ a *renaming for $\mathcal{F}$*.

We abuse notation and write $f(\mathcal{F})$ for the CAF obtained from renaming the arguments, i.e. $f(\mathcal{F})$ is the CAF $(f(F), cl_f) := (f(A), R_f, cl_f)$ where $(a, b) \in R_f$ iff $(f^{-1}(a), f^{-1}(b)) \in R_{\mathcal{F}}$ and $cl_f(f(a)) = cl_{\mathcal{F}}(a)$.

Example 4.3. Consider again our previous CAF $\mathcal{F}$. Let us assume we are given $A' = \{x_1, x_2, y_1, z_1, z_2\}$. The renaming f with $a_i \mapsto x_i$, $b_1 \mapsto y_1$ and $c_i \mapsto z_i$ induces the following CAF $f(\mathcal{F})$:



We observe that f does not change the structure of $\mathcal{F}$ on claim-level. In particular, $cl\text{-}stb_{cf}(\mathcal{F}) = cl\text{-}stb_{cf}(f(\mathcal{F}))$.

The last observation we made was no coincidence in the specific situation. More precisely, for the semantics considered in this paper, renaming does not change the meaning of our CAF.

Proposition 4.4. *For a CAF $\mathcal{F}$, an arbitrary set A' of arguments and a renaming f we have $\rho(\mathcal{F}) = \rho(f(\mathcal{F}))$ for any semantics ρ considered in this paper.*

Proof. We have $E \in \sigma(F)$ iff $f(E) \in \sigma(f(F))$ for the underlying AF and since all semantics are defined by selecting (subsets of) $\{cl(E) \mid E \in \sigma(F)\}$, the claim follows since $cl_{\mathcal{F}}(a) = cl_{\mathcal{F}}(f(a))$ for each argument a. $\square$

Having formally established that names of arguments do not change the given semantics, let us proceed with defining notions of equivalence that build upon this insight.

Definition 4.5. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *ordinary equivalent up to renaming* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv_{or}^{\rho} \mathcal{G}$, if there is some set A of arguments and some renaming $f : A_{\mathcal{F}} \to A$ for $\mathcal{F}$ s.t. $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$.

So, informally speaking, Definition 4.5 requires that $\mathcal{F}$ and $\mathcal{G}$ are equivalent, at least after the underlying arguments are relabeled in a suitable way. However, in Proposition 4.4 we have actually already established that this adjustment is superfluous for our semantics. More formally, we infer the following result.

Proposition 4.6. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_{or}^{\rho} \mathcal{G}$ iff $\mathcal{F} \equiv_{o}^{\rho} \mathcal{G}$ for any semantics ρ under consideration.*

Considering this result, it becomes apparent that we could also require that $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$ holds for any renaming, not just for one in particular.

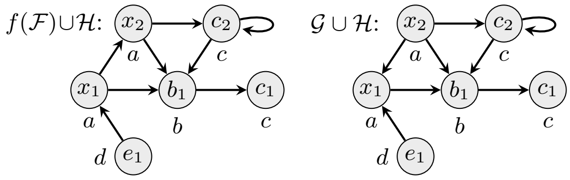
Proposition 4.7. *For two CAFs $\mathcal{F}$ and $\mathcal{G}$ we have that for all semantics considered in this paper $\mathcal{F} \equiv_{or}^{\rho} \mathcal{G}$ implies $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$ for any renaming f for $\mathcal{F}$.*

Now we utilize the notion of a renaming in order to define a strong equivalence-like relation which is more suitable than strong equivalence for situations like the one described in Example 4.1.

Definition 4.8. Two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ are *strongly equivalent up to renaming* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$, if there is a renaming $f : A_{\mathcal{F}} \to A_{\mathcal{F}}$ for $\mathcal{F}$ s.t. $\rho(f(\mathcal{F}) \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ for each CAF $\mathcal{H}$ which is compatible with $\mathcal{F}$ and $\mathcal{G}$.

Let us reconsider our motivating Example 4.1.

Example 4.9. Recall the CAFs $\mathcal{F}$ and $\mathcal{G}$ from before and consider a renaming f which maps a_i to x_i and leaves the remaining arguments unchanged. Augmenting both $f(\mathcal{F})$ and $\mathcal{G}$ with $\mathcal{H}$, we obtain the following desired situation:



Notice that Proposition 4.4 ensures that our renaming for $\mathcal{F}$ only prevents $\mathcal{H}$ from introducing a novel argument, while preserving the semantics of $\mathcal{F}$.

Strong equivalence up to renaming implies the usual strong equivalence. This can be obtained by setting $f = id$.

Proposition 4.10. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, if $\mathcal{F} \equiv^{\rho}_{s} \mathcal{G}$, then $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$.*

Even without using Proposition 4.4 explicitly we can infer that strong equivalence survives moving to a renamed version of f as well.

Proposition 4.11. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, if $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$, then $f(\mathcal{F}) \equiv^{\rho}_{sr} \mathcal{G}$ for any renaming f for $\mathcal{F}$.*

Proof. We have $\rho(g(\mathcal{F}) \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ for each $\mathcal{H}$ for some renaming g because we assume $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$. Since f is a bijection we find $\rho(g(f^{-1}(f(\mathcal{F}))) \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$, thus $g \circ f^{-1}$ is our witnessing renaming for $f(\mathcal{F}) \equiv^{\rho}_{sr} \mathcal{G}$. ☐

Let us now come to the kernels. Since our notion of strong equivalence up to renaming allows for changing the names of the arguments, we expect our kernels to behave similarly. More specifically, we also need to consider renamed versions of the CAFs before evaluating the kernels. However, checking strong equivalence up to renaming will surely require to take the structure of the CAFs into consideration. We thus define what we mean by a CAF isomorphism.

Definition 4.12. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *isomorphic* to each other iff there is a mapping $f : A_{\mathcal{F}} \to A_{\mathcal{G}}$ s.t. (1) f is a renaming for $\mathcal{F}$ and (2) for all $a, b \in A_{\mathcal{F}}$, $(a, b) \in R_{\mathcal{F}}$ iff $(f(a), f(b)) \in R_{\mathcal{G}}$. f is called *isomorphism* between $\mathcal{F}, \mathcal{G}$.

CAFs $\mathcal{F}$ and $f(\mathcal{F})$ from Example 4.3 are isomorphic. The given renaming f naturally is a CAF-isomorphism between $\mathcal{F}$ and $f(\mathcal{F})$. The following proposition collects basic properties of CAF isomorphisms.

Proposition 4.13. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, (a) if $\mathcal{F}$ and $\mathcal{G}$ are isomorphic, then $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for any considered semantics ρ; and (b) if f is a renaming for $\mathcal{F}$, then $\mathcal{F}$ and $f(\mathcal{F})$ are isomorphic.*

As it turns out, we obtain *exactly* the result we desire to: We check strong equivalence up to renaming by choosing the appropriate kernel for ρ, computing the kernels of $\mathcal{F}$ and $\mathcal{G}$ and then checking whether those are isomorphic to each other. Informally speaking, our tailored notion of equivalence which does not take the names of arguments into account yields the exact same kernels after relabeling the arguments in a suitable way.

Theorem 4.14. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$ iff $\mathcal{F}^{k(\rho)}$ and $\mathcal{G}^{k(\rho)}$ are isomorphic.*

Proof. ($\Leftarrow$) Let $\mathcal{F}^{k(\rho)}$ and $\mathcal{G}^{k(\rho)}$ be isomorphic, witnessed by the isomorphism f. We have $f(\mathcal{F}^{k(\rho)}) = \mathcal{G}^{k(\rho)}$; moreover, $\mathcal{F}^{k(\rho)} = \mathcal{G}^{k(\rho)}$ implies $(\mathcal{F} \cup \mathcal{H})^{k(\rho)} = (\mathcal{G} \cup \mathcal{H})^{k(\rho)}$ for any compatible CAF $\mathcal{H}$; extending f to $\mathcal{H}$ in a straightforward way yields $f((\mathcal{F} \cup \mathcal{H})^{k(\rho)}) = (\mathcal{G} \cup \mathcal{H})^{k(\rho)}$. Since $(\mathcal{F} \cup \mathcal{H})^{k(\rho)} = (\mathcal{G} \cup \mathcal{H})^{k(\rho)}$ implies $\rho(\mathcal{F} \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ our isomorphism ensures $\rho(\mathcal{F} \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$.

($\Rightarrow$) Now assume the kernels $\mathcal{F}^{k(\rho)}$ and $\mathcal{G}^{k(\rho)}$ are not isomorphic, i.e. for any renaming f, $f(\mathcal{F}^{k(\rho)}) \neq \mathcal{G}^{k(\rho)}$. Due to the properties of our kernel, there is some $\mathcal{H}$ s.t. $\rho(f(\mathcal{F}) \cup \mathcal{H}) \neq \rho(\mathcal{G} \cup \mathcal{H})$. ☐

Example 4.15. For our CAFs $\mathcal{F}$ and $\mathcal{G}$ from Example 4.1 we see that —given $\rho = \textit{cl-stb}_{cf}$— the kernels are isomorphic. Hence $\mathcal{F}$ and $\mathcal{G}$ are strongly equivalent up to renaming.

5 Computational Complexity

In this section we examine the computational complexity of deciding equivalence between two CAFs $\mathcal{F}$ and $\mathcal{G}$ for every equivalence notion which has been established in this paper. We assume the reader to be familiar with the polynomial hierarchy. Moreover, by $\text{QSAT}^{\exists}_{n}$ ($\text{QSAT}^{\forall}_{n}$) we denote the generic Σ^{P}_{n}-complete (Π^{P}_{n}-complete) problem, i.e. checking validity of a corresponding QBF. Our results reveal that ordinary equivalence can be computationally hard, up to the third level of the polynomial hierarchy for both variants of semi-stable and stage semantics as well as for i-preferred semantics. For the remaining semantics under consideration, the problem is Π^{P}_{2}-complete; the only exception is i-grounded semantics for which deciding ordinary equivalence is P-complete. Moreover, we show that deciding strong equivalence up to renaming extends the list of problems which lie in NP but are not known to be NP-complete.

First we present our complexity results for ordinary equivalence. We formulate the following decision problem:

VER-OE$_{\rho}$
Input: Two CAFs $\mathcal{F}, \mathcal{G}$.
Output: TRUE iff $\mathcal{F}, \mathcal{G}$ are ordinary equivalent w.r.t. ρ.

We obtain the following computational complexity results for deciding ordinary equivalence:

Theorem 5.1. VER-OE$_{\rho}$ *is*

- P-*complete for* $\rho = gr_c$;
- Π^{P}_{2}-*complete for* $\rho \in \{cf_c, ad_c, co_c, na_c, \textit{cl-pr}, \textit{cl-na}, stb_c, \textit{cl-stb}_{cf}, \textit{cl-stb}_{ad}, \}$; *and*
- Π^{P}_{3}-*complete for* $\rho \in \{pr_c, ss_c, stg_c, \textit{cl-stg}, \textit{cl-ss}\}$.

In the following we will provide proofs for the results from Theorem 5.1. To begin with, we show that verifying ordinary equivalence for i-grounded semantics is P-complete.

Proposition 5.2. *Deciding* VER-OE_{gr_c} *is* P*-complete.*

Proof. VER-OE_{gr_c} is in P since computing the grounded extensions of F and G and comparing the claims can be done in polynomial time. Hardness is by a reduction from the verification problem $Ver^{CAF}_{gr_c}$ for i-grounded semantics (which is P-complete by (Dvorák and Woltran 2020)) by setting $\mathcal{F} = \mathcal{F}$ and $\mathcal{G} = (S, \emptyset, id)$ for an instance $(\mathcal{F}, S)$ of $Ver^{CAF}_{gr_c}$. We obtain $gr_c(\mathcal{F}) = gr_c(\mathcal{G})$ iff $S = gr_c(\mathcal{F})$. □

Membership proofs for VER-OE_ρ, $\rho \neq gr_c$ are by standard guess-and-check procedures for the complementary problems: Guess a set of claims S and check whether it holds that $S \in \mathcal{F}$ as well as $S \notin \mathcal{G}$. For the semantics $\rho \in \{cf_c, ad_c, co_c, na_c, stb_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$, the latter requires two NP-oracle calls; for $\rho \in \{cl\text{-}pr, cl\text{-}na\}$ we require four NP-oracle calls (recall that verification for cl-preferred and cl-naive semantics is in D^P_1 (Dvořák et al. 2021)), which shows that VER-OE_ρ is in Π^P_2. For the semantics $\rho \in \{pr_c, ss_c, stg_c, cl\text{-}ss, cl\text{-}stg\}$, we require two Σ^P_2-oracle calls to check $S \in \mathcal{F}$ and $S \notin \mathcal{G}$; yielding Π^P_3-procedures for the decision problem VER-OE_ρ.

To show hardness of VER-OE_ρ for $\rho \neq gr_c$, we present reductions from $\text{QSAT}^\forall_2$ or $\text{QSAT}^\exists_2$, respectively. The overall idea is to construct two CAFs $\mathcal{F}$, $\mathcal{G}$ where $\rho(\mathcal{F})$ depends on the particular instance of the source problem while $\mathcal{G}$ serves as controlling entity. For a given instance $\Psi = Q_1X_1 \dots Q_nX_n\varphi$ of $\text{QSAT}^\forall_2$ or $\text{QSAT}^\exists_2$, respectively, we design the CAF $\mathcal{F}$ in a way such that $\rho(\mathcal{F})$ depends on the models of φ while $\mathcal{G}$ possesses every possible ρ-claim-set which can be obtained in $\mathcal{F}$ by varying φ, i.e., $\rho(\mathcal{G})$ is independent of the validity of Ψ. $\mathcal{F}$ is then constructed such that Ψ is valid iff $\rho(\mathcal{F}) = \rho(\mathcal{G})$ (if we reduce $\text{QSAT}^\forall_2$) or $\rho(\mathcal{F}) \neq \rho(\mathcal{G})$ (in case we reduce $\text{QSAT}^\exists_2$).

We will first discuss the hardness proofs for those semantics for which VER-OE_ρ is Π^P_2-complete. We outline the underlying aforementioned idea for i-stable semantics.

Proposition 5.3. *Deciding* VER-OE_ρ *is* Π^P_2*-hard for* $\rho \in \{stb_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$.

