Belief Base Revision as a Binary Operation on Implicant Sets: a finitary Approach

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Abstract

Belief revision has been considered on various language and theoretical levels. Alchourrón, Gärdenfors and Makinson considered revision operators as binary functions assigning a belief set to a belief set and a formula (Alchourron, Gärdenfors, and Makinson 1985). A more general formulation has been used by Lehmann and co-authors (Lehmann, Magidor, and Schlechta 2001) who formulated revision as a binary set operator, thus replacing both, the belief set and the formula by a set of sets (models). Katsuno and Mendelson in (Katsuno and Mendelzon 1991b) considered a more specific approach where the belief set is generated by a finite set of formulae thus being representable by one formula. We propose a new formulation of belief revision in terms of implicant sets, an implicant being a finite consistent set of literals. We define implicant revision functions as binary operators on implicant sets and we characterize implicant revision by postulates that specify properties every implicant revision function should have. We show that every function satisfying the implicant revision postulates satisfies the AGM postulates, but the opposite is not true: there is an AGM revision operator that does not satisfy all implicant revision postulates, thus implicant revision is stronger than AGM revision. Then we study the relation of implicant revison with more specific approaches, namely distance based operators (Schlechta and Lehmann). Implicant revision approach is more specific : there is a distance based revision function that does not satisfy all implicant revision postulates. Finally we study one important subclass of implicant revision operators, namely those that are invariant with respect to different equivalent implicant sets.

Introduction

The main objective of belief revision formalisms is to propose methods for incorporate new information in a belief set. The problem of how to characterize and to compute a belief set resulting from the incorporation of new information has been considered on various language and theoretical levels. The best known approach is due to Alchourrón, Gärdenfors and Makinson (AGM postulates) who considered revision operators as binary operators assigning a belief set to a belief set and a formula (Alchourron, Gärdenfors, and Makinson 1985; Gärdenfors 1988). They formulated postulates that specify properties a revision operator should have (AGM postulates).

A more general algebraic formulation replaces both, the belief set and the formula by a set of sets (models). This formulation is more general because in the infinite case there are model sets that are not representable by a formula set. Katsuno and Mendelson in (Katsuno and Mendelzon 1991b) reformulated the AGM postulates introducing a binary operator on formulas, for the more specific case where the belief set has a finite cover, that is, it is representable by a finite set of formulas (or by a formula). They propose a semantic characterization in terms of pre-orders (depending on formulas) on model sets.

This paper presents a new and more specific approach where revision occurs on implicant sets, an implicant being a finite set of literals. An implicant is equivalent to the conjunction of its elements and a set of implicants to the disjunction of the elements of this set. Thus implicant sets correspond to disjunctive normal forms (DNF) of formulas. We define implicant revision functions as binary operators on implicant sets and we characterize implicant revision by postulates that specify properties every implicant revision function should have. We show that every function satisfying the implicant revision postulates satisfies the AGM postulates, but the opposite is not true: we show that there is an AGM revision operator that does not satisfy all implicant revision postulates, thus implicant revision is stronger than AGM revision. Then we study more specific operators, namely those based on a distance function between models, (Lehmann, Magidor, and Schlechta 2001; Schlechta 2004). We show that implicant revision is more specific: there is a distance based revision function that does not satisfy all implicant revision postulates. Finally we study another important subclass of implicant revision operators, namely those that are invariant with respect to different equivalent implicant sets.

Our approach is situated somewhat between syntax and semantics. The DNF of a formula is a specific syntactic object that is easily obtained, by simple transformations, but also by some theorem prover. For example, every tableaux prover produces a disjunctive normal form of a formula. And every implicant of a formula that does not contain opposite literals (i.e. that is not closed) can be seen as a partial model of the formula, namely it represents all models that satisfy the literals it contains. Intuitively, our approach is due to the following observation. Revision is the process of integrating new information into a belief set Γ. When a new information μ comes about, we consider first the set $\Gamma \cup \{\mu\}$. There are two situations, either $\neg\mu \notin \Gamma$ or $\neg\mu \in \Gamma$. In the first case the AGM postulates tell us that $\Gamma{*}\mu$ is $\Gamma \cup \{\mu\}$, in the second case $\Gamma \cup \{\mu\}$ is inconsistent and a new set $\Gamma' \subseteq \Gamma$ must be found that does not contain $\neg\mu$ but contains most of the information from Γ that is consistent with μ. Consistency with μ can be fixed concretely by considering a set of implicants equivalent to Γ and those equivalent to μ. If their conjunction contains only closed implicants, Γ and μ have no common model. The models of the revised belief set are a subset of the models of μ. In order to obtain these models, we select implicants from μ augmented by non opposite parts from implicants from Γ.

Many specific revision operators have been defined in terms of models and taking into account the set of atoms in which two models differ (Bordiga 1985; Dalal 1988; Satoh 1988). These operators depend on the symmetrical differences between models, i.e. on the set of propositional variables in which the models differ. We show that the symetric differences between models are easily obtained by a simple operation on implicants.

The representation of a formula by a set of implicants, that is its disjunctive normal form, is not unique, unless we work with the set of all its prime implicants. But we can show that one class of implicant revision operators obtains equivalent revision results from equivalent implicant sets. Hence our approch does not systematically require to compute the prime implicants of the belief set and the revision formula. Several known revision operators (Dalal 1988; Satoh 1988)) can be computed in our formalism without generating prime implicants.

Finally we define three specific implicant revision operators all based on the symmetric difference between models. We minimize this difference in two ways, first by set inclusion of the symmetric differences (Satoh revision (Satoh 1988)) secondly by minimizing the number of elements where the two models differ (Dalal revision (Dalal 1988)). The third operator is based on literal weights where a weight is a natural number that indicates the importance of the literal. The idea is that the higher the weight the more important is the literal in the belief set and the less a user wants to give it up. It seems to us that this approach is more natural than those that weight whole interpretations. A single literal might be more concrete for a user than a model. We use this weight for revision by minimizing the sum of the weights of the symmetric difference thus giving up globally the less important information units.

Our system has been implemented by use of a tableaux prover. A a revised belief set can be obtained by calculating a tableau for the belief set formula and the revision formula and by suppressing opposite literals within the eventually closed tableau for these two formulas.

This paper is organized as follows. In the next section, we recall elements and results of theory revision. In the third part, we present the new implicant revision postulates and we situate them by comparing them formally to the AGM approach as well as to distance based revision. In the fourth part we study specific implicant revision operators . We then describe informally the implementation and give some complexity results. Finally some comparison with related approaches are presented.