Proof. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $\text{QSAT}^\forall_2$ where φ is identified with a set of clauses C over atoms in $V = Y \cup Z$. We construct two CAFs $\mathcal{F} = (A_\mathcal{F}, R_\mathcal{F}, cl_\mathcal{F})$ and $\mathcal{G} = (A_\mathcal{G}, R_\mathcal{G}, id)$. The CAF $\mathcal{F}$ is given by

$$A_\mathcal{F} = V \cup \bar{V} \cup C \text{ with } \bar{V} = \{\bar{v} \mid v \in V\};$$
$$R_\mathcal{F} = \{(v, cl) \mid cl \in C, v \in cl\} \cup \{(cl, cl) \mid cl \in C\} \cup \{(\bar{v}, cl) \mid cl \in C, \neg v \in cl\} \cup \{(v, \bar{v}), (\bar{v}, v) \mid v \in V\}$$

and $cl_\mathcal{F}(z) = cl_\mathcal{F}(\bar{z}) = z$ for $z \in Z$ and $cl_\mathcal{F}(a) = a$ else; that is, we introduce arguments for every clause and every literal; a literal argument attacks a clause argument if the corresponding literal is contained in the respective clause; moreover, the clauses are self-attacking and every literal and its negation attack each other. We assign every atom $z \in Z$ the same claim as its negation $\bar{z}$; the remaining arguments have their unique argument name as claim. The CAF $\mathcal{G}$ is given by $A_\mathcal{G} = V \cup \bar{Y}$; $R_\mathcal{G} = \{(y, \bar{y}), (\bar{y}, y) \mid y \in Y\}$. An example of the reduction is given in Figure 1. Observe that the i-stable (cl-stable) claim-sets of $\mathcal{G}$ are given by sets of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ for $Y' \subseteq Y$.

It can be shown that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ is i-stable (cl-stable) in $\mathcal{F}$ for every $Y' \subseteq Y$ iff Ψ is valid. By design of $\mathcal{G}$, the latter is satisfied iff the i-stable (cl-stable) extensions of $\mathcal{F}$ and $\mathcal{G}$ coincide. That is, Ψ is valid iff $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{stb_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$. □

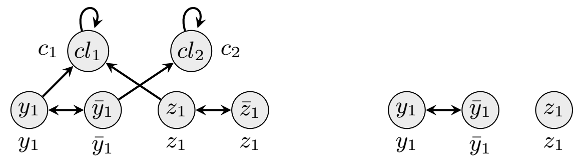


Figure 1: Reduction from the proof of Proposition 5.3 for a formula $\forall Y \exists Z \varphi(Y, Z)$ where φ is given by the clauses $\{\{y_1, z_1\}, \{\bar{y}_2\}\}$.

By modifying the constructions from the proof of Proposition 5.3 we obtain Π^P_2-hardness of VER-OE_{na_c}. For the construction of $\mathcal{F}$ in the Π^P_2-hardness proof of VER-OE_ρ, $\rho = \{cf_c, ad_c, cl\text{-}na, cl\text{-}pr\}$, we choose a slightly different approach: For an instance $\Psi = \forall Y \exists Z \varphi(Y, Z)$ of $\text{QSAT}^\forall_2$, we construct $\mathcal{F}$ such that each literal in a clause cl is represented by an argument having claim cl; we furthermore introduce arguments for each atom $y \in Y$ and its negation; finally, every two arguments representing negated literals attack each other. We construct $\mathcal{G}$ in a way such that $\rho(\mathcal{G})$ contains precisely the claim-sets $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup C$. Similar as above, it can be shown that Ψ is valid iff $\rho(\mathcal{F}) = \rho(\mathcal{G})$. An appropriate adaptation and claim-assignment of the standard construction as presented in (Dvorák and Dunne 2018, Reduction 3.6) yields Π^P_2-hardness for i-complete semantics.

Turning now to the Π^P_3-hardness results, we adjust our general reduction scheme by targeting inequality of $\rho(\mathcal{F})$ and $\rho(\mathcal{G})$ in case the given instance Ψ of $\text{QSAT}^\exists_3$ is valid. As an example, we present the construction from the Π^P_3-hardness proof for cl-semi-stable and cl-stage semantics.

Proposition 5.4. *Deciding* VER-OE_ρ *is* Π^P_3*-hard for* $\rho \in \{cl\text{-}ss, cl\text{-}stg\}$.

Proof. Consider an instance $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ of $\text{QSAT}^\exists_3$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. We can assume that there is $y_0 \in Y$ with $y_0 \in cl$ for all $cl \in C$ (otherwise we can add such a y_0 without changing the validity of Ψ). We write $\bar{v}$ to denote $\neg v$ for an atom $v \in V$, moreover, let $V' = X \cup Y$. We construct CAFs $\mathcal{F} = (A_\mathcal{F}, R_\mathcal{F}, cl_\mathcal{F})$ and $\mathcal{G} = (A_\mathcal{G}, R_\mathcal{G}, id)$ as follows: The CAF $\mathcal{F}$ is given by

$$A_\mathcal{F} = V \cup \bar{V} \cup \mathcal{C} \cup \{\varphi_1, \varphi_2\} \cup \{d_v, d_{\bar{v}} \mid v \in V' \cup \bar{V}'\};$$
$$R_\mathcal{F} = \{(a, cl) \mid cl \in C, a \in cl, a \in V \cup \bar{V}\} \cup \{(cl, \varphi) \mid cl \in C\} \cup \{(a, d_a), (d_a, d_a) \mid a \in V' \cup \bar{V}'\} \cup \{(\varphi_1, \varphi_2), (\varphi_2, \varphi_2)\} \cup \{(v, \bar{v}), (\bar{v}, v) \mid v \in V\};$$

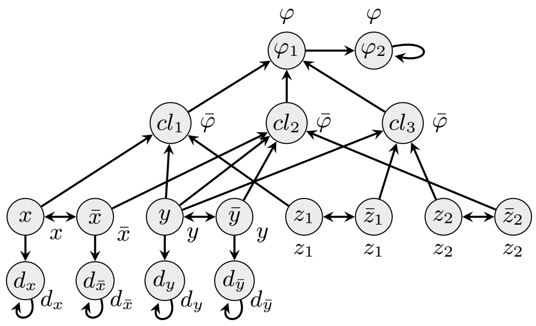


Figure 2: Construction of the CAF $\mathcal{F}$ from the proof from Proposition 5.4 for the formula $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ with clauses $\{\{z_1, x, y\}, \{\neg x, \neg y, \neg z_2, y\}, \{\neg z_1, z_2, y\}\}$.

$cl_{\mathcal{F}}(v) = cl_{\mathcal{F}}(\bar{v}) = v$ for $v \in Y \cup Z$; $cl_{\mathcal{F}}(cl) = \bar{\varphi}$ for $cl \in C$; $cl_{\mathcal{F}}(\varphi_1) = cl_{\mathcal{F}}(\varphi_2) = \varphi$; and $cl_{\mathcal{F}}(a) = a$ otherwise. An example of this construction is given in Figure 2. We observe that each set $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ is cl-semi-stable (cl-stage) in $\mathcal{F}$ for every $X' \subseteq X$ (remember that there is $y_0 \in Y$ which attacks every clause $cl \in C$).

We define $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, id)$ such that it has the cl-semi-stable (cl-stage) claim-sets $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for every $X' \subseteq X$, $e \in \{\varphi, \bar{\varphi}\}$ with $A_{\mathcal{G}} = V \cup \bar{X} \cup \{\varphi, \bar{\varphi}\}$, and $R_{\mathcal{G}} = \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi)\}$. It is easy to see that $\mathcal{G}$ possesses exactly the desired cl-semi-stable (cl-stage) claim-sets.

It can be checked that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ is cl-semi-stable (cl-stage) in $\mathcal{F}$ for every $X' \subseteq X$ iff Ψ is not valid. The former is satisfied iff $\mathcal{F}$ and $\mathcal{G}$ possess the cl-semi-stable (cl-stage) claim-sets. Thus Ψ is valid iff $\rho(\mathcal{F}) \neq \rho(\mathcal{G})$ for $\rho \in \{cl\text{-}ss, cl\text{-}stg\}$. $\square$

Π_3^{P}-hardness of ordinary equivalence for i-semi-stable and i-stage semantics is by adapting the Π_3^{P}-hardness proof of the concurrence problem for semi-stable semantics, i.e., deciding whether $ss_c(\mathcal{F}) = cl\text{-}ss(\mathcal{F})$ for a CAF $\mathcal{F}$ (Dvořák et al. 2021, Proposition 6). For i-preferred semantics, we modify the standard reduction for preferred semantics (cf. (Dvorák and Dunne 2018, Reduction 3.7)) via an appropriate claim-assignment. This concludes the proof of Theorem 5.1.

Remark 5.5. The computational complexity results from Theorem 5.1 extend to ordinary equivalence up to renaming by Proposition 4.6 for any semantics under consideration.

Having established complexity results for ordinary equivalence it remains to discuss the computational complexity of strong equivalence and strong equivalence up to renaming.

VER-SE$_\rho$

Input: Two CAFs $\mathcal{F}$, $\mathcal{G}$.
Output: TRUE iff $\mathcal{F}$, $\mathcal{G}$ are strongly equivalent w.r.t. ρ.

Recall that in Section 3, we have shown that strong equivalence of two CAFs $\mathcal{F}$ and $\mathcal{G}$ can be characterized via syntactic equivalence of their kernels. Since the computation and comparison of the kernels of $\mathcal{F}$ and $\mathcal{G}$ can be done in polynomial time, we obtain tractability of strong equivalence for every semantics under consideration.

Theorem 5.6. *The problem* VER-SE$_\rho$ *can be solved in polynomial time for any semantics ρ considered in this paper.*

Finally, we consider strong equivalence up to renaming. An analogous decision problem be formulated as follows:

VER-SER$_\rho$

Input: Two CAFs $\mathcal{F}$, $\mathcal{G}$.
Output: TRUE iff $\mathcal{F}$, $\mathcal{G}$ are strongly equivalent up to renaming w.r.t. ρ.

As outlined above, the computation of the kernels lies in P and is therefore negligible; the complexity of verifying strong equivalence up to renaming thus stems entirely from deciding whether two labelled graphs (i.e., the kernels of the given CAFs) are isomorphic. As a consequence we obtain that the complexity of VER-SER$_\rho$ coincides with the complexity of the famous graph isomorphism problem.

Theorem 5.7. *The problem* VER-SER$_\rho$ *is exactly as hard as the graph isomorphism problem for any semantics ρ considered in this paper.*

It is well-known that the graph isomorphism problem lies in NP but is not known to be NP-complete (although the latter is considered unlikely (Schöning 1988)).

6 Conclusion and Future Work

In this paper, we considered ordinary and strong equivalence as well as novel equivalence notions based on argument renaming for CAFs w.r.t. all semantics for CAFs which have been considered in the literature so far and provided a complexity analysis of all considered equivalence notions.

Our characterization results for strong equivalence are in line with existing studies for related argumentation formalisms (Oikarinen and Woltran 2011; Dvorák, Rapberger, and Woltran 2020b); in addition, we adapt an argument-independent view by considering equivalence under renaming. Equivalence of logic-based argumentation has been studied in (Amgoud, Besnard, and Vesic 2014); they show that under certain conditions on the underlying logic, unnecessary arguments can be removed while retaining (strong) equivalence. In contrast to their work, our studies are independent of the underlying formalism of the instantiated argumentation system as we do not impose any further constraints on the arguments or their claims; in this way, it is even possible to test equivalence between argumentation systems stemming from entirely different base formalisms.

For future work, we want to extend our strong equivalence studies by considering certain constraints of the framework modifications. What has been commonly investigated in the literature are *normal expansions* where attacks can only be introduced if they involve newly added arguments (observe that in the proof of Theorem 3.17, the expansion in case (a) satisfy this criteria while $\mathcal{H}$ in case (b) introduces also new attacks between existing arguments). We moreover want to adapt our strong equivalence notion to arbitrary CAFs, not only compatible ones, by relaxing the notion of framework expansions. Another point on our agenda is to consider certain sub-classes of CAFs, which have been introduced in the literature, e.g., well-formed CAFs which impose restrictions on the attack relation.

Acknowledgements

This work was supported by the German Federal Ministry of Education and Research (BMBF, 01/S18026A-F) by funding the competence center for Big Data and AI "ScaDS.AI" Dresden/Leipzig and by the Austrian Science Fund (FWF) through projects W1255-N23 and Y698.

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A Ordinary equivalence relations and counter-examples (cf. Proposition 3.3)

Theorem A.1. *The results depicted in Table 1 hold.*

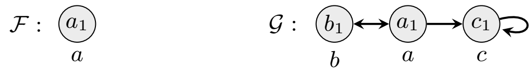
In contrast to the positive results, counter-examples carry over from known results for AFs (cf. (Oikarinen and Woltran 2011)) as the following lemma shows.

Lemma A.2. *For two AF semantics σ and τ, if $\sigma(F) = \sigma(G) \not\Rightarrow \tau(F) = \tau(G)$ for some AFs F, G, then $\sigma_c(\mathcal{F}) = \sigma_c(\mathcal{G}) \not\Rightarrow \tau_c(\mathcal{F}) = \tau_c(\mathcal{G})$ for some CAFs $\mathcal{F}$, $\mathcal{G}$.*

Proof. Consider such two AFs F and G with $\sigma(F) = \sigma(G)$ and $\tau(F) \neq \tau(G)$ and let $\mathcal{F} = (F, id)$, $\mathcal{G} = (G, id)$. □

Example 3.4 (ctd.) Observe $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{cf_c, ad_c, cl\text{-}na, cl\text{-}pr\}$; the claim-extensions of $\mathcal{F}$ and $\mathcal{G}$ disagree for the remaining semantics $\rho' \neq \rho$, yielding counter-examples for $\rho(\mathcal{F}) = \rho(\mathcal{G}) \Rightarrow \rho'(\mathcal{F}) = \rho'(\mathcal{G})$.

Example A.3. Consider the following two CAFs $\mathcal{F}$ and $\mathcal{G}$:

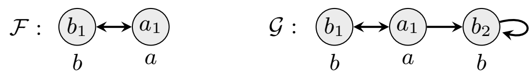


We have $\rho(\mathcal{F}) = \rho(\mathcal{G}) = \{\{a\}\}$ for the semantics $\rho \in \{stb_c, ss_c, stg_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}, cl\text{-}ss, cl\text{-}stg\}$ and disagree for any semantics $\rho' \neq \rho$ under consideration. Thus $\mathcal{F} \equiv_o^\rho \mathcal{G}$ does not imply $\mathcal{F} \equiv_o^{\rho'} \mathcal{G}$ for any semantics $\rho' \neq \rho$.

Example A.4. Let $\mathcal{F}$, $\mathcal{G}$ as in Example A.3 and let $\mathcal{F}'$ with $R_{\mathcal{F}'} = R_{\mathcal{F}} \setminus \{(b_1, a_1)\}$. Then $\rho(\mathcal{F}) \neq \rho(\mathcal{G})$ for $\rho \in \{cf_c, na_c, cl\text{-}na\}$ and $\rho'(\mathcal{F}) = \rho'(\mathcal{G})$ for any semantics $\rho' \neq \rho$ under consideration, yielding the respective counter-examples.

Example A.5. Consider the CAFs $\mathcal{F}$, $\mathcal{G}$ from Example A.3 and let $cl_{\mathcal{G}}(c_1) = b$, then $\rho(\mathcal{F}) = \rho(\mathcal{G}) = \{\{a\}\}$ (only) for $\rho \in \{stb_c, ss_c, stg_c\}$. In this case we have counter-examples for $\mathcal{F} \equiv_o^\rho \mathcal{G} \Rightarrow \mathcal{F} \equiv_o^{\rho'} \mathcal{G}$ for any semantics $\rho' \neq \rho$.

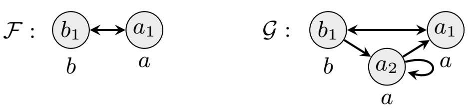
Example A.6. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:



$\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{ad}, cl\text{-}ss, cl\text{-}stg\}$ (not exclusively) but $\rho'(\mathcal{F}) \neq \rho'(\mathcal{G})$ for $\rho' \in \{stb_c, ss_c, stg_c\}$.

Example A.7. Consider the CAFs $\mathcal{F}$, $\mathcal{G}$ from Example A.6 and let $cl_{\mathcal{G}}(b_2) = c$. Then $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{cf_c, na_c, ad_c, gr_c, co_c, pr_c, cl\text{-}na, cl\text{-}pr\}$ and $\rho'(\mathcal{F}) \neq \rho'(\mathcal{G})$ for $\rho' \neq \rho$.

Example A.8. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:



Then $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{cf_c, cl\text{-}stb_{cf}, cl\text{-}stg, na_c, cl\text{-}na, gr_c\}$ which is not the case for the remaining semantics.

Example A.9. Consider the CAF $\mathcal{G}$ from Example A.8 and let $\mathcal{F} = (\{b\}, \emptyset, id)$, then $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for $\rho \in \{ad_c, co_c, pr_c, stb_c, ss_c, stg_c, cl\text{-}pr, cl\text{-}stb_{ad}, cl\text{-}ss\}$ and $\rho'(\mathcal{F}) \neq \rho'(\mathcal{G})$ for $\rho' \neq \rho$.

Example A.10. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:

Then $\rho(\mathcal{F}) = \rho(\mathcal{G}) = \emptyset$ for $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$ while $\rho'(\mathcal{F}) = \{\{b\}\} \neq \{\{c\}\} = \rho'(\mathcal{G})$ for $\rho' \in \{cl\text{-}stg, cl\text{-}ss\}$.

Example A.11. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:

Then $\rho(\mathcal{F}) = \rho(\mathcal{G}) = \{\{b\}\}$ for $\rho \in \{cl\text{-}stg, cl\text{-}ss\}$ while $\rho'(\mathcal{F}) = \emptyset \neq \{\{b\}\} = \rho'(\mathcal{G})$ for $\rho' \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$.

B Differnt arguments or self-attacks - criteria to disprove strong equivalence (Proofs of Lemmata 3.13 and 3.14)

We will make use of the following lemma.

Lemma B.1. *Given a CAF $\mathcal{F} = (F, cl)$, a set of claims $S \subseteq cl(A)$. Then $S \subseteq S'$ for some $S' \in stb_c(\mathcal{F})$ implies that for all semantics $\rho \neq \{gr_c\}$ under consideration, there is $S'' \in \rho(\mathcal{F})$ with $S \subseteq S''$.*

Proof. The statement follows from known relations between semantics and since the cl-preferred (cl-naive) claim-sets are precisely the subset-maximal i-preferred (i-naive) claim-sets. Thus $S \in pr_c(\mathcal{F})$ implies there is $S' \in cl\text{-}pr(\mathcal{F})$ with $S' \subseteq S$ and $S \in na_c(\mathcal{F})$ implies there is $S' \in cl\text{-}na(\mathcal{F})$ with $S' \subseteq S$. □

Lemma 3.13. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $A_{\mathcal{F}} \neq A_{\mathcal{G}}$ implies $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$ for any considered semantics ρ.*

Proof. W.l.o.g., we may assume that there is $a \in A_{\mathcal{F}}$ with $a \notin A_{\mathcal{G}}$. To prove the statement, we distinguish the following cases: (a) $(a, a) \notin R_{\mathcal{F}}$ and (b) $(a, a) \in R_{\mathcal{F}}$.

- In case $(a, a) \notin R_{\mathcal{F}}$, we recall the following construction: For a fresh argument x and a fresh claim c, let $\mathcal{H} = (A_{\mathcal{H}}, R_{\mathcal{H}}, cl_{\mathcal{H}})$ with

  $$A_{\mathcal{H}} = (A_{\mathcal{F}} \cup A_{\mathcal{G}} \cup \{x\}) \setminus \{a\};$$
  $$R_{\mathcal{H}} = \{(x, b) \mid b \in (A_{\mathcal{F}} \cup A_{\mathcal{G}}) \setminus \{a\}\};$$

  and $cl_{\mathcal{H}}(b) = cl_{\mathcal{F}}(b)$ for $b \in A_{\mathcal{F}} \cup A_{\mathcal{G}}$ and $cl_{\mathcal{H}}(x) = c$; that is, we introduce a new argument having a fresh claim c which attacks every argument except a. Observe that $\{cl_{\mathcal{H}}(a), c\} \in gr_c(\mathcal{F} \cup \mathcal{H})$ and $\{cl_{\mathcal{H}}(a), c\} \in stb_c(\mathcal{F} \cup \mathcal{H})$ since $\{a, x\}$ is conflict-free, and x is unattacked and attacks all remaining arguments except a in $F \cup H$; thus there is $S \in \rho(\mathcal{F} \cup \mathcal{H})$ with $\{cl_{\mathcal{H}}(a), c\} \subseteq S$ for every semantics ρ under consideration by Lemma B.1. On the other hand, $\{cl_{\mathcal{H}}(a), c\} \notin cf(\mathcal{G} \cup \mathcal{H})$ since x attacks every occurrence of $cl_{\mathcal{H}}(a)$ in $\mathcal{G}$; therefore, $\{cl_{\mathcal{H}}(a), c\} \notin \rho(\mathcal{G} \cup \mathcal{H})$.

	cf_c	na_c	ad_c	co_c	gr_c	pr_c	stb_c	ss_c	stg_c	$cl\text{-}na$	$cl\text{-}pr$	$cl\text{-}stb_{cf}$	$cl\text{-}stb_{ad}$	$cl\text{-}ss$	$cl\text{-}stg$
cf_c	✓	3.4	A.2	A.2	A.2	A.2	A.2	A.2	A.2	✓	A.8	3.4	3.4	3.4	3.4
na_c	✓	✓	A.2	A.2	A.2	A.2	A.2	A.2	A.2	✓	A.8	A.7	A.8	A.8	A.7
ad_c	A.2	A.2	✓	A.2	A.2	A.2	A.2	A.2	A.2	A.4	✓	3.4	3.4	3.4	3.4
co_c	A.2	A.2	A.2	✓	✓	A.2	A.2	A.2	A.2	A.4	✓	A.7	A.7	A.7	A.7
gr_c	A.2	A.2	A.2	A.2	✓	A.2	A.2	A.2	A.2	A.4	A.8	A.7	A.7	A.7	A.7
pr_c	A.2	A.2	A.2	A.2	A.2	✓	A.2	A.2	A.2	A.4	✓	A.7	A.7	A.7	A.7
stb_c	A.2	A.2	A.2	A.2	A.2	A.2	✓	A.2	A.2	A.3	A.3	A.5	A.5	A.5	A.5
ss_c	A.2	A.2	A.2	A.2	A.2	A.2	A.2	✓	A.2	A.3	A.3	A.5	A.5	A.5	A.5
stg_c	A.2	A.2	A.2	A.2	A.2	A.2	A.2	A.2	✓	A.3	A.3	A.5	A.5	A.5	A.5
$cl\text{-}na$	✓	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	✓	A.8	3.4	3.4	3.4	3.4
$cl\text{-}pr$	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	A.4	✓	3.4	3.4	3.4	3.4
$cl\text{-}stb_{cf}$	A.3	A.3	A.3	A.3	A.3	A.3	A.6	A.6	A.6	A.3	A.3	✓	A.8	A.8	A.10
$cl\text{-}stb_{ad}$	A.3	A.3	A.3	A.3	A.3	A.3	A.6	A.6	A.6	A.3	A.3	A.9	✓	A.10	A.9
$cl\text{-}ss$	A.3	A.3	A.3	A.3	A.3	A.3	A.6	A.6	A.6	A.3	A.3	A.9	A.11	✓	A.9
$cl\text{-}stg$	A.3	A.3	A.3	A.3	A.3	A.3	A.6	A.6	A.6	A.3	A.3	A.11	A.8	A.8	✓

Table 1: Ordinary equivalence relations. ✓ indicates $\rho(\mathcal{F}) = \rho(\mathcal{G}) \Rightarrow \rho'(\mathcal{F}) = \rho'(\mathcal{G})$ for the semantics ρ, ρ' and for any two CAFs $\mathcal{F}$, $\mathcal{G}$ (cf. Prop. 3.3). The given references for the other cases refer to the respective counter-examples.

- Now, let $(a, a) \in R_{\mathcal{F}}$. For a fresh argument x and a fresh claim c, let $\mathcal{H} = (A_{\mathcal{H}}, R_{\mathcal{H}}, cl_{\mathcal{H}})$ with

$$A_{\mathcal{H}} = A_{\mathcal{F}} \cup A_{\mathcal{G}} \cup \{x\};$$
$$R_{\mathcal{H}} = \{(x, b) \mid b \in (A_{\mathcal{F}} \cup A_{\mathcal{G}}) \setminus \{a\}\};$$

and $cl_{\mathcal{H}}(b) = cl_{\mathcal{F}}(b)$ for $b \in A_{\mathcal{F}}$, $cl_{\mathcal{H}}(b) = cl_{\mathcal{G}}(b)$ for $b \in A_{\mathcal{G}}$; and $cl_{\mathcal{H}}(x) = c$; that is, the new argument x attacks every argument in $A_{\mathcal{F}} \cup A_{\mathcal{G}}$ except a. Observe that a is unattacked in $\mathcal{G} \cup \mathcal{H}$ since a is a newly introduced argument in $\mathcal{G} \cup \mathcal{H}_1$ by assumption $a \notin A_{\mathcal{G}}$. Therefore $\{cl_{\mathcal{H}}(a), c\} \in gr_c(\mathcal{G} \cup \mathcal{H})$ since $\{a, x\}$ is conflict-free and unattacked; moreover, $\{cl_{\mathcal{H}}(a), c\} \in stb_c(\mathcal{G} \cup \mathcal{H})$ since $\{a, x\}$ is conflict-free and attacks all remaining arguments in $G \cup H$. By Lemma B.1, $\{cl_{\mathcal{H}}(a), c\}$ is thus contained in some ρ-claim-set for every semantics ρ under consideration. On the other hand, $\{cl_{\mathcal{H}}(a), c\} \notin cf(\mathcal{F} \cup \mathcal{H})$ since every realisation of $\{cl_{\mathcal{H}}(a), c\}$ is conflicting: a is self-attacking and x attacks every other occurrence of $cl_{\mathcal{H}}(a)$. Thus $\{cl_{\mathcal{H}}(a), c\} \notin \rho(\mathcal{F} \cup \mathcal{H}_{\mathcal{H}})$ for each considered semantics ρ.

In both cases, we found a witness $\mathcal{H}$ showing that $\rho(\mathcal{F} \cup \mathcal{H}) \neq \rho(\mathcal{G} \cup \mathcal{H})$. Thus the statement follows. □

Lemma 3.14. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $(a, a) \in R_{\mathcal{F}} \Delta R_{\mathcal{G}}$ implies $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$ for any semantics ρ under consideration.*

Proof. By Lemma 3.13, we may assume that $A_{\mathcal{F}} = A_{\mathcal{G}} (= A)$, i.e., a is contained in both CAFs $\mathcal{F}$ and $\mathcal{G}$. W.l.o.g., let $(a, a) \in R_{\mathcal{F}}$ and $(a, a) \notin R_{\mathcal{G}}$. Now, for a fresh argument x and a fresh claim c, let $\mathcal{H} = (A, R_{\mathcal{H}}, cl_{\mathcal{H}})$ with

$$R_{\mathcal{H}} = \{(x, b) \mid b \in A \setminus \{a\}\}$$

and $cl_{\mathcal{H}}(b) = cl_{\mathcal{F}}(b)$ for $b \in A$ and $cl_{\mathcal{H}}(x) = c$. Then $\{cl_{\mathcal{H}}(a), c\}$ has no cf-realisation in $\mathcal{F} \cup \mathcal{H}$ since a is self-attacking and x attacks every remaining occurrence of $cl_{\mathcal{H}}(a)$ in $\mathcal{F} \cup \mathcal{H}$. On the other hand, $\{cl_{\mathcal{H}}(a), c\} \in gr_c(\mathcal{G} \cup \mathcal{H})$ and $\{cl_{\mathcal{H}}(a), c\} \in stb_c(\mathcal{G} \cup \mathcal{H})$ since $\{a, x\}$ is conflict-free and attacks every other argument, moreover, x is unattacked. By Lemma B.1, for all semantics ρ, there is $S \in \rho(\mathcal{G} \cup \mathcal{H})$ which contains $\{cl_{\mathcal{H}}(a), c\}$. Thus $\mathcal{F} \not\equiv_s^\rho \mathcal{G}$. □

C Towards characterizing strong equivalence for cl-cf-stable and cl-stage semantics (Proofs of the Lemmata 3.15 and 3.16)

Lemma 3.15. *For any CAF $\mathcal{F}$, $\rho(\mathcal{F}) = \rho(\mathcal{F}^{csk})$ for the semantics $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stg\}$.*

Proof. Consider a CAF $\mathcal{F} = (F, cl)$. We show (a) $cf(F) = cf(F^{csk})$ and (b) for all $E \in cf(F)$, $E^{\dotplus}_{\mathcal{F}} = E^{\dotplus}_{\mathcal{F}^{csk}}$.