Preliminaries and Notation We consider a finite propositional language over a finite set of propositional variables $\mathcal{P}$. $\mathcal{M}$ is the set of all interpretations, $\mathcal{F}$ the set of all formulas and LIT the set of all literals, i.e. $LIT = \mathcal{P} \cup \{\neg a : a \in \mathcal{P}\}$. We call $\neg l$ also the opposite of l. $l \in LIT$ is identified with $\neg\neg l$, etc. as well as $\neg l$ with $\neg\neg\neg l, \ldots$, etc. For a formula $\phi \in \mathcal{F}$, $[|\phi|] = \{m \in \mathcal{M} : m \models \phi\}$ is the set of models of ϕ (the set of interpretations that satisfy ϕ). An interpretation can be identified with the set of propositional variables it evaluates to true. Then $\mathcal{M} = 2^{\mathcal{P}}$[1]. Given a set of interpretations $M \subseteq \mathcal{M}$, $FOR(M)$ is a formula whose set of models is M.

The consequence relation is noted "$\models$", i.e. $\phi \models \psi$ iff $[|\phi|] \subseteq [|\psi|]$ and we denote $Cn(\phi) = \{\psi : \phi \models \psi\}$ the set of consequences of formula ϕ. $Eq(\phi)$ denotes the class of formulas logically equivalent to ϕ. We call *belief base* any set of formulas (not necessarily deductively closed) and *belief set* a deductively closed set of formulas.

Background

The AGM postulates for belief revision

Alchourron, Gärdenfors and Makinson proposed the well-known AGM-postulates for theory revision (Alchourron, Gärdenfors, and Makinson 1985). Here a revision is defined as an operator on a *belief base* Γ and a formula μ and the result of the revision is a belief base denoted $\Gamma{*}\mu$. AGM postulates express what properties a revision operator should have. Revision comes with another operation, expansion, that simply adds a new information to a belief set regardless of whether the result is inconsistent. Expansion is noted $+$ and it holds that $\Gamma + \phi = Cn(\Gamma \cup \{\phi\})$.

Definition 1 (Revision operator) *Let Γ be a belief set and $\phi, \phi_1, \phi_2, \psi \in \mathcal{F}$. We call AGM revision operator every operator $*$ for which the following basic postulates hold*

K$*$1 $\Gamma{*}\phi$ *is a belief set.*

K$*$2 $\phi \in \Gamma{*}\phi$

K$*$3 $\Gamma{*}\phi \subseteq \Gamma + \phi$

K$*$4 *If $\Gamma + \phi$ is satisfiable, then $\Gamma{*}\phi \supseteq \Gamma + \phi$*

K$*$5 *If ϕ is satisfiable, then $\Gamma{*}\phi$ is satisfiable*

K$*$6 *If $\phi_1 \equiv \phi_2$ then $\Gamma{*}\phi_1 \equiv \Gamma{*}\phi_2$*

Two additional postulates are normally considered that deal with the relation between revising with a conjunction and revising with each of the conjuncts subsequently.

K$*$7 $\Gamma{*}(\phi \wedge \psi) \subseteq (\Gamma{*}\phi) + \psi$

K$*$8 *If $(\Gamma{*}\phi)+\psi$ is satisfiable then $(\Gamma{*}\phi)+\psi \subseteq \Gamma{*}(\phi\wedge\psi)$*

We note AGM the set of AGM-revision operators.

[1] 2^M is the power set of set M

Here we will use a characterization of revision operators in terms of model sets identifying a formula (or a formula set) with the set of models satisfying it. In view of postulate K$*$2, a revision operator selects models from the revision formula as result of the revision operation. This formulation yields a rather simple algebraic characterization of revision operators. In the following, $\mathcal{U}$ can be considered as a set of model sets.

Definition 2 (Revision function) *Le $\mathcal{U}$ be a set of sets, $N, M, L \in \mathcal{U}$. $sm : \mathcal{U} \times \mathcal{U} \longrightarrow \mathcal{U}$ is a revision function on $\mathcal{U}$ iff for any $M, N, L \in \mathcal{U}$*

(S1) $sm(M, N) \subseteq N$

(S2) $M \cap N \subseteq sm(M, N)$

(S3) *if $M \cap N \neq \emptyset$ then $sm(M, N) \subseteq M \cap N$*

(S4) *if $N \neq \emptyset$ then $sm(M, N) \neq \emptyset$*

The two additional postulates are:

(S5) $sm(M, N) \cap L \subseteq sm(M, N \cap L)$

(S6) *if $sm(M, N) \cap L \neq \emptyset$ then $sm(M, N \cap L) \subseteq sm(M, N) \cap L)$*

The following correspondence results are straightforward. They rely on the fact that we treat with finite sets of propositional variables. If $\mathcal{P}$ is not finite $FOR(M)$ is not always defined and the result of a revision function is eventually a set of models that cannot be represented by a formula (or by a set of formulas) (Lehmann, Magidor, and Schlechta 2001). A revision operator $*$ is called *definabiblity preserving* when $FOR(sm([|\Gamma|], [|\mu|]))$ is defined (i.e. is a formula).

Theorem 1 *1 Let $*$ be an AGM revision operator. Then the function sm defined by $sm(M, N) = [|Cn(FOR(M)) * FOR(N)|]$ is a revision function on $2^{\mathcal{M}}$.*

2 Let sm be a revision function on $2^{\mathcal{M}}$. Then the operator $*$ defined by $\Gamma * \mu = FOR(sm([|\Gamma|], [|\mu|]))$ is an AGM revision operator.

Specific revision operators have been proposed that are based on the distance between the set of models of a belief set and of a revision or an update formula (Dalal 1988; Forbus 1989). All these approaches define some distance function between models, that induces a distance between model sets. The models of the belief set resulting from the revision operation is then the set of models of the revision formula that are closest to the models of the original base. For example, Dalal uses the Hamming distance between sets, that is the number of propositional variables in wich the two interpretations differ. Lehmann et al. characterize formally the class of distance based revision operators (Lehmann, Magidor, and Schlechta 2001) by means of distance spaces. Intuitively, $\Gamma * \phi$ is a belief set that contains ϕ and is as close as possible to Γ. According to K$*$2, every model of $\Gamma * \phi$ is a model of ϕ, that means that the models of the revised belief set $\Gamma * \phi$ are a subset of the models of ϕ. Distance based revision relies on a distance function between models and is defined semantically as a binary function on sets of models.

Definition 3 *A* distance space *is a pair (Δ, d) where Δ is a non-empty set and d is a* distance function *from $\Delta \times \Delta$ to $\mathbb{R}^{\geq 0}$. We say that d respects identity if it satisfies the following condition:*

$$\forall x, y \in \Delta,\ \ d(x, y) = 0 \textit{ iff } x = y \qquad \text{(id)}$$

The distance between two sets $A \subseteq \Delta$ and $B \subseteq \Delta$ is defined by

$$d(A, B) = \inf\{d(x, y) : x \in A, y \in B\} \qquad \text{(inf)}$$

Definition (inf) is not a distance function since $d(A, B) = 0$ iff $A \cap B \neq \emptyset$. As a special case of Definition (inf), the distance between an *indiviual* $w \in \Delta$ and a set $A \in \Delta$ is $d(\{w\}, A)$ and is noted

$$d(w, A) = \inf\{d(w, y) : y \in A\}$$

Hence $d(w, A) = 0$ iff $w \in A$. If $A = \emptyset$, then $d(w, A) = \infty$. (Δ, d) is called a *min-space* if additionally it satisfies the following condition on the existence of a minimum of a set of distances:
If $A \neq \emptyset$ and $B \neq \emptyset$ then $\exists x_0 \in A \, \exists y_0 \in B$ such that $d(x_0, y_0) = d(A, B)$.

Definition 4 (Distance based revision function) *Given a distance space (Δ, d) and $A, B \subseteq \Delta$, $A \mid_d B = \{b \in B : \exists a \in A \, \forall a' \in A, b' \in B \; d(a, b) \leq d(a', b')\}$.*

Remark 1 $A \mid_d B = \{b \in B : d(A, b) = d(A, B)\}$

Definition 5 *Given a belief set Γ, a formula ϕ and a distance based revision function $\mid_d$, the corresponding distance based revision operator $*_d$ is defined by $\Gamma *_d \phi = FOR([|\Gamma|] *_d [|\phi|])$.*

Theorem 2 *(Schlechta 2004) A distance based revision operator $*_d$ satisfies the AGMpostulates if*

1. *d respects identity (id),*
2. *(Δ, d) is a min space*
3. *$*_d$is definability preserving.*

The opposite is not true: there is an AGM revision operator that is not distance based as shown in (Lehmann, Magidor, and Schlechta 2001). Distance based revision takes into account disjunctive information (Lehmann, Magidor, and Schlechta 2001): If a revision operator $*$ is distance based then $(\Gamma * (\phi \vee \psi)) * \mu$ is $(\Gamma * \phi) * \mu$ or it is $(\Gamma * \psi) * \mu$ or it is $(\Gamma * \phi) * \mu \cup (\Gamma * \psi) * \mu$.

Implicant revision

Here we introduce implicant revision. In its formulation it is more specific than revision since it is defined on formulas under a specific syntactic form.

Notations An *implicant* is a finite set of literals. We note $\mathcal{T}$ the set of all implicants. An implicant is called *closed* when it contains a literal l and its negation $\neg l$ and it is called *open* otherwise. A set of implicants is closed when all of its elements are closed and it is open when at least one of its elements is open. We can think of an implicant as being a formula, namely the conjunction of its literals and of a set of implicants as of a disjunction, namely the disjunction of the conjunction of its elements. By abuse of notation, we will write t (resp. T) to design the formula that corresponds to t (resp. T). A set T of implicants is closed iff $[|T|] = \emptyset$. We will use the following operation on pairs of implicant sets. Let be $S, T \subseteq \mathcal{T}$. $S \otimes T = \{s \cup t : s \in S \text{ and } t \in T\}$. $S \otimes T$ corresponds to the conjunction of the corresponding formulas. Let $S^o = \{s : s \in S \text{ and } s \text{ not closed }\}$ be the set of implicants of S that are open.

The following is easy to see:

Fact 1 *Let S, T be implicants. Then*

1. $S \otimes T \leftrightarrow S \wedge T$
2. $[|S \otimes T|] = [|S|] \cap [|T|]$
3. $[|S^o|] = [|S|]$ *and* $\models S^o \leftrightarrow S$

In the finite case, every interpretation m can be represented by an implicant $m^I = m \cup \{\neg a : a \notin m\}$[2]. For a set of interpretations M, we set $M^I = \{m^I : m \in M\}$ and we have $[|m^I|] = \{m\}$ and $[|M^I|] = M$.

Definition 6 (Implicant Revision Function) *Let be $S, T, R \in 2^{\mathcal{T}}$. An implicant revision function on $2^{\mathcal{T}}$ is a binary operator $st : 2^{\mathcal{T}} \times 2^{\mathcal{T}} \longrightarrow 2^{\mathcal{T}}$ for which the following basic postulates hold:*

(ST1) *For all $v \in st(S, T)$ there is $t \in T$, $s \in S$ and $s' \subseteq s$ such that $v = t \cup s'$.*

(ST2) *If $S \otimes T$ is not closed then $st(S, T) = S \otimes T$.*

(ST3) *If T is not closed then $st(S, T)$ is not closed.*

(ST4) *If $[|S_1|] = [|S_2|]$ and $[|T_1|] = [|T_2|]$ then $[|st(S_1, T_1)|] = [|st(S_2, T_2)|]$.*

One additional postulate is usually considered, that deals with successive application of function st vs. application of st to a "conjunction" of literal sets.

(ST5) *If $st(S, T) \otimes R$ is open then $st(S, T) \otimes R \equiv st(S, T \otimes R)$*

It is not difficult to see that postulate (ST2) corresponds to K∗4 (resp. (S3)), (ST3) corresponds to K∗5 (resp. (S4)), (ST4) corresponds to K∗6, and (ST5) corresponds to K∗7 and K∗8 (resp. (S5) and (S6)). The difference between implicant revision and "standard" revision relies on the first postulate (ST1) that replaces K∗2 (or (S1): according to (S1) every model of the revised belief set is a model of the revision formula. Postulate (ST1) is somewhat stronger: it says something about which models of the revision formula are to be selected for becoming models of the revised base. When we revise a belief set seen as a set of implicants S by a new information, given as a set of implicants T then we choose a subset among the implicants of T and eventually augment each implicant choosen by subsets of implicants in S "compatible" (not contradictory) with that implicant. In other words, we try to "keep" as many literals occurring in implicants of the original belief set as possible. This means that an important syntactical aspect intervenes in implicant revision. The AGM postulates do not at all take into account the specific atoms within models for "choosing" one model instead of another one within a revision result. Postulate (ST4), that corresponds to K∗6, requires the independence of revision from syntax. One way to satisfy always (ST4) is to work on the set of all prime implicants of the belief set and of the revision formula. However, there is a special class of implicant revision function, that we will present in the next section, that produces equivalent revision results when applied to equivalent implicant sets without requiring to work with prime implicants. Most related approaches to implicant or implicate revision work with prime implicants (Marchi, Bittencourt, and Perrussel 2010; Bienvenu, Herzig, and Qi 2008). Our approach is different in this respect.