(a) Clearly, $cf(F^{csk}) \subseteq cf(F)$. Now, let $E \in cf(F)$ and assume $E \notin cf(F^{csk})$, that is, there is $a, b \in E$ such that $(a, b) \in R^{csk}$. But then either $(a, a) \in R$ or $\{(b, a), (b, b)\} \cap R \neq \emptyset$ by the definition of the cf-stable kernel, contradiction to the conflict-freeness of E in F.

(b) Now, let $E \in cf(F)$ and observe that $E^{\dotplus}_{\mathcal{F}} \subseteq E^{\dotplus}_{\mathcal{F}^{csk}}$. Now, let $c \in E^{\dotplus}_{\mathcal{F}^{csk}}$ and assume $c \notin E^{\dotplus}_{\mathcal{F}}$, that is, there is $b \in A$ with $cl(b) = c$ which is not attacked by E in $\mathcal{F}$ but there is $a \in E$ such that $(a, b) \in R^{csk}$. By definition of the cf-stable kernel, either $(a, a) \in R$ or $cl(a) = cl(b)$ and $(b, a) \in R$ or $(b, b) \in R$, contradiction to E being conflict-free in F^{csk}. □

Lemma 3.16. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ implies $(\mathcal{F} \cup \mathcal{H})^{csk} = (\mathcal{G} \cup \mathcal{H})^{csk}$ for any CAF $\mathcal{H}$ compatible with $\mathcal{F}$ and $\mathcal{G}$.*

Proof. First observe that (i) $\mathcal{F} \cup \mathcal{H} \subseteq \mathcal{F}^{csk} \cup \mathcal{H}^{csk} \subseteq (\mathcal{F} \cup \mathcal{H})^{csk}$ for every two CAFs $\mathcal{F}$, $\mathcal{H}$. Moreover, (ii) $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ implies that $\mathcal{F}, \mathcal{G}$ contain the same self-attacks by definition of the cf-stable kernel.

Now, suppose $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ and let $(a, b) \in (\mathcal{F} \cup \mathcal{H})^{csk}$. We show that $(a, b) \in (\mathcal{G} \cup \mathcal{H})^{csk}$ (the other direction is analogous): In case $(a, b) \in \mathcal{F} \cup \mathcal{H}$, we have $(a, b) \in \mathcal{F}^{csk} \cup \mathcal{H}^{csk}$ by (i). Since $\mathcal{F}^{csk} \cup \mathcal{H}^{csk} = \mathcal{G}^{csk} \cup \mathcal{H}^{csk}$ we conclude

$(a,b) \in (\mathcal{G} \cup \mathcal{H})^{csk}$. In case $(a,b) \notin \mathcal{F} \cup \mathcal{H}$, either $(a,a) \in \mathcal{F} \cup \mathcal{H}$ or $cl(a) = cl(b)$ and $\{(b,b),(b,a)\} \cap (\mathcal{F} \cup \mathcal{H}) \neq \emptyset$. In case $(a,a) \in \mathcal{F} \cup \mathcal{H}$ ($(b,b) \in \mathcal{F} \cup \mathcal{H}$), we are done since $(a,a) \in \mathcal{G} \cup \mathcal{H}$ ($(b,b) \in \mathcal{G} \cup \mathcal{H}$) by (ii). Now, suppose $cl(a) = cl(b)$ and $(b,a) \in \mathcal{F} \cup \mathcal{H}$, then $(b,a) \in \mathcal{F}^{csk} \cup \mathcal{H}^{csk}$ by (i), thus also $(b,a) \in \mathcal{G}^{csk} \cup \mathcal{H}^{csk}$ by assumption $\mathcal{F}^{csk} = \mathcal{G}^{csk}$. In case $(b,a) \in \mathcal{G} \cup \mathcal{H}$, we get $(a,b) \in (\mathcal{G} \cup \mathcal{H})^{csk}$; else we have $cl(a) = cl(b)$ and $\{(a,a),(b,b),(a,b)\} \cap (\mathcal{G} \cup \mathcal{H}) \neq \emptyset$. By definition of the cf-stable kernel we get $(a,b) \in (\mathcal{G} \cup \mathcal{H})^{csk}$. □

D Strong equivalence results (Proofs of the Theorems 3.20, 3.21, and 3.22)

Theorem 3.20. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv^{\rho}_{s} \mathcal{G}$ iff $F^{ak} = G^{ak}$ for $\rho \in \{cl\text{-}stb_{ad}, cl\text{-}ss\}$.*

Proof. First suppose $F^{ak} = G^{ak}$ and let $\mathcal{H}$ be a CAF compatible with $\mathcal{F}$, $\mathcal{G}$. By Lemma 3.19, and since $F \cup H = (F \cup H)^{ak}$ by known results for AF strong equivalence (Oikarinen and Woltran 2011, Lemma 5), we get $\rho(\mathcal{F} \cup \mathcal{H}) = \rho((\mathcal{F} \cup \mathcal{H})^{ak}) = \rho((\mathcal{G} \cup \mathcal{H})^{ak}) = \rho(\mathcal{G} \cup \mathcal{H})$. Therefore, $\mathcal{F} \equiv^{\rho}_{s} \mathcal{G}$.

Now assume $\mathcal{F}^{ak} \neq \mathcal{G}^{ak}$. We may assume $\rho(\mathcal{F}^{ak}) = \rho(\mathcal{G}^{ak})$ by Lemma 3.19; moreover, $A_{\mathcal{F}} = A_{\mathcal{G}} (= A)$ by Lemma 3.13; also, $\mathcal{F}$ and $\mathcal{G}$ contain the same self-attacks by Lemma 3.14. Thus there is $(a,b) \in R^{ak}_{\mathcal{F}} \Delta R^{ak}_{\mathcal{G}}$; w.l.o.g., let $(a,b) \in R^{ak}_{\mathcal{F}}$. We distinguish three cases: (a) $(a,a) \notin R_{\mathcal{F}^{ak}}$; (b) $(a,a) \in R_{\mathcal{F}^{ak}}$ and $cl(a) \neq cl(b)$; and (c) $(a,a) \in R_{\mathcal{F}^{ak}}$ and $cl(a) = cl(b)$.

(a) In case $(a,a) \notin R_{\mathcal{F}^{ak}}$, let $\mathcal{H}1 = (A \cup \{x,y\}, R_1, cl_1)$ with

$$R_1 = \{(b,y)\} \cup \{(x,h) \mid h \in A \setminus \{a,b\}\}$$

and $cl_1(h) = cl_{\mathcal{F}}(h)$ if $h \in A$ and $cl_1(x) = c$, $cl_1(y) = d$ for newly introduced arguments x, y and fresh claims c, d.

Clearly, $\{a,x,y\} \in stb(F^{ak} \cup H_1)$ since a defends y against b and x attacks every remaining argument. Consequently, $\{cl_1(a), c, d\} \in stb_c(\mathcal{F}^{ak} \cup \mathcal{H}_1) \subseteq \rho(\mathcal{F}^{ak} \cup \mathcal{H}_1)$. On the other hand, we have that $\{cl_1(a), c, d\}$ is not admissible in $\mathcal{G}^{ak} \cup \mathcal{H}_1$ since it has no ad-realisation in $G^{ak} \cup H_1$: Clearly, every candidate set must contain x, y, which are the only arguments having claims c, d. The only cf-realisation of $\{cl_1(a), c, d\}$ is $\{a,x,y\}$ since every other argument is attacked by x. Observe that y is not defended against b by $\{a,x,y\}$ in $G^{ak} \cup H_1$, thus $\{cl_1(a), c, d\} \notin \rho(\mathcal{G}^{ak} \cup \mathcal{H}_1)$.

(b) In case $(a,a) \in R_{\mathcal{F}^{ak}}$, $cl(a) \neq cl(b)$, let $\mathcal{H}_2 = (A \cup \{x\}, R_2, cl_2)$ with

$$R_2 = \{(x,h) \mid h \in A \setminus \{a,b\}\}$$

for a fresh argument x with $cl_2(h) = cl_{\mathcal{F}}(h)$ for $h \in A$ and $cl_2(x) = cl_{\mathcal{F}}(a)$. First observe that $(b,b) \notin R^{ak}_{\mathcal{F}}$ (and thus also not in $R^{ak}_{\mathcal{G}}$), otherwise $(a,b) \notin R^{ak}_{\mathcal{F}}$ by definition of the ad-stable kernel. It follows that $E = \{b,x\}$ is admissible in $G^{ak} \cup H_2$ since a does not attack b and x attacks each remaining argument. Let $S = cl_2(E)$ and observe that $S \cup E^{+}_{\mathcal{G}^{ak} \cup \mathcal{H}_2} = S \cup cl_2(A \setminus \{a\}) = cl_2(A)$ since $cl_2(a) \in S$. Thus we have $S \in \rho(\mathcal{G}^{ak} \cup \mathcal{H}_2)$. On the other hand, $S \notin ad_c(\mathcal{F}^{ak} \cup \mathcal{H}_2)$: Consider a cf-realisation D of S. In case $x \notin D$, we have that D is not defended against x in $F^{ak} \cup H_2$ since x attacks any potential realization of $cl_2(a)$ in F which is not self-attacking. Now assume $x \in D$, then also $b \in D$, since x attacks any other possible choice of $cl_2(b)$ in F. In this case we have that D is not defended against a in $G^{ak} \cup H_2$ and thus $S \notin ad_c(\mathcal{F}^{ak} \cup \mathcal{H}_2)$. It follows that $\rho(\mathcal{F}^{ak} \cup \mathcal{H}_2) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H}_2)$.

(c) Now assume $(a,a) \in R_{\mathcal{F}^{ak}}$ and $cl(a) = cl(b)$. Let $\mathcal{H}_3 = (A \cup \{x,y\}, R_3, cl_3)$ with

$$R_3 = \{(x,y),(y,x)\} \cup \{(y,h \mid h \in A \cup \{x\}\} \cup \{(x,h) \mid h \in A \setminus \{a,b\}\}$$

and $cl_3(h) = cl_{\mathcal{F}}(h)$ if $h \in A$ and $cl_3(x) = c$, $cl_3(y) = d$ for newly introduced arguments x, y and fresh claims c, d, that is, $\mathcal{H}_3$ coincides with the construction $\mathcal{H}_1$ from case (a) in the Proof of Theorem 3.17. The argument y guarantees that $cl\text{-}stb_{ad}(\mathcal{F}^{ak} \cup \mathcal{H}_3) \neq \emptyset$ and $cl\text{-}stb_{ad}(\mathcal{G}^{ak} \cup \mathcal{H}_3) \neq \emptyset$ since in both $\mathcal{F}^{ak} \cup \mathcal{H}_3$ and $\mathcal{G}^{ak} \cup \mathcal{H}_3$, the claim-set $\{d\}$ is i-stable witnessed by y with claim d which attacks every argument in $A \cup \{x\}$. Moreover, we have that $\{cl_3(b), c\} \in cl\text{-}stb_{ad}(G^{ak} \cup H_3)$ (and thus $\{cl_3(b), c\} \in cl\text{-}ss(G^{ak} \cup H_3)$) since $\{b,x\}$ is conflict-free and defends itself in $G^{ak} \cup H_3$—recall that $(b,b),(a,b) \notin R^{ak}_{\mathcal{G}}$ and x attacks every remaining argument except a. Since $cl_3(a) = cl_3(b)$ it follows that $\{b,x\}$ has full claim-range. On the other hand, we have that $\{cl_3(b), c\}$ has no ad-realisation in $F^{ak} \cup H_3$: Clearly, each candidate must contain x which is the only argument having claim c. Thus $\{b,x\}$ is the only cf-realisation of $\{cl_3(b), c\}$ in $F^{ak} \cup H_3$. Observe that $\{b,x\}$ is not admissible since b is not defended against the attack from a. We obtain $\rho(\mathcal{F}^{ak} \cup \mathcal{H}_3) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H}_3)$.

In every case, we have found a witness $\mathcal{H}$ showing $\rho(\mathcal{F}^{ak} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H})$. By Lemma 3.19, we get $\rho(\mathcal{F} \cup \mathcal{H}) = \rho((\mathcal{F} \cup \mathcal{H})^{ak}) = \rho(\mathcal{F}^{ak} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H}) = \rho((\mathcal{G} \cup \mathcal{H})^{ak}) = \rho(\mathcal{G} \cup \mathcal{H})$. It follows that $\mathcal{F} \not\equiv^{s}_{\rho} \mathcal{G}$. □

Theorem 3.21. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv^{\sigma_c}_{s} \mathcal{G}$ iff $F \equiv^{\sigma}_{s} G$ for any considered AF semantics σ.*

Proof. Clearly, $F \equiv^{\sigma}_{s} G$ implies $\mathcal{F} \equiv^{\sigma_c}_{s} \mathcal{F}$ since $\sigma(F \cup H) = \sigma(G \cup H)$ implies $\sigma_c(\mathcal{F} \cup \mathcal{H}) = \sigma_c(\mathcal{G} \cup \mathcal{H})$ for any CAF $\mathcal{H}$ which is compatible with $\mathcal{F}$, $\mathcal{G}$.