Observe that the postulates do not prevent st(S,T) of containing closed implicants. Since a closed implicant has no model, it can be deleted within st(S,T) and the corresponding formulas are equivalent according to fact 1.

Implicant revision satisfies the AGM postulates. Here we show that every implicant revision function defines a revision function. Revision function postulates (S1) to (S6) are satisfied by every implicant revision function and by theorem 1 it follows that an implicant revision function satisfies the AGM postulates.

Theorem 3 *Let st be an implicant revision function on $2^{\mathcal{T}}$. Then the function $sm : 2^{\mathcal{M}} \times 2^{\mathcal{M}} \longrightarrow 2^{\mathcal{M}}$ defined by*

$$sm(M, N) = [|st(M^I, N^I)|] \qquad \text{(smt)}$$

is a revision function.

Proof: We show that S1 to S6 hold for the function sm.
S1: Let $m \in sm(M, N)$. Then $m \in [|st(M^I, N^I)|]$ by the equation (smt). Then there is $v \in st(M^I, N^I)$ such that $m \in [|v|]$. By ST1 there is $s \in M^I, t \in N^I, s' \subseteq s$ such that $v = t \cup s'$. Since $t \subseteq v$, $[|v|] \subseteq [|t|]$ from which $[|v|] \subseteq N$ which entails $m \in N$.

S2 and S3: Let $M \cap N \neq \emptyset$, then $M^I \otimes N^I$ is not closed there therefore $st(M^I, N^I) = M^I \otimes N^I$ by ST2. Hence $sm(M, N) = [|st(M^I, N^I)|] = [|M^I \otimes N^I|] = [|M|] \cap [|N|]$.

S4: Let be $N \neq \emptyset$. Then N^I is not closed and by ST3 $st(M^I, N^I)$ is not closed. Hence $[|st(M^I, N^I)|] \neq \emptyset$ and $sm(M, N) \neq \emptyset$ by the equation (smt).

S5 and S6: Let be $sm(M, N) \cap L \neq \emptyset$. Then $[|st(M^I, N^I)|] \cap L \neq \emptyset$ by equation (smt). Therefore $[|st(M^I, N^I) \otimes L^I|] \neq \emptyset$ by fact 1, 2. Therefore $st(M^I, N^I) \otimes L^I)$ is open and by (ST3), $st(M^I, N^I) \otimes L^I = st(M^I, N^I \otimes L^I)$, i.e. $[|st(M^I, N^I) \otimes L^I|] = [|st(M^I, N^I \otimes L^I)|]$, from which $[|st(M^I, N^I)|] \cap L = sm(M, [|N^I \otimes L^I)|] = sm(M, N \cap L)$. Hence we get the result $sm(M, N) \cap L = sm(M, N \cap L)$.

Q.E.D. The opposite of theorem 3 does not hold. There is an AGM revision operator that is not an implicant revision operator.

Theorem 4 *There is a revision operator that is not an implicant revision operator.*
Proof: We define the following revision operator sm_0 that fails to satisfy (ST1). Let $M = \{m_1, m_2, m_3, m_4\}$ be the models over the propositional language $\{p, q\}$, such that

[2] remember that we identify an interpretation with the set of atoms evaluated to true.

$m_1 = \emptyset$, $m_2 = \{p\}$, $m_3 = \{q\}$ and $m_4 = \{p, q\}$. Let be $X, Y \subseteq M$. We define a binary set operator sm_0 by.

$sm_0(X, Y) = X \cap Y$ *if* $X \cap Y \neq \emptyset$

$sm_0(X, Y) = Y$ *if* $card(Y) \leq 1$

For all other cases, namely when $X \cap Y = \emptyset$ or $card(Y) > 1$, define

$sm_0(\{m_1\}, X) = sm_0(\{m_2\}, X) = \{m_3\}$ *if* $m_3 \in X$

$sm_0(\{m_1\}, \{m_2, m_4\}) = \{m_2\}$

$sm_0(\{m_2\}, \{m_1, m_4\}) = \{m_1\}$

$sm_0(\{m_3\}, X) = \{m_4\}$, *if* $m_4 \in X$

$sm_0(\{m_3\}, \{m_1, m_2\}) = sm_0(\{m_4\}, \{m_1, m_2\}) = \{m_1, m_2\}$

$sm_0(\{m_4\}, X) = \{m_3\}$ *if* $m_3 \in X$

$sm_0(\{m_1, m_2\}, \{m_3, m_4\}) = sm_0(\{m_1, m_3\}, \{m_2, m_4\}) = sm_0(\{m_2, m_3\}, \{m_1, m_4\}) = \{m_4\}$

$sm_0(\{m_1, m_4\}, \{m_2, m_3\}) = sm_0(\{m_2, m_4\}, \{m_1, m_3\}) = \{m_3\}$

$sm_0(\{m_3, m_4\}, \{m_1, m_2\}) = \{m_1, m_2\}$

It is not difficult to check that sm_0 satisfies the revision postulates (S1) to (S6). Now, consider $sm_0(\{m_1, m_2\}, \{m_3, m_4\}) = \{m_4\}$. We have also $sm_0(\{m_1\}, \{m_3, m_4\}) = sm_0(\{m_2\}, \{m_3, m_4\}) = \{m_3\}$. sm_0, interpreted as an implicant revision function violates postulate (ST1): we have $FOR(\{m_1, m_2\}) = \neg q$ and $FOR(\{m_3, m_4\}) = q$, and $FOR(\{m_4\}) = p \wedge q$. In terms of implicants we have then $st_0(\neg q, q) = \{p, q\}$ And there is no $s' \subseteq \{\neg q\}$ such that $\{p, q\} = \{q\} \cup s'$. sm_0 defines a revision operator that cannt be defined by an implicant revision function. Q.E.D.

One might wonder how (and why) the result of revising belief set $Cn(\neg q)$ by q can result in $Cn(\{p, q\})$. But $\neg q$ has two models, m_1 and m_2. m_2 satisfies p and according to (ST4) $st_0(\{\neg q\}, X) = st_0(\{(\neg q, p), (\neg q, \neg p)\}, X)$.

There is also a distance based revision operator that is not an implicant revision operator.

Theorem 5 *There is a distance based revision operator that is not an implicant revision operator.*

Proof: We consider the same language $\{p, q\}$ as in theorem 4 with the same model set M. Suppose that M is in the plane as in figure 1 and d is the geometrical distance.

Then d is symmetric and identity respecting and

$d(m_1, m4) = d(m_2, m4) = \sqrt{2}$

$d(m_1, m3) = d(m_2, m3) = \sqrt{5}$

$d(m_3, m_4) = 1$

$d(m_1, m_2) = 2$

The revision function defined by d is not sm_0: we have $\{m_1\} *_d \{m_3, m_4\} = \{m_4\}$ but $sm_0(\{m_1\}, \{m_3, m_4\}) = \{m_3, \}$. Then $FOR(\{m_1, m_2\}) = Eq(\neg q)$, $FOR(\{m_3, m_4\}) = Eq(q)$ and $FOR(\{m_4\}) = Eq(p \wedge q)$. Then $Cn(\neg q) *_d q = Cn(p \wedge q)$. Any implicant revision function st_d, that satisfies (ST4), must be such that $st_d(\neg q, q) = \{(p, q)\}$ violating (ST1). According to (ST1)we cannot have but $st(\neg q, q) = \{(q)\}$, which is not possible. Q.E.D.

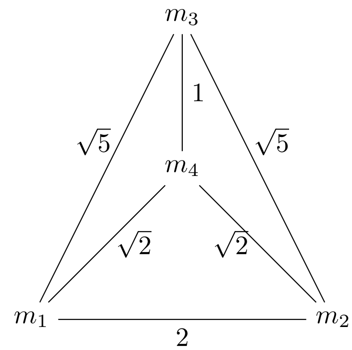

Figure 1: Distance function d on

Theorems 3, 4 and 5 show the following relationships between the three classes of revision operators:

- $AGM \supsetneq IBR$
- $DBR \not\subset IBR$
- $AGM \supsetneq DBR$ (Lehmann, Magidor, and Schlechta 2001)

Syntax independent implicant revision functions

The postulates for implicant revision provide a general framework for implicant revision. According to postulate (ST1), every implicant of $v \in st(S, T)$ is the union of an implicant of T and a subset of an implicant of S. We presuppose that every implicant $v \in st(S, T)$ is open, since eventually closed elements can be ignored. This means that every v is a subset of $s \cup t$, containing t for some $s \in S$ and $t \in T$. Since v must be open, the subset of s is obtained by suppressing, at least, all elements in s opposite to elements in t. Moreover, not every element t of T will be a subset of some element of $st(S, T)$. This means that every implicant revision function st is a choice function on $2^{\mathcal{T}} \times 2^{\mathcal{T}}$ that chooses for every $S, T \in 2^{\mathcal{T}}$ open subsets from the set $\{s' \cup t : s' \subseteq s, s \in S, t \in T\}$ according to (ST1). Specific implicant revision functions are frequently based on some minimization criterion, namely it selects the implicants that are closest to implicants from the original belief set S.

It turns out that some of the known revision operators are implicant revision operators. This is not a surprise because the application of the choice function can be considered as defining a preference relation between implicants, where v is closer to S than w when it contains "more literals" from s than from t.

In this section we show specific implicant revision operators. They depend all on a model distance minimization that is easily obtained by operations on implicants. Hence we present first these operations.

Notation Given a literal l, we note $|l|$ the atom of l, i.e. $|a| = |\neg a| = a$ for $a \in \mathcal{P}$. We note $\bar{l}$ the literal opposite to l, i. e $\bar{l} = \neg l$. For an implicant t, we note $|t| = \{|l| : l \in t\}$ and $\bar{t} = \{\bar{l} : l \in t\}$. Subsequently, we make use of the set of literals $s \cap \bar{t}$ that is the literals in s whose opposite literal belongs to t. It turns out that the propositional variables "corresponding" to these literals are precisely the minimal symmetric differences between the models of s and the models of t. Note that $s \cup t$ is open iff $s \cap \bar{t} = \emptyset$. The symmetric difference of the sets M and N is noted $M \Delta N$, i.e. $M \Delta N = \{x : (x \in M \text{ and } x \notin N), \text{ or } (x \in N \text{ and } x \notin M)\}$.

Lemma 1 *Let be $s, t \in \mathcal{T}$. Then*
(i) $|s \cap \bar{t}\,| = |t \cap \bar{s}\,|$ and
(ii) $|s \cap \bar{t}\,| = Min_{\subseteq}(\{m \Delta n : m \in [|s|] \text{ and } n \in [|t|]\})$
(iii) $(t \cup s) \setminus (s \cap \bar{t}\,) = t \cup (s \setminus \bar{t}\,)$
Proof:

(i) is obvious. For (ii), we first show that for all $m \in [|s|]$ and $n \in [|t|]$, $|s \cap \bar{t}\,| \subseteq m \div n$. Let be $a \in |s \cap \bar{t}\,|$. Then either $a \in s$ and $\neg a \in t$ or $\neg a \in s$ and $a \in t$. Then we have that $a \in m$ and $a \notin n$ or $a \notin m$ and $a \in n$, hence $a \in m \div n$. This shows that $|s \cap \bar{t}\,|$ is also included in every minimal $m \div n$. Now it is sufficient to show that $|s \cap \bar{t}\,|$ is also the symmetric difference between two models. Hence we must show that there are interpretations $m \in [|s|]$ and $n \in [|t|]$, such that $m \div n = |s \cap \bar{t}\,|$. Given an implicant u, let be $u^+ = u \cap \mathcal{P}$. Then we define $m = (s^+ \cup \{a \in t^+ : \neg a \notin s\}$ and $n = (t^+ \cup \{a \in s^+ : \neg a \notin t\}$. Obviously, $m \in [|s|]$ and $n \in [|t|]$. Now it is sufficient to show that $m \div n \subseteq |s \cap \bar{t}\,|$. Let be $a \in m \div n$. Then either $a \in m$ and $a \notin n$ or $a \notin m$ and $a \in n$. Consider the first case: Then we have (i) $a \in s^+$ or (ii) $a \in t^+$ and $\neg a \notin s$, but since $a \notin n$, (ii) is not possible. Hence $a \in s^+$. Since $a \notin n$, $a \notin t^+$ and either $a \notin s^+$, which is not possible, or $\neg a \in t$. It follows that $a \in |s \cap \bar{t}\,|$. The other case, $a \notin m$ and $a \in n$ goes through analogously.