For the other direction, let $F \not\equiv^{\sigma}_{s} G$. We may assume $A_{\mathcal{F}} = A_{\mathcal{G}} (= A)$ by Lemma 3.13; moreover $F^{k(\sigma)} \neq G^{k(\sigma)}$ for the respective kernel which characterises the semantics σ. Since $\mathcal{F}$ and $\mathcal{G}$ agree on their arguments, there must be some attack $(a,b) \in R^{k(\sigma)}_{\mathcal{F}} \Delta R^{k(\sigma)}_{\mathcal{G}}$. W.l.o.g., let $(a,b) \in R^{sk}_{\mathcal{F}}$. By Lemma 3.14, we have $a \neq b$.

1. First, let $\sigma \in \{stb, stg\}$. Recall that stable and stage semantics are characterised via the stable kernel. Since $(a,b) \in R^{sk}_{\mathcal{F}}$, we conclude that a is not self-attacking in F (which implies $(a,a) \notin R_{\mathcal{G}}$ by Lemma 3.14).

For fresh arguments x, y, z and fresh claims c, d, e, let $\mathcal{H}_1 = (A \cup \{x, y, z\}, \{(b, z)\} \cup \{(x, h) \mid h \in (A \cup \{y\}) \setminus \{a, b\}\} \cup \{(y, h) \mid h \in A \cup \{x, z\}\}, cl_1)$ with $cl_1(h) = cl_{\mathcal{F}}(h)$ for $h \in A$, $cl_1(x) = c$, $cl_1(y) = d$, and $cl_1(z) = e$. First observe that $\{y\} \in stb(F^{sk} \cup H_1) \cap stb(G^{sk} \cup H_1)$ and thus $stb(F^{sk} \cup H_1) = stg(F^{sk} \cup H_1)$; analogously for $G^{sk} \cup H_1$. Moreover, $\{a, x, z\} \in stb(F^{sk} \cup H_1)$ since x attacks any remaining argument; thus $\{cl_1(a), c, e\} \in stb_c(\mathcal{F}^{sk} \cup \mathcal{H}_1)$. On the other hand, $\{cl_1(a), c, e\}$ has no stb-realisation in $\mathcal{G}^{sk} \cup \mathcal{H}_1$ since $\{a, x, z\}$ does not attack b; every other realisation of $\{cl_1(a), c, e\}$ in $\mathcal{G}^{sk} \cup \mathcal{H}_1$ is conflicting since z is attacked by b and x attacks every remaining argument.

2. Next we consider the semantics which are charactierised by the admissible kernel, i.e., let $\sigma \in \{ad, pr, ss\}$. Since $(a, b) \in R^{ak}_{\mathcal{F}}$, we have either (a) $(a, a) \in R^{ak}_{\mathcal{F}}$ and $\{(b, a), (b, b)\} \notin R^{ak}_{\mathcal{F}}$; or (b) $(a, a) \notin R^{ak}_{\mathcal{F}}$.
   (a) For a fresh argument x and a fresh claim c, let $\mathcal{H}_2 = (A \cup \{x\}, \{(x, h) \mid h \in A \setminus \{a, b\}\}, cl_2)$ with $cl_2(h) = cl_{\mathcal{F}}(h)$ for $h \in A$ and $cl_2(x) = c$. Then $\{b, x\} \in ad(G^{ak} \cup H_2)$ since b is not attacked by a in G^{ak} and defended against any other potential attack by x; moreover, $\{b, x\}$ semi-stable in $G^{ak} \cup H_2$ since there is no other set $D \subseteq A \cup \{x\}$ with $x \in D^{\oplus}_{\mathcal{G}^{ak} \cup \mathcal{H}_2}$ (besides $\{x\}$ which is a proper subset of $\{b, x\}$). Thus $\{cl_2(b), c\} \in \sigma_c(\mathcal{G}^{ak} \cup \mathcal{H}_1)$. On the other hand, $\{b, x\} \notin ad(F^{ak} \cup H_2)$ since b is not defended against a in $F^{ak} \cup H_2$. Thus $\{cl_2(b), c\} \notin \sigma_c(\mathcal{F}^{ak} \cup \mathcal{H}_1)$.
   (b) In case $(a, a) \notin R_{\mathcal{F}}$, consider construction $\mathcal{H}_1$ from (1). $\{cl_1(a), c, e\} \in \sigma_c(\mathcal{F}^{ak} \cup \mathcal{H}_1)$ since $\{cl_1(a), c, e\} \in stb_c(\mathcal{F}^{ak} \cup \mathcal{H}_1)$; on the other hand, $\{cl_1(a), c, e\}$ has no ad-realisation in $\mathcal{G}^{ak} \cup \mathcal{H}_1$ since z is not defended against b; every other realisation of $\{cl_1(a), c, e\}$ in $\mathcal{G}^{ak} \cup \mathcal{H}_1$ is conflicting since z is attacked by b and x attacks every remaining argument.
3. For $\sigma = co$, we have either $(a, a) \notin R^{ak}_{\mathcal{F}}$ or $(b, b) \notin R^{ak}_{\mathcal{F}}$. The case $(a, a) \notin R^{ak}_{\mathcal{F}}$ is analogous to (2.b). It remains to discuss the case $(b, b) \notin R^{ak}_{\mathcal{F}}$. For fresh arguments x, y and fresh claims c, d, let $\mathcal{H}_3 = (A \cup \{x, y\}, \{(y, a), (y, y)\} \cup \{(x, h) \mid h \in A \setminus \{a, b\}\}, cl_3)$ with $cl_3(h) = cl_{\mathcal{F}}(h)$ for $h \in A$, $cl_3(x) = c$, $cl_3(y) = d$. Then $\{cl_3(b), c\} \in co_c(\mathcal{G}^{ak} \cup \mathcal{H}_3)$ since $\{b, x\}$ is conflict-free and x defends b against each attack; moreover, a is not defended by $\{b, x\}$ against y. On the other hand, $\{cl_3(b), c\} \notin co_c(\mathcal{F}^{ak} \cup \mathcal{H}_3)$ since the only conflict-free sets containing x are $\{b, x\}$, which is not defended against a; $\{x\}$, which does not realise $cl_3(b)$; and $\{a, x\}$, which is not defended against y (and a has potentially a different claim than b).
4. For $\sigma = grd$, either (a) $(b, b) \in R^{ak}_{\mathcal{F}}$ and $\{(b, a), (a, a)\} \notin R^{ak}_{\mathcal{F}}$; or (b) $(b, b) \notin R^{ak}_{\mathcal{F}}$. Observe that (b) coincides with (3) where we constructed an expansion $\mathcal{H}_3$ yielding different i-grounded claim-sets in $\mathcal{F}^{ak} \cup \mathcal{H}_3$ and $\mathcal{G}^{ak} \cup \mathcal{H}_3$. It remains to discuss the case $(b, b) \in R^{ak}_{\mathcal{F}}$. For fresh arguments x, y and fresh claims c, d, let $\mathcal{H}_4 = (A \cup \{x, y\}, \{(b, y)\} \cup \{(x, h) \mid h \in A \setminus \{a, b\}\}, cl_3)$ with $cl_4(h) = cl_{\mathcal{F}}(h)$ for $h \in A$, $cl_4(x) = c$, $cl_4(y) = d$. Then x is unattacked and defends a in $\mathcal{F}^{ak} \cup \mathcal{H}_4$, which in turn defends y. Thus $\{cl_4(a), c, d\} \in gr_c(\mathcal{F}^{ak} \cup \mathcal{H}_4)$. On the other hand, we have $\{cl_4(a), c, e\} \notin gr_c(\mathcal{G}^{ak} \cup \mathcal{H}_4)$ since y is not defended against b.

This concludes the proof for the semantics $\sigma \in \{stb, stg, ad, pr, ss, gr, co\}$: In every case, we have found a witness $\mathcal{H}$ showing $\rho(\mathcal{F}^{ak} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H})$. By Lemma 3.19, we get $\rho(\mathcal{F} \cup \mathcal{H}) = \rho((\mathcal{F} \cup \mathcal{H})^{ak}) = \rho(\mathcal{F}^{ak} \cup \mathcal{H}) \neq \rho(\mathcal{G}^{ak} \cup \mathcal{H}) = \rho((\mathcal{G} \cup \mathcal{H})^{ak}) = \rho(\mathcal{G} \cup \mathcal{H})$. Consequently, $\mathcal{F} \not\equiv^s_\rho \mathcal{G}$.

Next we discuss conflict-free and naive semantics.

5. For $\sigma \in \{cf, na\}$, first notice that we can assume $\sigma_c(\mathcal{F}) = \sigma_c(\mathcal{G})$ otherwise let $\mathcal{H} = (\emptyset, \emptyset, \emptyset)$; furthermore, we can assume $\sigma(F) \neq \sigma(G)$; otherwise consider instead $\mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \cup \mathcal{H}$ for a compatible CAF $\mathcal{H}$ with $\sigma_c(\mathcal{F} \cup \mathcal{H}) \neq \sigma_c(\mathcal{G} \cup \mathcal{H})$.
   First consider the case that there is some $E \in \sigma(F) \Delta \sigma(G)$ such that E is not conflict-free in F (or G, respectively). W.l.o.g., let $E \in \sigma(F)$ such that E is subset-minimal among $\sigma(F) \Delta \sigma(G)$, i.e., there is no $E' \in \sigma(F) \Delta \sigma(G)$ with $E' \subsetneq E$; otherwise, exchange the roles of F and G. For a fresh argument x and a fresh claim c, let $\mathcal{H}_5 = (A \cup \{x\}, \{(x, b) \mid b \in A \setminus E, cl_5)$ with $cl_5(b) = cl_{\mathcal{F}}(b)$ for $b \in A$ and $cl_5(x) = c$. Then $cl_5(E) \cup \{c\} \in na(\mathcal{F} \cup \mathcal{H}_5)$ but $\{cl_5(E) \cup \{c\}$ has no cf-realisation in $\mathcal{G} \cup \mathcal{H}_5$ since every subset of E is conflicting and x attacks all remaining arguments, thus $cl_5(E) \cup \{c\} \notin \sigma_c(\mathcal{G} \cup \mathcal{H}_5)$. Observe that this suffices to conclude the proof for conflict-free semantics.
   For naive semantics, assume that for all $E \in \sigma(F) \Delta \sigma(G)$, $E \in cf(F) \cap cf(G)$. We derive a contradiction: W.l.o.g., let $E \in \sigma(F)$ such that E is subset-minimal among $\sigma(F) \Delta \sigma(G)$. Since E is conflict-free in G, there is some $E' \in na(G)$ with $E \subseteq E'$. But then $E' \in cf(G)$ and thus $E \in cf(F)$ by assumption, contradiction to E being a subset-maximal conflict-free extension in F.

We have shown $\mathcal{F} \not\equiv^{\sigma_c}_s \mathcal{G}$ for every semantics σ under consideration. □

Theorem 3.22. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv^{cl\text{-}\sigma}_s \mathcal{G}$ iff $\mathcal{F} \equiv^{\sigma_c}_s \mathcal{G}$ for $\sigma \in \{na, pr\}$.*

Proof. If $\mathcal{F} \equiv^{\sigma_c}_s \mathcal{G}$, then $\sigma_c(\mathcal{F} \cup \mathcal{H}) = \sigma_c(\mathcal{G} \cup \mathcal{H})$ for every compatible CAF $\mathcal{H}$. $\mathcal{F} \equiv^{cl\text{-}\sigma}_s \mathcal{G}$ follows since $cl\text{-}\sigma(\mathcal{F} \cup \mathcal{H})$ are the subset-maximal i-naive claim-sets of $\mathcal{F} \cup \mathcal{H}$ and, analogously, $cl\text{-}\sigma(\mathcal{G} \cup \mathcal{H})$ are the subset-maximal i-naive claim-sets of $\mathcal{G} \cup \mathcal{H}$.

Now assume $\mathcal{F} \not\equiv^{\sigma_c}_s \mathcal{G}$ and let $\sigma = pr$ (the proof for $\sigma = na$ is analogous). We may assume $A_{\mathcal{F}} = A_{\mathcal{G}} (= A)$ (by Lemma 3.13); also, $pr_c(\mathcal{F}) \neq pr_c(\mathcal{G})$(otherwise consider instead $\mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \cup \mathcal{H}$ for a compatible CAF $\mathcal{H}$ with $pr_c(\mathcal{F} \cup \mathcal{H}) \neq pr_c(\mathcal{G} \cup \mathcal{H})$). The latter implies $ad(F) \neq ad(G)$. Consider a subset-minimal set $E \in ad(F) \Delta ad(G)$, i.e., there is no $E' \in ad(F) \Delta ad(G)$ with $E' \subsetneq E$. W.l.o.g., let $E \in ad(F)$.

In case there is no $D \in ad(F) \cap ad(G)$ with $D \subsetneq E$, we consider the following construction: For a fresh argument x and a fresh claim c, let $\mathcal{H}_1 = ((A \cup \{x\}, \{(x, b) \mid b \in (A \setminus$

$E\}, cl_1)$ with $cl_1(b) = cl_{\mathcal{F}}(b)$ for $b \in A$ and $cl_1(x) = c$. Then $E \cup \{x\} \in ad(F \cup H)$ since $E \cup \{x\}$ is conflict-free and defends itself, thus $cl(E) \cup \{c\} \in ad_c(\mathcal{F} \cup \mathcal{H}_1)$. Also observe that there is no other admissible set D with $D \nsubseteq E \cup \{x\}$ which contains x, thus $cl(E) \cup \{x\}$ is a subset-maximal i-admissible set in $\mathcal{F} \cup \mathcal{H}_1$. On the other hand, $cl(E) \cup \{x\}$ has no ad-realisation in $\mathcal{G} \cup \mathcal{H}_1$ since no subset of E is admissible in G by minimality of E and x attacks every remaining argument. Thus $cl(E) \cup \{c\} \notin cl\text{-}pr(\mathcal{G} \cup \mathcal{H}_1)$.

Observe that for naive semantics, this concludes the proof since by minimality of E, we can always find a conflict-free set E such that there is no $D \in cf(F) \cap cf(G)$ with $D \subsetneq E$.

In case of preferred semantics, we now assume that the assumption is not satisfied, i.e., there is $D \in ad(F) \cap ad(G)$ with $D \subsetneq E$. There is some $a \in E$ such that $a \notin D$ for any $D \in ad(F) \cap ad(G)$ with $D \subsetneq E$: Otherwise every argument $a \in E$ is contained in some admissible set $D \subsetneq E$, and thus $\bigcup\{D \in ad(G) \cap ad(F) \mid D \subsetneq E\} = E$, i.e., the union of all admissible sets contained in E coincides with E, which implies E is admissible in G, contradiction to the assumption. We consider the following construction: For fresh arguments x, y and fresh claims c, d, let $\mathcal{H}_2 = (A \cup \{x, y\}, \{(a, y)\} \cup \{(y, b) \mid b \in E\} \cup \{(x, b) \mid b \in (A \setminus E)\}, cl_2)$ with $cl_1(b) = cl_{\mathcal{F}}(b)$ for $b \in A$, $cl_2(y) = d$ and $cl_2(x) = c$. First observe that there is no $D \subsetneq E$ such that $D \in ad(F \cup H_2)$ (or $D \in ad(G \cup H_2)$ by the choice of a: y attacks every argument $b \in E$ and a is the only argument which defends E against y. Similar as above, we conclude that $cl(E) \cup \{c\} \in cl\text{-}pr(\mathcal{F} \cup \mathcal{H}_2)$ since E is admissible in $F \cup H_2$ and x attacks every remaining argument; on the other hand, $cl(E) \cup \{c\} \notin cl\text{-}pr(\mathcal{G} \cup \mathcal{H}_2)$ since no subset D of E is admissible in G.

In every case, we have found a witness $\mathcal{H}$ such that $cl\text{-}\sigma(\mathcal{F} \cup \mathcal{H}) \neq cl\text{-}\sigma(\mathcal{G} \cup \mathcal{H})$, thus $\mathcal{F} \not\equiv_s^{cl\text{-}\sigma} \mathcal{G}$. □

E Computational Complexity of VER-OE$_\rho$ (Proof details of Theorem 5.1)

We make use of (variants of) the following standard construction which has been already defined in the proof of Proposition 5.3.

Reduction 1. For a formula φ which is given by a set of clauses C over atoms in V we construct an AF $F = (A, R)$ with

$$A = V \cup \bar{V} \cup C \text{ with } \bar{V} = \{\bar{v} \mid v \in V\};$$
$$R = \{(v, cl) \mid cl \in C, v \in cl\} \cup \{(cl, cl) \mid cl \in C\} \cup \{(\bar{v}, cl) \mid cl \in C, \neg v \in cl\} \cup \{(v, \bar{v}), (\bar{v}, v) \mid v \in V\}.$$

Proposition 5.3. *Deciding* VER-OE$_\rho$ *is* Π_2^{P}*-hard for* $\rho \in \{stb_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$.

Proof. Let $\rho = stb_c$ and let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of QSAT$_2^\forall$ where φ is given by a set of clauses C over atoms in $V = Y \cup Z$. We restate the constructions of the two CAFs $\mathcal{F} = (A_{\mathcal{F}}, R_{\mathcal{F}}, cl_{\mathcal{F}})$, $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, id)$: For $\mathcal{F}$ we have $A_{\mathcal{F}} = A$, $R_{\mathcal{F}} = R$ where (A, R) is the AF from Reduction 1; $cl_{\mathcal{F}}(z) = cl_{\mathcal{F}}(\bar{z}) = z$ for $z \in Z$ and $cl_{\mathcal{F}}(a) = a$ else. The CAF $\mathcal{G}$ is given by $A_{\mathcal{G}} = Y \cup \bar{Y} \cup Z$ and $R_{\mathcal{G}} = \{(y, \bar{y}), (\bar{y}, y) \mid y \in Y\}$. We observe that $stb_c(\mathcal{G}) = \{Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \mid Y' \subseteq Y\}$.

We show Ψ is valid iff $stb_c(\mathcal{F}) = stb_c(\mathcal{G})$.

First assume Ψ is valid, let $Y' \subseteq Y$ and consider a model $M = Y' \cup Z'$ of φ. Then the set of arguments $E = M \cup \{\bar{v} \mid v \notin M\}$ is stable in $\mathcal{F}$: Clearly, E is conflict-free; moreover, E attacks every $cl \in C$ since every clause cl is satisfied by M: In case there is $v \in M$ with $v \in cl$ we have $v \in E$ with $(v, cl) \in R_{\mathcal{F}}$; in case there is $\neg v \in M$ we have $\bar{v} \in E$ with $(\bar{v}, cl) \in R_{\mathcal{F}}$. Since $cl_{\mathcal{F}}(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ we have shown that every such claim-set is contained in $stb_c(\mathcal{F})$; $stb_c(\mathcal{F}) = stb_c(\mathcal{G})$ thus follows.

Now assume $stb_c(\mathcal{F}) = stb_c(\mathcal{G})$. Let $Y' \subseteq Y$, let E be a stb_c-realisation of $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ and let $Z' = E \cap Z$. We show that $M = Y' \cup Z'$ is a model of φ: Consider an arbitrary clause $cl \in C$. By assumption that E is stable in F there is some $a \in E$ such that $(a, cl) \in R_{\mathcal{F}}$. In case $a = v$ for some atom $v \in V$ we have $v \in cl$; in this case $v \in M$ and thus cl is satisfied. In case $a = \bar{v}$ for some atom v we have $\neg v \in cl$; in this case $v \notin M$ since $\bar{v} \in E$ and thus cl is satisfied. We obtain that M is a model of φ. We have shown that for any $Y' \subseteq Y$ there is $Z' \subseteq Z$ such that $Y' \cup Z'$ is a model of φ; i.e., Ψ is valid.

Π_2^{P}-hardness of VER-OE$_\rho$ for $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{ad}\}$ follows since $stb_c(\mathcal{F}) = cl\text{-}stb_{cf}(\mathcal{F}) = cl\text{-}stb_{ad}(\mathcal{F})$ and $stb_c(\mathcal{G}) = cl\text{-}stb_{cf}(\mathcal{G}) = cl\text{-}stb_{ad}(\mathcal{G})$. □

Proposition E.1. *Deciding* VER-OE$_\rho$ *is* Π_2^{P}*-hard,* $\rho \in \{cf_c, ad_c\ cl\text{-}na, cl\text{-}pr\}$.

Proof. We will first show the statement for cl-naive semantics: Consider an instance $\Psi = \forall Y \exists Z \varphi(Y, Z)$ of QSAT$_2^\forall$, where φ is a 3-CNF, given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in $V = Y \cup Z$. We construct two CAFs $\mathcal{F} = (A_{\mathcal{F}}, R_{\mathcal{F}}, cl_{\mathcal{F}})$, $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, id)$. The CAF $\mathcal{F}$ is given by

$$A_{\mathcal{F}} = Y \cup \bar{Y} \cup \{v_i \mid v \in cl_i, cl_i \in C\} \cup \{\bar{v}_i \mid \neg v \in cl_i, cl_i \in C\};$$
$$R_{\mathcal{F}} = \{(v_i, \bar{v}_j), (\bar{v}_j, v_i), (v, \bar{v}_i), (\bar{v}_i, v), (v_i, \bar{v}), (\bar{v}, v_i) \mid v \in V; i, j \leq n\};$$

and $cl_{\mathcal{F}}(v_i) = cl_{\mathcal{F}}(\bar{v}_i) = i$, $cl_{\mathcal{F}}(y) = y$, and $cl_{\mathcal{F}}(\bar{y}) = \bar{y}$. We construct a CAF $\mathcal{G}$ having the cl-naive claim-sets $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \ldots, n\}$ for every $Y' \subseteq Y$ by setting $A_{\mathcal{G}} = Y \cup \bar{Y} \cup \{1, \ldots, n\}$ and $R_{\mathcal{G}} = \{(y, \bar{y}), (\bar{y}, y) \mid y \in Y\}$.

First assume Ψ is valid. Fix some $Y' \subseteq Y$. Since Ψ is valid, there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. Let $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{v_i \mid v \in M\} \cup \{\bar{v}_i \mid v \notin M\}$. E is conflict-free since conflicts appear only between arguments representing negated literals; moreover, since M is a model of φ, we have that for each clause cl_i there is either a positive literal $v \in cl_i$ with $v \in M$ or a negative literal $\bar{v} \in cl_i$ with $v \notin M$. Thus $\{1, \ldots, n\} \subseteq cl_{\mathcal{F}}(E)$; moreover, $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq cl_{\mathcal{F}}(E)$. $S = cl_{\mathcal{F}}(E)$ is a maximal cl-conflict-free claim-set since $S \cup \{c\} \notin cf_c(\mathcal{F})$ for any $c \in (Y \cup \bar{Y}) \setminus S$ as each realization of $S \cup \{c\}$

contains $y, \bar{y}$ for some $y \in Y$. It follows that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1,\ldots,n\} \in \textit{cl-na}(\mathcal{F})$ for every $Y' \subseteq Y$. Moreover, there is no other cl-naive claim-set of $\mathcal{F}$ since every proper superset has no cf-realisation in $\mathcal{F}$ as outlined above. We have shown $\textit{cl-na}(\mathcal{F}) = \textit{cl-na}(\mathcal{G})$ in case Ψ is valid.

Now assume $\textit{cl-na}(\mathcal{F}) = \textit{cl-na}(\mathcal{G})$ and fix some $Y' \subseteq Y$. Let E be some cf-realisation of $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1,\ldots,n\}$, let $Z' = \{z \mid z_i \in E\}$ and let $M = Y' \cup Z'$. Now, consider an arbitrary clause cl_i. Since E cf-realises S, there is some argument with claim i in E, that is, either $v_i \in E$ or $\bar{v}_i \in E$ for some $v \in Y \cup Z$ (observe that $y_i \in E$ iff $y \in E$ and $\bar{y}_i \in E$ iff $\bar{y} \in E$, thus a further case distinction for $y \in Y$, $\bar{y} \in \bar{Y}$ is not required). In the former case, we have $v \in M$ and thus M satisfies cl_i, in the latter case $v \notin M$ and thus cl_i is satisfied. We obtain that M is a model of φ.

Since conflict-free semantics satisfy downward closure (each subset of a conflict-free set is conflict-free), we have $cf_c(\mathcal{F}) = cf_c(\mathcal{G})$ iff $\textit{cl-na}(\mathcal{F}) = \textit{cl-na}(\mathcal{G})$ and thus the statement follows for i-conflict-free semantics. By symmetry of $\mathcal{F}$ and $\mathcal{G}$ we furthermore have $ad(F) = cf(F)$ and $ad(G) = cf(G)$ which implies $ad_c(\mathcal{F}) = cf_c(\mathcal{F})$, $ad_c(\mathcal{G}) = cf_c(\mathcal{G})$, $\textit{cl-pr}(\mathcal{F}) = \textit{cl-na}(\mathcal{F})$, and $\textit{cl-pr}(\mathcal{G}) = \textit{cl-na}(\mathcal{G})$. Thus Π_2^{P}-hardness of VER-OE_ρ for i-admissible and cl-preferred semantics follow. $\square$

Proposition E.2. *Deciding* VER-OE_{na_c} *is* Π_2^{P}*-hard.*

Proof. Consider an instance $\Psi = \forall Y \exists Z \varphi(Y,Z)$ of $\text{QSAT}_2^\forall$, where φ is a 3-CNF, given by a set of clauses $C = \{cl_1,\ldots,cl_n\}$ over atoms in $V = Y \cup Z$. We construct two CAFs $\mathcal{F} = (A_\mathcal{F}, R_\mathcal{F}, cl_\mathcal{F})$, $\mathcal{G} = (A_\mathcal{G}, R_\mathcal{G}, id)$, where $\mathcal{F}$ modifies the standard construction (A,R) (cf. Reduction 1) as follows:

$$
\begin{aligned}
&A_\mathcal{F} = A \cup Y_2 \cup \bar{Y}_2 \cup Z_2;\\
&R_\mathcal{F} = (R \setminus \{(cl,cl) \mid cl \in C\}) \cup\\
&\qquad \{(y_2,\bar{y}_2) \mid y_2 \in Y_2\} \cup (y,\bar{y}_2),(y_2,\bar{y}) \mid y \in Y\};
\end{aligned}
$$

and $cl_\mathcal{F}(y_2) = y$, $cl_\mathcal{F}(\bar{y}_2) = \bar{y}$ for $y_2 \in Y_2$; $cl_\mathcal{F}(z_2) = cl_\mathcal{F}(\bar{z}_2) = z$ for $z_2 \in Z_2$; $cl_\mathcal{F}(cl) = \bar{\varphi}$ for $cl \in C$; $cl_\mathcal{F}(a) = a$ else.

Observe that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\bar{\varphi}\}$ is i-naive for every $Y' \subseteq Y$: Let $E = Y_2' \cup \{\bar{y}_2 \mid y_2 \notin Y_2'\} \cup Z_2 \cup C \cup E'$ with $Y_2' \subseteq Y_2$ and $E' \subseteq V \cup \bar{V}$ is a non-conflicting subset-maximal set of arguments which do not attack any $cl \in C$. E is conflict-free and subset-maximal by the choice of E'; moreover, $cl_\mathcal{F}(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\bar{\varphi}\}$.

We construct $\mathcal{G} = (A_\mathcal{G}, R_\mathcal{G}, cl_\mathcal{G})$ such that $na_c(\mathcal{G}) = \{Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\bar{\varphi}\} \mid Y' \subseteq Y\} \cup \{Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \mid Y' \subseteq Y\}$. Let

$$
\begin{aligned}
&A_\mathcal{G} = Y_1 \cup \bar{Y}_1 \cup Y_2 \cup \bar{Y}_2 \cup Z \cup \{\bar{\varphi}\};\\
&R_\mathcal{F} = \{(y_i,\bar{y}_i) \mid y_i \in Y_i, i \leq 2\} \cup\\
&\qquad \{(a,b) \mid a \in Y_1 \cup \bar{Y}_1, b \in Y_2 \cup \bar{Y}_2 \cup \{\bar{\varphi}\}\};
\end{aligned}
$$

and $cl_\mathcal{G}(y_i) = y$, $cl_\mathcal{G}(\bar{y}_i) = \bar{y}$ for $y_i \in Y_i$; $cl_\mathcal{G}(z) = z$, $cl_\mathcal{G}(\bar{z}) = \bar{z}$ for $z \in Z$; $cl_\mathcal{G}(\bar{\varphi}) = \bar{\varphi}$. It can be checked that $\mathcal{G}$ has precisely the desired i-naive extensions.