Q.E.D.

Example 1 *Let be $s = \{a, \neg b, c\}$ and $t = \{\neg a, b, d\}$. Then $s \cap \bar{t} = \{a, \neg b\}$ and $Min_{\subseteq}(\{m \Delta n : m \in [|s|] \text{ and } n \in [|t|]\}) = \{a, b\}$*

There is exactly one $\subseteq$ $-$minimal set among $\{m \Delta n : m \in [|s|] \text{ and } n \in [|t|]\})$, namely $|s \cap \bar{t}\,|$ and in view of lemma 1 the sets $s \cap \bar{t}$ and $t \cap \bar{s}$ denote a sort of symmetric difference set between the implicants s and t.

Minimizing symmetric differences

Any formula ϕ is the disjunction of a finite set of implicants. This defines the set of its models as a finite union of model sets (not necessarily disjoint), namely the models of its implicants. In order to obtain the $\subseteq$-minimal symmetric differences of all the models of ϕ it is sufficient to obtain the minimal sets among the $|s \cap \bar{t}\,|$ for all $s \in S$ and $t \in T$.

We will need the following lemma to show that this minimization obtains the global minimal symmetric differences for all models of the formula[3].

Lemma 2 *Let M be a set that is a finite union of sets that are not necessarily disjoint, $M = A_1 \cup \cdots \cup A_k$ and let $\leq$ be a pre-order on M for which the limit assumption holds (i.e. the infimum of $\mathcal{M}$ is a minimum). Then*

$$Min_{\leq}(M) = Min_{\leq}(\bigcup_{i=1}^{k} Min_{\leq} A_i)$$

Proof: "$\subseteq$": Let be $a \in Min_{\leq}(M)$. Then $a \in M$ and there is no $b \in M$ such that $b < a$. There is i, $1 \leq i \leq k$ such that $a \in A_i$ and $A_i \subseteq M$. Therefore there is no $b \in A_i$ such that $b < a$. Hence $a \in Min_{\leq} A_i$.

"$\supseteq$": Let be $a \in Min_{\leq}(\bigcup_{i=1}^{k} Min_{\leq} A_i)$ and suppose that $a \notin Min_{\leq}(M)$. Then there is $b \in M$ and $b < a$. Hence there is i such that $b \in A_i$ and $b < a$. If $b \in Min A_i$ then $b \not< a$ which is a contradiction. Hence $b \notin Min_{\leq} A_i$ and by the limit assumption, there is $c \in Min_{\leq} A_i$ and $c < b$. But then $c < a$ by the transitivity of $\leq$, contradicting the minimality of a in $\bigcup_{i=1}^{k} Min_{\leq} A_i$. Q.E.D.

Corollary 1 *Let be $M = A_1 \cup \cdots \cup A_k = B_1 \cup \cdots \cup B_l$ two different non-empty partitions of a set M and let $\leq$ be a pre-order on M for which the limit assumption holds (i.e. the infimum of $\mathcal{M}$ is a minimum). Then $Min_{\leq}(\bigcup_{i=1}^{k} Min_{\leq} A_i) = Min_{\leq}(\bigcup_{i=1}^{l} Min_{\leq} B_i)$*

The following theorem shows that the set of minimal elements of the symmetric differences between two model sets can be obtained by minimizing all sets $|t \cap \bar{s}\,|$.

Theorem 6 *Let $S, T \subseteq \mathcal{T}$ and $MinI = Min_{\subseteq}(\{|s \cap \bar{t}\,| : s \in S \text{ and } t \in T\})$. Then*

$$MinI = Min_{\subseteq}(m \Delta n : m \in [|S|] \text{ and } n \in [|T|]) \qquad \text{(M)}$$

Proof: Consider the set $DIFF = \{m \Delta n : m \in [|S|], n \in [|T|]\}$ and observe that $DIFF = \bigcup_{s \in S, t \in T} \{m \Delta n : m \in [|s|], n \in [|t|]\}$. $DIFF$ is ordered by $\subseteq$ and $\forall s \in S, t \in T, |s \cap \bar{t}\,| = Min_{\subseteq}\{m \Delta n : m \in [|s|] \text{ and } n \in [|t|]\}$ by lemma 1, 2. (M) follows by applying the lemma 2. Q.E.D.

Theorem 6 shows that we obtain all minimal symmetric difference sets between the models of a formula ϕ and a formula ψ as the $\subseteq$ $-$smallest sets $|s \cap \bar{t}\,|$ for implicants s of ϕ and t of ψ. And we obtain them independently on the particular implicant sets involved whenever they have the same models. This shows that for revision functions based on the symmetric difference of models it is not necessary to use all prime implicants of a formula. Any set of implicants is sufficient.

Theorem 6 can be used for all revision operators that are defined using the symmetric differences between models. Here we discuss two of them.

Weighting literals

The revision function we propose takes into account that for a user the different entities may have different importance. Using this function presupposes that an agent associates weights to literals that measure the "importance" of

[3] We do not claim originality of this lemma

the literal in the belief base . The weight of a set of literals is simply the sum of its elements.

Definition 7 *w is a weighting function on LIT if $w : LIT \longrightarrow \mathcal{N}$, where $\mathcal{N}$ is the set of natural numbers. w can be extended to a function on implicants and on propositional models. Let $t \in \mathcal{T}$ be an implicant. Then $w(t) = \sum_{l \in t} w(l)$ and for a model $m \in \mathcal{M}$, $w(m) = \sum_{p \in m} w(p)$.*

Fact 2 *Let w be a weighting function on LIT and $m, n, m', n' \in \mathcal{M}$. Then we have:*
If $m\Delta n \subseteq m'\Delta n'$ then $w(m\Delta n) \leq w(m'\Delta n')$

Lemma 3 *Let w be a weighting function on LIT and $s, t \in \mathcal{T}$ and $S, T \subseteq \mathcal{T}$. Then*

1. *$w(s \cap \bar{t}\,) = Min_{\leq}\{w(m\Delta n) : m \in [|s|]$ and $n \in [|t|]\}$*
2. *$Min_{\leq}\{w(m\Delta n) : m \in [|S|]$ and $n \in [|T|]\} = Min_{\leq}\{w(s \cap \bar{t}\,) : s \in S$ and $t \in T\}$*

This follows easily by lemma 1,(ii) and fact **??** and lemma 2.

The corresponding revision function is then:

$st_w(S, T) = \{t \cup (s \setminus \bar{t}\,) : t \in T, s \in S$ and $w(s \cap \bar{t}\,) = MinW\}$, where $MinW = Min_{\leq}\{w(s \cap \bar{t}\,) : s \in S$ and $t \in T\}$.

It is not difficult to see that st_w is an implicant revision function (for any weighting function w).

Dalal Revision

In (Dalal 1988) a revision operator is defined that is based on the Hamming distance of sets. For two interpretations this is the number of propositional variables that are assigned different truth values by the two interpretations. Dalal revision is obtained as a special case of weighting based revision, by setting $w(l) = 1$ for all $l \in LIT$.

Definition 8 *Let Γ be a belief set and ϕ a formula. The Dalal revision operator $*_{d_h}$ is defined by*

$[|\Gamma *_{d_h} \phi|] = \{n \in [|\phi|] : \exists m \in [|\Gamma|], d_h(m, n) = Min_{\leq}\{d_h(k, l) : k \in [|\Gamma|], l \in [|\phi|]\}$, where $d_h(m, n) = card\{p : p \in m$ iff $p \notin n\}$ is the Hamming distance between sets m and n.

We observe that:

1. Given two implicants s and t, $Min_{\leq}\{d_h(m, n) : m \in [|s|], n \in [|t|]\} = card(s \cap \bar{t}\,)$ (by lemma 1)
2. Given two sets of implicants, S, T, $Min_{\leq}\{d_h(m, n) : m \in [|S|], n \in [|T|] = Min_{\leq}\{card(s \cap \bar{t}\,) : s \in S, t \in T\}$ (by lemma 2).

This means that we obtain Dalal revision by minimizing the sets $s \cap \bar{t}$ for all $s \in S$ and $t \in T$ according to their cardinality. The revision result is then the set of implicants $t \cup (s \setminus \bar{t}\,)$ such that $card(s \cap \bar{t}\,) = Min_{\leq}\{card(s \cap \bar{t}\,) : s \in S, t \in T\}$.

Theorem 7 *Let Γ be a belief set, ϕ a formula and S (resp. T) a set of implicants equivalent to Γ (resp. ϕ). Then $st_d(S, T) = \{t \cup (s \setminus \bar{t}\,) : t \in T, s \in S$ and $card(s \cap \bar{t}\,) = Min_{\leq}\{card(s \cap \bar{t}\,) : s \in S, t \in T\}$ is an implicant revision function and $[|\Gamma *_{d_h} \phi|] = st_d(S, T)$.*

Satoh Revision

Satoh in (Satoh 1988) defines a revision operator that minimizes symmetric differences according to set inclusion. $st_{sa}(S, T) = \{t \cup (s \setminus \bar{t}\,) : t \in T, s \in S$ and $s \cap \bar{t} \in Min_{\subseteq}\{|s \cap \bar{t}\,| : s \in S, t \in T\}\}$

st_{sa} satisfies postulates (ST1) to (ST4) but fails to satisfy (ST5) as shows the following example (Katsuno and Mendelzon 1991b).

Example 2 $S = \{\{p, q, r, s\}, \{\neg p, \neg q, \neg r, \neg s\}\}$
$T = \{\{\neg p, \neg q, r, s\}, \{p, \neg q, \neg r, \neg s\}, \{\neg p, \neg q, r, s\}\}$
$U = \{\{\neg p, \neg q, r, s\}, \{p, \neg q, \neg r, \neg s\}\}$
Then $st_{sa}(S, T) \otimes U = \{\{p, \neg q, \neg r, \neg s\}\}$ but $st_{sa}(S, T \otimes U) = \{\{\neg p, \neg q, r, s\}, \{p, \neg q, \neg r, \neg s\}\}$.

Therefor st_{sa} is not an implicant revision operator.

Implementation and complexity considerations

Implicant revision postulates suggest to consider specific revision operations that are defined as operations on implicants. They indicate how to calculate effectively results of revision operations. Given a belief base with finite cover ϕ and a revision information μ we first produce the DNF of ϕ and μ and we obtain the implicant sets S and T. Then we apply operations on the implicants of ϕ and μ in order to obtain a revised base as a new implicant set.

Here we give the informal algorithm for calculating the implicant functions st_w and st_d, as well as st_{sa}, that is not an implicant revision operator since it fails to satisfy (ST5). Let formula ψ, represent a belief base and μ represent a new information. Let n be the maximal length of the formulas ($n = max(length(\psi), length(\mu))$.

1. Generate a disjunctive normal form of ψ and μ, yielding implicant sets S equivalent to ψ and T equivalent to μ. This yields $O(2^n)$ implicants.
2. Determine the symmetric differences $|s \cap \bar{t}\,|$ for all $s \in S$ and $t \in T$. The number of operations is $O(2^n \times 2^n)$.
3. Determine the $\leq\ -$minimal elements of the set $\{w(|s \cap \bar{t}\,|) : s \in S$ and $t \in T\}$. The number of operations is linear in the number of elements, i.e. $O(2^n \times 2^n)$.
4. The revision result is the set $st_w(S, T) = \{v : v = t \cup (s \setminus \bar{t}\,)$ and $w(s \cap \bar{t}\,) \in MinW\}$.

As pointed out above, For Satoh the points 3. is:

3' Determine the $\subseteq\ -$minimal elements of the set $\{|s \cap \bar{t}\,| : s \in S$ and $t \in T\}$. The number of operations is polynomial (quadratic) in the number of elements, i.e. $O((2^n \times 2^n))^2$.

4 . The revision result is the set $st_{sa}(S, T) = \{v : v = t \cup (s \setminus \bar{t}\,)$ and $w(|s \cap \bar{t}\,|) \in MinI\}$.

The problem of deciding whether a formula belongs to a knowledge base after revision resides on the second level of the polynomial hierarchy (Eiter and Gottlob 1992). Here we give the number of steps for calculating the set $st(S, T)$, where S and T are the set of implicants of formulas ϕ and μ. Let Γ be a formula representing a belief base and μ the new infomation. Let be n the length of the formula $\Gamma \wedge \mu$. Then, the number of conjuncts of a DNF of $\Gamma \wedge \mu$ is $O(2^n)$. To determine the $\leq$-smallest element (or elements) within the subsets $s \cap \bar{t}$ we have to compare all branches pairwise, that gives a polynomial time algorithm (in an exponential number of branches). These complexity results result from two factors:

- Obtaining implicants of a formula is NP-complete.
- The second factor results from the fact that our algorithm compares subsets in an exponential number of implicants (w.r.t. the number of propositional variables).