We show that Ψ is valid iff $na_c(\mathcal{F}) = na_c(\mathcal{G})$. First, assume Ψ is valid and fix some $Y' \subseteq Y$. There is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. Let $E = M \cup \{\bar{v} \mid v \notin M\} \cup Y_2' \cup \{\bar{y}_2 \mid y_2 \notin Y_2'\} \cup Z_2$. E is conflict-free; moreover, E is subset-maximal among conflict-free sets since any other argument $a \in A_\mathcal{F} \setminus E$ is in conflict with E: Since M is a model of φ, we have that for each clause cl_i there is either a positive literal $v \in cl$ with $v \in M$ or a negative literal $\bar{v} \in cl$ with $v \notin M$; that is, each cl is attacked in $\mathcal{F}$. Also, E contains either v or $\bar{v}$ for any atom $v \in Y \cup Z \cup Y_2$, thus any argument representing a literal in $\mathcal{F}$ which is not a member of E is attacked by E. It follows that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \in na_c(\mathcal{F})$ for every $Y' \subseteq Y$. Each i-naive claim-set is thus either of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\bar{\varphi}\}$ or $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$. Consequently, $na_c(\mathcal{F}) = na_c(\mathcal{G})$ in case Ψ is valid.

Now assume $na_c(\mathcal{F}) = na_c(\mathcal{G})$ and fix $Y' \subseteq Y$. Consider a na_c-realisation E of $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ and let $Z' = E \cap Z$. We show $M = Y' \cup Z'$ is a model of φ: Consider an arbitrary clause $cl \in C$. Since E is a subset-maximal conflict-free set of arguments we have $E \cup \{cl\}$ is conflicting; that is, there is $a \in E$ such that a attacks cl. In case $a = v$ for some atom v we have $v \in cl$; in case $a = \bar{v}$ for some v we have $\bar{v} \in cl$. In the former case, $v \in M$ and thus cl is satisfied, in the latter case we have $v \notin M$ and thus cl is satisfied. We obtain that M is a model of φ. $\square$

We present a modification of the standard reduction for AFs (cf. (Dvorák and Dunne 2018, Reduction 3.6)) .

Reduction 2. For a formula φ which is given by a set of clauses C over atoms in V, let (A', R') be given as in Reduction 1. We construct an AF $F = (A,R)$ with $A = A' \cup \{\varphi, \bar{\varphi}\}$ and

$$R = R' \cup \{(cl,\varphi) \mid cl \in C\} \cup \{(\varphi,\bar{\varphi}),(\bar{\varphi},\varphi)\}.$$

Proposition E.3. *Deciding* VER-OE_{co_c} *is* Π_2^{P}*-hard.*

Proof. Consider an instance $\Psi = \forall Y \exists Z \varphi(Y,Z)$ of $\text{QSAT}_2^\forall$, where φ is given by a set of clauses $C = \{cl_1,\ldots,cl_n\}$ over atoms in $V = Y \cup Z$. We may assume that $Z \neq \emptyset$; i.e., there is some $z_0 \in Z$. We construct two CAFs $\mathcal{F} = (A_\mathcal{F}, R_\mathcal{F}, cl_\mathcal{F})$, $\mathcal{G} = (A_\mathcal{G}, R_\mathcal{G}, cl_\mathcal{G})$, where $\mathcal{F}$ is a modification of the standard construction (A,R) (cf. Reduction 2) with

$$
\begin{aligned}
&A_\mathcal{F} = A \cup \{d_v \mid v \in V\};\\
&R_\mathcal{F} = R \cup \{(d_v,d_v),(v,d_v),(\bar{v},d_v),\\
&\qquad (d_v,a) \mid v \in V, a \in V \cup \bar{V}\};
\end{aligned}
$$

$cl_\mathcal{F}(\bar{\varphi}) = z_0$, $cl_\mathcal{F}(z) = cl_\mathcal{F}(\bar{z}) = z$ and $cl_\mathcal{F}(a) = a$ else. We observe that $co_c(\mathcal{F})$ contains $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ for each $Y' \subseteq Y$ as well as $\emptyset$. A witness is given by the complete extension $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\bar{\varphi}\}$. Moreover, since at least one of $v, \bar{v}$ has to be contained in a complete extension E in order to be defended we observe that no subset of any $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$, $Y' \subseteq Y$, is i-complete in $\mathcal{F}$.

The CAF $\mathcal{G}$ is given by

$$A_{\mathcal{G}} = Y \cup \bar{Y} \cup Z \cup \{\varphi, \bar{\varphi}, d_\varphi\} \cup \{d_y \mid y \in Y\};$$
$$R_{\mathcal{G}} = \{(y, \bar{y}), (\bar{y}, y) \mid y \in Y\}) \cup \{(d_v, d_v), (v, d_v), (\bar{v}, d_v), (d_v, a) \mid v \in Y \cup \{\varphi\}, a \in A_{\mathcal{G}}\};$$

and $cl_{\mathcal{G}}(\bar{\varphi}) = z_0$ and $cl_{\mathcal{G}}(a) = a$ else. Observe that $\mathcal{G}$ contains the i-complete claim-sets $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\varphi\} \cup Z$ and $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ for $Y' \subseteq Y$ as well as the empty claim-set $\emptyset$. Given a complete extension $E \neq \emptyset$ of G, we observe that either y or $\bar{y}$ is contained in E for every $y \in Y$ since every $a \in Y \cup \bar{Y} \cup \{\varphi\} \cup Z$ must be defended against d_y; similarly, either φ or $\bar{\varphi}$ is contained in E. Thus there is some $Y' \subseteq Y$ such that $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq E$. In case $\varphi \in E$ we have that E is of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\varphi\} \cup Z$ for some $Y' \subseteq Y$ since each $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\varphi\}$ defends itself and Z in G; in case $\bar{\varphi} \in E$ we have that E is of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\bar{\varphi}\} \cup Z$ for some $Y' \subseteq Y$ since each $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\bar{\varphi}\}$ defends itself and Z in G. In the former case, $cl_{\mathcal{G}}(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\varphi\} \cup Z$, in the latter case, $cl_{\mathcal{G}}(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$.

We show Ψ is valid iff $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\varphi\} \in co_c(\mathcal{F})$ for each $Y' \subseteq Y$.

Assume Ψ is valid; fix some $Y' \subseteq Y$. Then there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. We show that $E = M \cup \{\bar{v} \mid v \notin M\} \cup \{\varphi\}$ is complete in F: E is conflict-free; moreover, since M is a model of φ we have that each clause $cl \in C$ is attacked; consequently, E defends φ against each attack. E contains each defended argument since it attacks any remaining argument $a \notin E$ in F. Thus $cl_{\mathcal{F}}(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{\varphi\} \cup Z \in co_c(\mathcal{F})$. As Y' was arbitrary, we have shown $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\varphi\} \in co_c(\mathcal{F})$ for each $Y' \subseteq Y$.

Now assume $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\varphi\} \in co_c(\mathcal{F})$ for each $Y' \subseteq Y$. Fix some $Y' \subseteq Y$ and let E be the complete realization of $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\varphi\}$ in $\mathcal{F}$. We show that $M = Y' \cup Z'$ with $Z' = E \cap Z$ is a model of φ: From $\varphi \in E$ we obtain that every clause $cl \in C$ is attacked; that is, for every $cl \in C$, there is $a \in E$ with $(a, cl) \in R_{\mathcal{G}}$. In case $a = v$ for some $v \in V$, we have $v \in M \cap cl$; in case $a = \bar{v}$ for some $v \in V$ we have $\neg v \in cl$ and $v \notin M$—in both cases, cl is satisfied, thus M is a model of φ. It follows that Ψ is valid.

As outlined above, $co_c(\mathcal{F})$ contains $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z$ for each $Y' \subseteq Y$, moreover, $\emptyset \in co_c(\mathcal{F})$ and $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup Z \cup \{\varphi\} \in co_c(\mathcal{F})$ for each $Y' \subseteq Y$ iff Ψ is valid. By design of $\mathcal{G}$ we obtain Ψ is valid iff $co_c(\mathcal{F}) = co_c(\mathcal{G})$. □

To show Π_3^{P}-hardness of VER-OE$_{ss_c}$ and VER-OE$_{stg_c}$, we will make use of the following reduction, taken from (Dvořák et al. 2021).

Reduction 3. Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^{\exists}$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. Let $V' = X \cup Y$ and let $\bar{x}$ denote $\neg x$. We can assume that there is $y_0 \in Y$ with $y_0 \in cl$ for all $cl \in C$ (otherwise we can add such a y_0 without changing the validity of Ψ). Let $\mathcal{F} = (A, R, cl)$ be given by

$$A = V \cup \bar{V} \cup \mathcal{C} \cup \{d_1, d_2, \varphi, \bar{\varphi}\} \cup \{d_v, d_{\bar{v}} \mid v \in V'\};$$
$$R = \{(a, cl) \mid cl \in C, a \in cl, a \in V \cup \bar{V}\} \cup \{(cl, \varphi) \mid cl \in C\} \cup \{(a, d_a), (d_a, d_a) \mid a \in V' \cup \bar{V}'\} \cup \{(d_i, d_j) \mid i = 1, 2\} \cup \{(v, \bar{v}), (\bar{v}, v) \mid v \in V\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi), (\varphi, d_1)\};$$

and $cl(v) = cl(\bar{v}) = v$ for $v \in Y \cup Z$; $cl(cl_i) = \bar{\varphi}$ for $i \leq n$; $cl(d_1) = cl(d_2) = d$; and $cl(a) = a$ else.

Proposition E.4. *Deciding* VER-OE$_\rho$ *is* Π_3^{P}*-hard,* $\rho \in \{ss_c, stg_c\}$.

Proof. Consider an instance $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ of QSAT$_3^{\exists}$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. Let $\mathcal{F}$ be given as in (Dvořák et al. 2021) (cf. Reduction 3). In (Dvořák et al. 2021), it has been shown that $cl\text{-}ss(\mathcal{F}) = cl\text{-}stg(\mathcal{F})$, moreover, Ψ is not valid iff $ss_c(\mathcal{F}) = cl\text{-}ss(\mathcal{F}) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$. It suffices to construct $\mathcal{G}$ in such a way that $\rho(\mathcal{G}) = cl\text{-}ss(\mathcal{F})$: Then Ψ is not valid iff $ss_c(\mathcal{G}) = cl\text{-}ss(\mathcal{F}) = ss_c(\mathcal{F})$. We construct such a CAF $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, id)$ by setting

$$A_{\mathcal{G}} = X \cup \bar{X} \cup Y \cup Z \cup \{\varphi, \bar{\varphi}\}, \text{ and}$$
$$R_{\mathcal{G}} = \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi)\}.$$

It is easy to see that $\mathcal{G}$ possesses exactly the desired i-semi-stable claim-sets.

This concludes the proof for i-semi-stable semantics. Π_3^{P}-hardness of VER-OE$_{stg_c}$ follows from the fact that $ss_c(\mathcal{F}) = stg_c(\mathcal{F})$ and $ss_c(\mathcal{G}) = stg_c(\mathcal{G})$. □

We recall the construction of $\mathcal{F}$ from the proof of Proposition 5.4:

Reduction 4. Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^{\exists}$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. Let $V' = X \cup Y$ and let $\bar{x}$ denote $\neg x$. We can assume that there is $y_0 \in Y$ with $y_0 \in cl$ for all $cl \in C$ (otherwise we can add such a y_0 without changing the validity of Ψ). Let $\mathcal{F}' = (A', R', cl')$ be given as in Reduction 3. We define $\mathcal{F} = (A, R, cl)$ with

$$A = (A' \setminus \{d_1, d_2, \varphi, \bar{\varphi}\}) \cup \{\varphi_1, \varphi_2\};$$
$$R = R'|_A \cup \{(cl, \varphi_1) \mid cl \in C\} \cup \{(\varphi_1, \varphi_2), (\varphi_2, \varphi_2)\};$$

and $cl'(a) = cl(a)$ for $a \in A' \setminus \{d_1, d_2, \varphi, \bar{\varphi}\}$, $cl(\varphi_1) = cl(\varphi_2) = \varphi$.

Lemma E.5. *Let* $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ *be an instance of* QSAT$_3^{\exists}$ *and let* $\mathcal{F} = (A, R, cl)$ *be given as in Reduction 4. Then each cl-semi-stable and each cl-stage claim-set of* $\mathcal{F}$ *is of the form* $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ *for some* $X' \subseteq X$ *and for* $e \in \{\varphi, \bar{\varphi}\}$.

Proof. We will first prove the statement for cl-semi-stable semantics: As $cl\text{-}ss(\mathcal{F}) \subseteq pr_c(\mathcal{F})$, it suffices to prove the statement for each i-preferred claim-set S. First observe that S cannot contain both $a, \bar{a}$ for $a \in X \cup \{\varphi\}$ since there is no cf_c-realization containing both $a, \bar{a}$. It remains to show that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \subseteq S$ for some $X' \subseteq X$ and

$e \in \{\varphi, \bar{\varphi}\}$. S contains $X' \cup \{\bar{x} \mid x \notin X'\}$ for some $X' \subseteq X$: Assume there is $x \in X$ such that $x, \bar{x} \notin S$. Consider a pr_c-realization E of S and let $D = E \cup \{x\}$. D is conflict-free since $\bar{x}, d_x \notin E$, moreover, $cl_i \notin E$ for each clause cl with $(x, cl) \in R$, since cl is not defended against the attack (x, cl). Also, D is admissible since E does not contain the only attacker $\bar{x}$ of x and $D \supset E$, contradiction to E being preferred in F. S contains $Y \cup Z$: Assume there is $v \in Y \cup Z$ such that $v \notin S$. Consider a pr_c-realization E of S and let $D = E \cup \{v\}$. D is admissible since $\bar{v} \notin E$ by assumption $v \notin S$ and $D \supset E$, contradiction to E being preferred in F. S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. Consider a pr_c-realization E of S; in case there is $cl \in C$ such that $cl \notin E^+_{\mathcal{F}}$, let $E' = E \cup \{cl\}$. Then cl is defended against every attack since E contains either v or $\bar{v}$ for every V thus E' is admissible and $E \subsetneq E'$, contradiction to E being preferred in F. In case there is no such $cl \in C$ we have φ is defended. Then $E' = E \cup \{\varphi\}$ is admissible and properly extends E, contradiction to E being preferred in F.