Better bounds can perhaps be obtained by addressing restrictions on the syntactic form of the belief set and of the revision formula.

Related work

Specific implicant revision operators defined for knowledge bases in specific syntactical forms have been proposed by several authors (Marchi, Bittencourt, and Perrussel 2010; Bienvenu, Herzig, and Qi 2008). Marchi et al. define a specific prime implicant revision method. Their system presupposes a knowledge base in clausal form and they propose to weighten a literal by counting the number of its occurrences within all clauses: the number of occurrences indicates the importance of the literal. The weight of an implicant is then the sum of the weights of the literals it contains. Their algorithm is based on prime implicants; from a belief base first the set of all its prime implicants must be calculated. The base resulting from a revision is in general not more in form of prime implicants and those have to be recalculated after every revision step, in the case of multiple revisions. Since the weight of a literal and of a prime implicant depends on the number of occurrences in the clauses of the belief base, it will probably change after each revision step. Hence for multiple revisions, new clauses must be obtained from the revision result and new weights must be calculated. The authors have extensively studied the performances of their system with many benchmark examples. The differences with our approach are the following:

- Marchi and co-authors use prime implicants, we do not use prime implicants but implicants.
- The revision result in (Marchi, Bittencourt, and Perrussel 2010) is in the form of implicants for both approaches.
- Multiple revision steps need a new treatment of the belief base in (Marchi, Bittencourt, and Perrussel 2010): prime implicants, clauses and weights must be recalculated. In our approach nothing must be redone, multiple revision steps can naturally occur.
- We have a more global approach that comports many revision approaches and we have clearly situated our class of revision functions. Marchi et al. calculate essentially one specific distance based revision operator.
- Marchi et al. have tested their approach on many benchmark examples, we only have a tableaux based system that can compute revised bases.

In our approach, we do not use prime implicants but implicants, multiple revision steps can naturally occur, we have a more global approach that comports many revision approaches and we have clearly situated our class of revision functions. Marchi et al. calculate essentially one specific distance based revision operator.

Bienvenu et al. (Bienvenu, Herzig, and Qi 2008) propose a prime implicate-based revision operator that is a full meet operator. Their algorithm is based on the set of all prime implicates of a formula K, $\Pi(K)$. The revision operator is defined by $K *_{\Pi} \phi = \phi \wedge \bigvee(\Pi(K) \bot \neg\phi)$ where $K \bot \phi$ is the set of maximal subsets of K consistent with $\neg\phi$. Again, one problem is that $\Pi(K)$ has to be recalculated after every revision step since the revised belief base is not more the conjunction of all its prime implicates. Our approach is more general (and simpler), since we do not need a belief base in normal form and we do not need to calculate the prime implicants (or prime implicates) of the belief base and the revision formula.

DelVal (Val 1991) present a framework where revision can be expressed in terms of DNF. He shows how to treat most of the known distance based revision operators.

Delgrande and Schaub present a "consistency based" approach to express belief change in general (contraction and revision) (Delgrande and Schaub 2003). Concerning revision, a belief change scenario is given by a pair (Γ, ϕ), where Γ is the belief set and ϕ is the revision formula. In order to revise Γ by ϕ, they start with ϕ, that must belong to the revised base according to postulate K$*$2, and then include as much as possible of Γ. Technically they express Γ and ϕ in different languages, associating a new propositional variable p' to every $p \in \mathcal{P}$. A belief base extension of scenario(Γ, ϕ) is then obtained from the set $\Gamma' \cup \{\phi\} \cup E$ where $E \subseteq \{p \leftrightarrow p' : p \in \mathcal{P}\}$ is a maximal subset of $\{p \leftrightarrow p' : p \in \mathcal{P}\}$ such that $\Gamma' \cup \{\phi\} \cup E$ is consistent. The belief base extension is then $E \cap \mathcal{L}(\mathcal{P})$. A belief change scenario can be expressed in this framework by using a selection function c on the set $\mathcal{E}$ of all extensions. c selects a subset of $\mathcal{E}$ and the revision result is the intersection of the selected subsets: $\Gamma *_c \phi = \bigcap c(\mathcal{E})$. Delgrande and Schaub present one revision operator that is the same than our implicant revision function st_{sa}. This is the operator that selects all extensions yielding their disjunction. The following result shows the precise relationship between Delgrande Schaub's consistency-based approach and our IBR approach.

Theorem 8 *Let $E \subseteq \mathcal{F}$ be a belief set extension on (Γ, ϕ). Then there is an implicant revision function st such that $[|E|] = [|v|]$ for some $v \in st(\Gamma^I, \phi^I)$.*

Conclusion

In this paper we defined a new class of belief set revision operators. They work on a specific syntactical formula representation, namely a DNF or implicant sets, and we have defined a specific set of postulates that define implicant revision. We have situated our implicant revision operators w.r.t. the AGM approach and distance based revision. We have shown that several known approaches to theory revision are implicant revisions.

In this paper we only address one type of theory change operators, namely revision. We think that we can also capture other belief change functions such as update (Katsuno and Mendelzon 1991a; Winslett 1988; Chou and Winslett 1991), erasure or contraction (Gärdenfors 1988) but also merging (Konieczny and Pérez 1998) and iterated revision (Darwiche and Pearl 1997).

Our revision functions can naturally be calculated by use of a tableaux prover. The branches of a tableau for a formula ϕ are implicants of the formula. In this case, the tableaux prover is used classically and revision results are "calculated" by operations on tableau branches, that are the operations on implicants described in this paper. One of our forthcoming research will be the definition of tableaux rules that give directly a decision procedure for the problem wether a revised knowledge base $\Gamma * \phi$ contains some formula ψ. Delgrande, Jin and Pelletier in (**?**) have investigated rules for calculating updates that break up the update formula. We think of tableaux rules that break up the belief base formula according to the new information. Our new implicant revision postulate (ST1) suggests that the revised belief base is composed of implicants that contain implicants of the revision formula and parts of implicants of the belief base. Thus, a decision procedure for the problem whether a formula is entailed by a revised belief base must modify the original belief base according to the contradictions with elements of the revision formulas.

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