For cl-stage semantics, consider some $S \in cl\text{-}stg(\mathcal{F})$. We will first show that S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $cl(E) \cup E^{\ddagger}_{\mathcal{F}}$ is maximal among conflict-free claim-sets. In case $\varphi \in cl(E) \cup E^{\ddagger}_{\mathcal{F}}$ we have $\varphi \in E$ since there is no conflict-free set of arguments which attacks every occurrence of φ, contradiction to $\varphi \notin S$. In case $\varphi \notin cl(E) \cup E^{\ddagger}_{\mathcal{F}}$ there is some argument $a \in E$ which is in conflict with φ, by construction, $a = cl$ for some $cl \in C$. It follows that either φ or $\bar{\varphi}$ is contained in S. To show that S contains $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z$, consider again some $cl\text{-}stg$-realisation E of S and assume there is $v \in V$ such that $v, \bar{v} \notin E$. Then E is in conflict with both $v, \bar{v}$ otherwise $cl(E \cup \{v\})$ would properly extend the claim-range of S, contradiction to $S \in cl\text{-}stg(\mathcal{F})$. Also, the attacking arguments are in C by construction, thus $\bar{\varphi} \in S$. Let $E' = (E \setminus v^+_{\mathcal{F}}) \cup \{v\}$, that is, we remove every $cl \in C$ which is in conflict with v and add v. E' is conflict-free since $\bar{v} \notin E$ and every argument which is attacked by v has been removed; also observe that we remove only arguments $cl \in C$ since $\bar{v} \notin E$ and every other argument which is attacked by v (in case there is some) is self-attacking. Now, if there is some $cl \in C$ such that $E'' = \{cl\} \cup E'$ is conflict-free, we have found a set of arguments whose claim-range properly extends the claim-range of E: Observe that $cl_{\mathcal{F}}(E'') = cl_{\mathcal{F}}(E) \cup \{v\}$ and $E''^{\ddagger}_{\mathcal{F}} \supseteq E^{\ddagger}_{\mathcal{F}}$, moreover, $v \notin E^{\ddagger}_{\mathcal{F}}$. In case there is no such $cl \in C$ which can be added to E' we have that E' attacks every occurrence of $\bar{\varphi}$ thus $\bar{\varphi} \in E'^{\ddagger}_{\mathcal{F}}$. Thus we have $cl_{\mathcal{F}}(E') \cup E'^{\ddagger}_{\mathcal{F}} \supset cl_{\mathcal{F}}(E) \cup E^{\ddagger}_{\mathcal{F}}$. In both cases, we have derived a contradiction to S being cl-stage in $\mathcal{F}$. □

Proposition 5.4. *Deciding* VER-OE$_\rho$ *is* Π_3^{P}*-hard for* $\rho \in \{cl\text{-}ss, cl\text{-}stg\}$.

Proof. Proof by reduction from QSAT$_3^{\exists}$: Consider an instance $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ of QSAT$_3^{\exists}$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. We assume that there is some $y_0 \in Y$ which appears in every clause $cl \in C$. Let $\mathcal{F} = (A_{\mathcal{F}}, R_{\mathcal{F}}, cl_{\mathcal{F}})$ be defined as in Reduction 4. By Lemma E.5, each cl-semi-stable (cl-stage) claim-set of $\mathcal{F}$ is of the form $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for $X' \subseteq X$, $e \in \{\varphi, \bar{\varphi}\}$. Additionally, we observe that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\} \in \rho(\mathcal{F})$, $\rho \in \{cl\text{-}ss, cl\text{-}stg\}$, for every $X' \subseteq X$ since there is $y_0 \in Y$ which attacks each clause $cl \in C$.

We define $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, cl_{\mathcal{G}})$ such that $\mathcal{G}$ has exactly the cl-semi-stable (cl-stage) claim-sets $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for every $X' \subseteq X$, $e \in \{\varphi, \bar{\varphi}\}$ by setting

$$A_{\mathcal{G}} = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi, \bar{\varphi}\};$$
$$R_{\mathcal{G}} = \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi)\};$$

and $cl_{\mathcal{G}} = id$.

Now assume Ψ is valid. Then there is $X' \subseteq X$ such that $\Psi' = \forall Y \exists Z \varphi(X', Y, Z)$ is valid ($\varphi(X', Y, Z)$ is the formula which arises after replacing each $x \in X$ with $\top$ in case $x \in X'$ and $\bot$ if $x \notin X'$). We show that $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \notin cl\text{-}ss(\mathcal{F})$ (the proof for $\rho = cl\text{-}stg$ is analogous): Towards a contradiction, assume there is a $cl\text{-}ss$-realization E of S. Then there is $cl \in C$ such that $cl \in E$, moreover, $\varphi \notin E^{\ddagger}_{\mathcal{F}}$ since φ_2 is not attacked by E in $\mathcal{F}$. Let $Y' = E \cap Y$ and consider the set $D = M \cup \{\bar{v} \mid v \notin M\} \cup \{\varphi\}$, where $M = X' \cup Y' \cup Z'$ is a model of φ (since Ψ' is valid, there is such a $Z' \subseteq Z$). It can be checked that D is admissible; moreover, D attacks the same dummy-arguments $d_v, d_{\bar{v}}$ for $v \in V$. Since D contains φ, it follows that $cl_{\mathcal{F}}(D) \cup D^{\ddagger}_{\mathcal{F}} \supset cl_{\mathcal{F}}(E) \cup E^{\ddagger}_{\mathcal{F}}$, i.e., the claim-range of D is a proper superset of the claim-range of E, contradiction to $S \in cl\text{-}ss(\mathcal{F})$.

In case Ψ is not valid, one can show that for all $X' \subseteq X$, $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \in cl\text{-}ss(\mathcal{F})$ (the proof is analogous for cl-stage claim-sets): Let $X' \subseteq X$. Since Ψ is not valid, there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $X' \cup Y' \cup Z'$ is not a model of φ. Fix such a $Y' \subseteq Y$ and some $Z' \subseteq Z$ and let $E = V' \cup \{\bar{v} \mid v \notin V'\} \cup \{cl\}$, where $V = X \cup Y' \cup Z'$ and $cl \in C$ is some clause which is unattacked by $V' \cup \{\bar{v} \mid v \notin V'\}$. E $cl\text{-}ss$-realises $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ since for all $Z'' \subseteq Z$, there is some $cl' \in C$ such that $D = V' \cup \{\bar{v} \mid v \notin V'\}$, $V = X \cup Y' \cup Z''$ does not attack cl. That is, there is no conflict-free set of arguments D which contains $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y' \cup \{\bar{y} \mid y \notin Y'\}$ and has both φ and $\bar{\varphi}$ in its claim-range. Thus $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \in cl\text{-}ss(\mathcal{F})$ for every $X' \subseteq X$ and therefore $cl\text{-}ss(\mathcal{F}) = cl\text{-}ss(\mathcal{G})$. □

Proposition E.6. *Deciding* VER-OE$_{pr_c}$ *is* Π_3^{P}*-hard.*

Proof. We show hardness via a reduction from QSAT$_3^{\exists}$. Consider an instance $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ of QSAT$_3^{\exists}$, where φ is given by a set of clauses C over atoms in $V = X \cup Y \cup Z$. W.l.o.g., we can assume there is $y_0 \in Y$ which is contained in each clause $cl \in C$. We construct two CAFs $\mathcal{F} = (A_{\mathcal{F}}, R_{\mathcal{F}}, cl_{\mathcal{F}})$, $\mathcal{G} = (A_{\mathcal{G}}, R_{\mathcal{G}}, cl_{\mathcal{G}})$. Let (A, R) be given as in Reduction 1. We construct $\mathcal{F}$ with

$$A_{\mathcal{F}} = A \cup \{\varphi, \bar{\varphi}\};$$
$$R_{\mathcal{F}} = R \cup \{(cl, \varphi) \mid cl \in C\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \bar{\varphi})\} \cup \{(\bar{\varphi}, z), (\bar{\varphi}, \bar{z}) \mid z \in Z\};$$

and $cl_{\mathcal{F}}(y) = cl_{\mathcal{F}}(\bar{y}) = y$ for $y \in Y$ and $cl_{\mathcal{F}}(v) = v$ else; that is, $\mathcal{F}$ is the standard construction for preferred semantics on AF level.

We construct the CAF $\mathcal{G}$ such that $pr_c(\mathcal{G}) = \{V' \cup \{\bar{v} \mid v \notin V'\} \cup Y \cup \{\varphi\} \mid V' \subseteq X \cup Z\} \cup \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \mid X' \subseteq X\}$. This can be achieved by setting

$$A_{\mathcal{G}} = X_i \cup \bar{X}_i \cup Y \cup Z \cup \bar{Z} \cup \{\varphi\}$$

for two copies $X_i, \bar{X}_i$, $i \leq 2$, of X and $\bar{X}$, respectively;

$$\begin{aligned} R_{\mathcal{G}} = \{&(v_i, \bar{v}_j), (\bar{v}_i, v_j) \mid v_i, v_j \in X_1 \cup X_2\} \cup \\ \{&(v, \bar{v}), (\bar{v}, v) \mid v \in Z\} \cup \\ \{&(a, b), (b, a) \mid a \in A' \cup \{\varphi\}, b \in X_2 \cup \bar{X}_2\} \end{aligned}$$

where $A' = X_1 \cup \bar{X}_1 \cup Z \cup \bar{Z}$; moreover, $cl_{\mathcal{G}}(x_i) = x$, $cl_{\mathcal{G}}(\bar{x}_i) = \bar{x}$, and $cl_{\mathcal{G}}(a) = a$ for all remaining $a \in A_{\mathcal{G}}$.

First observe that $\{V' \cup \{\bar{v} \mid v \notin V'\} \cup Y \cup \{\varphi\} \mid V' \subseteq X \cup Z\} \subseteq pr_c(\mathcal{F})$ since $y_0 \in cl$ for every clause cl, that is, for every atom $v \in V \setminus \{y_0\}$, we can choose either v or $\bar{v}$ as long as y_0 is contained in $E \subseteq A_{\mathcal{F}}$, we have that E defends φ against each attack.

In case Ψ is not valid, consider some $X' \subseteq X$. Since Ψ is not valid, there is some $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, some clause $cl \in C$ is not satisfied. It follows that $E = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y' \cup \{\bar{y} \mid y \notin Y'\}$ is preferred in F: Clearly, E is conflict-free and defends itself. Now assume there is $a \in A \setminus E$ such that $E \cup \{a\} \in ad(F)$. In case $a = \varphi$ we have that each $cl \in C$ is attacked, that is, for every clause $cl \in C$ there is $v \in X' \cup Y'$ such that either $v \in X' \cup Y'$ with $v \in cl$ or $v \notin X' \cup Y'$ with $\neq v \in cl$. Thus $X' \cup Y'$ is a model of φ, contradiction to Ψ being not valid. Observe that the case $a \in Z \cup \bar{Z}$ requires $\varphi \in E$, otherwise a is not defended against $\bar{\varphi}$. We have thus shown that $cl(E) = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \in pr_c(\mathcal{F})$ for every $X' \subseteq X$.

We show that every i-preferred set of $\mathcal{F}$ is either of the form (a) $V' \cup \{\bar{v} \mid v \notin V'\} \cup Y \cup \{\varphi\}$ for some $V' \subseteq X \cup Z$ or (b) $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y$ for some $X' \subseteq X$. As outlined above, any such set is i-preferred in $\mathcal{F}$, thus it remains to show that there is no other i-preferred set in $\mathcal{F}$. First notice that each i-preferred claim-set of $\mathcal{F}$ contains $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y$ for some $X' \subseteq X$ since every preferred set E of F contains $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y' \cup \{\bar{y} \mid y \notin Y'\}$ for some $X' \subseteq X$, $Y' \subseteq Y$ by construction. Now assume there is $S \subseteq cl(A_{\mathcal{F}})$ such that $S \in pr_c(\mathcal{F})$ and S is not of the form (a) or (b). Let E be a pr_c-realisation of S. First assume $\varphi \notin E$. Then $z, \bar{z} \notin E$ for any $z \in Z$ since φ is the only argument which defends $z, \bar{z}$ against $\bar{\varphi}$. By the above consideration there are $X' \subseteq X$, $Y' \subseteq Y$ such that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq E$. Observe that $a \notin E$ for any $a \in (X \setminus X') \cup \{\bar{x} \mid x \in X'\} \cup (Y \setminus Y') \cup \{\bar{y} \mid y \in Y'\}$ since $v, \bar{v}$ are mutually attacking for any $v \in X \cup Y$. Since every remaining argument is either attacked by E or self-attacking it follows that $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y$. In case $\varphi \in E$, we have that every $z, \bar{z}$ is defended against $\bar{\varphi}$. Thus E contains either z or $\bar{z}$ for every $z \in Z$ by subset-maximality of E. Thus there is $Z' \subseteq Z$ such that $E = V' \cup \{\bar{v} \mid v \notin V'\} \cup \{\varphi\}$. Since every remaining argument is either attacked by E or self-attacking, we have $S = V' \cup \{\bar{v} \mid v \notin V'\} \cup Y \cup \{\varphi\}$ for some $V' \subseteq X \cup Z$. It follows that $pr_c(\mathcal{F}) = pr_c(\mathcal{G})$.

Now assume $pr_c(\mathcal{F}) = pr_c(\mathcal{G})$ and consider some $X' \subseteq X$. Let E be a pr_c-realisation of $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y$ and let $Y' = E \cap Y$. We show that for all $Z' \subseteq Z$, $X' \cup Y' \cup Z'$ is not a model of φ. Fix some $Z' \subseteq Z$ and let $M = X' \cup Y' \cup Z'$. Since E is preferred in $\mathcal{F}$ we have that φ is not defended by $E \cup Z' \cup \{\bar{z} \mid z \notin Z'\}$; i.e., there is some $cl \in C$ such that $E \cup Z' \cup \{\bar{z} \mid z \notin Z'\}$ does not attack cl. Consequently, for all $v \in V$, in case $v \in cl$ we have $v \notin M$, and in case $\neq v \in cl$ we have $v \in M$. It follows that M is not a model of φ.

It follows that Ψ is not valid iff $pr_c(\mathcal{F}) = pr_c(\mathcal{G})$. $\square$