Preferential model semantics, argumentation frameworks and closure properties

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Abstract

Preferential models provide a semantics for non-monotonic reasoning systems. Moreover, they enable the study of closure properties at the level of the semantics.

Argumentation frameworks are an abstraction of argumentation systems that enable the specification of justified arguments through an argumentation semantics. Since an argumentation framework abstracts from the premisses and the conclusions of arguments, closure properties cannot be addressed.

This paper proposes a less abstract *suppositional argumentation framework* that enables the study of closure properties. The closure properties *Cumulativity* and *Loop* are investigated using a preferential model semantics that is defined for a suppositional argumentation framework.

Introduction

Argumentation systems are now a day often used as a proof theory for non-monotonic logics, and are used in the study of human argumentation, for instance in legal reasoning. An *argumentation framework* offers an abstract representation of an argumentation system (Dung 1995). An argumentation framework abstracts from the conclusion supported by an argument and from the reasoning process that leads to the conclusion. It only keeps track of attack relations between arguments. Several *argumentation semantics* (Dung 1995; Baroni, Giacomin, and Guida 2005; Caminada 2006; Baroni and Giacomin 2007; Bench-Capon and Dunne 2007; Dung, Mancarella, and Toni 2007) have been defined for argumentation frameworks, specifying sets of justified arguments which support the conclusions we may accept to believe.

Argumentation with defeasible arguments is a special case of *non-monotonic reasoning*. The *preferential model semantics* is the standard semantics of the underlying non-monotonic logics (Kraus, Lehmann, and Magidor 1990; Makinson 1988; 1994). This suggests that there must be a relation between the preferential model semantics and argumentation frameworks. The research on which this paper reports, studies this relation.

The first results concerning the relation between the preferential model semantics and argumentation frameworks were presented in (Roos 2010). The paper describes a preferential model semantics for argumentation frameworks and shows that well known argumentation semantics (Dung 1995) correspond to restrictions on the preference relation.

Closure properties The preferential model semantics can be used to study the closure properties of a corresponding reasoning process. Closure properties are desirable properties of the consequence operator $C_<$. Given a set of facts Σ and a preference relation generated by the available knowledge, the preferential model semantics specifies a set of preferred conclusions $C_<(\Sigma)$.

Closure properties are related to rationality postulates described in (Caminada and Amgoud 2007). The main difference is that closure properties focus on (non-)monotonic reasoning systems in general while the rationality postulates focus on argumentation systems.

An argumentation framework abstracts from the structure of an argument and the way the argument is constructed. So, we neither know the facts that have been used in the construction of an argument, nor the conclusion supported by the argument. Because the facts are hidden within the arguments, we cannot investigate closure properties; i.e., the relation between the conclusions supported by justified arguments and a set of facts Σ used to construct a set of arguments.

To overcome this limitation, we will introduce a less abstract *suppositional argumentation framework*. The idea is that we construct arguments for different sets of facts Σ. In other words: construct arguments *supposing* that the facts Σ hold in the world. This resembles the suppositional arguments introduced by Pollock (1992). Hence the name *suppositional argumentation framework*. Note that we still abstract from the structure of an argument and the way the argument is constructed.

In this paper we will focus on two closure properties: *Cumulativity* and *Loop*. Cumulativity states that the confirmation of a believed conclusion should not change the set of conclusions while Loop states that equivalent representations should result in the same set of conclusions. Other closure properties such as *Supra Classicality*, *Absorption* and *Distribution* will not be addressed because of space limitations.

The relevance of Cumulativity Although Cumulativity states that the confirmation of a believed conclusion should not change the set of conclusions, several researchers doubt whether Cumulativity is desirable.[1] They point out that an argument based on a fact φ is stronger than an argument based an a justified sub-argument supporting φ. As a result the set of conclusions may change after adding the fact φ. Moreover, some researcher point out that the absence of Cumulativity in many systems has not been a reason to abandon these systems.[1]

Another argument in favor of Cumulativity is the equivalence of Cumulativity with *Reciprocity*

if $\Sigma \subseteq C(\Gamma)$ and $\Gamma \subseteq C(\Sigma)$, then $C(\Gamma) = C(\Gamma)$.

in the presence of *Inclusion* ($\Sigma \subseteq C(\Sigma)$). Reciprocity says that two equivalent representations should result in the same set of conclusions. If one agrees that equivalent representations must lead to the same set of conclusions, Cumulativity must hold. Note that Loop is a stronger version of Reciprocity.

Below we will see that Cumulativity holds if certain types of self-attacking arguments are disallowed. This suggests that the absence of Cumulativity is a problem of knowledge represented in a system. The system in which the knowledge is represented is therefore not to blame.

Paper outline First, the definitions of argumentation semantics and of preferential model semantics are given. Next the suppositional argumentation frameworks are introduced and a preferential model is defined. Subsequently, the closure properties Cumulativity and Loop are addressed. Finally, the relation with the argumentation semantics is described and related work is discussed.

Preliminaries

This section reviews the definitions of the argumentation semantics and the preferential model semantics.

Argumentation semantics We use Dung's argumentation framework as a starting point (Dung 1995).

Definition 1 *An argumentation framework is a couple $AF = \langle \mathcal{A}, \longrightarrow \rangle$ where $\mathcal{A}$ is a finite set of arguments and $\longrightarrow \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation over the arguments.*

For convenience, we extend the attack relation $\longrightarrow$ to sets of arguments.

Definition 2 *Let $A \in \mathcal{A}$ be an argument and let $\mathcal{S}, \mathcal{P} \subseteq \mathcal{A}$ be two sets of arguments. We define:*

- *$\mathcal{S} \longrightarrow A$ iff for some $B \in \mathcal{S}$, $B \longrightarrow A$.*
- *$A \longrightarrow \mathcal{S}$ iff for some $B \in \mathcal{S}$, $A \longrightarrow B$.*
- *$\mathcal{S} \longrightarrow \mathcal{P}$ iff for some $B \in \mathcal{S}$ and $C \in \mathcal{P}$, $B \longrightarrow C$.*

We wish to select a coherent subset $\mathcal{E}$ of the set of arguments $\mathcal{A}$ of the argumentation framework $AF = \langle \mathcal{A}, \longrightarrow \rangle$. Such a set of arguments $\mathcal{E}$ is called an *argument extension*. The arguments of an argument extension support propositions that give a coherent description of what might hold in the world. Clearly, a basic requirement of an argument extension is being *conflict-free*; i.e., no argument in an argument extension attacks another argument is the argument extension. Beside being conflict-free, we will use the notion of an admissible set of arguments and the notion of an argument that is acceptable w.r.t. a set of arguments. Defense against attacking arguments forms the basis of both notions. An admissible set of arguments defends itself against all attacking arguments. An argument is acceptable w.r.t. a set of arguments if the argument is defended by that set against all attacking arguments.

Definition 3 *Let $AF = \langle \mathcal{A}, \longrightarrow \rangle$ be an argumentation framework and let $\mathcal{S} \subseteq \mathcal{A}$ be a set of arguments.*

- *$\mathcal{S}$ is* conflict-free *iff $\mathcal{S} \not\longrightarrow \mathcal{S}$.*
- *$\mathcal{S}$ is admissible iff $\mathcal{S}$ is conflict-free and for every argument $A \in \mathcal{A}$: if $A \longrightarrow \mathcal{S}$, then $\mathcal{S} \longrightarrow A$.*
- *$A \in \mathcal{A}$ is acceptable w.r.t. $\mathcal{S}$ iff for every argument $B \in \mathcal{A}$, if $B \longrightarrow A$, then $\mathcal{S} \longrightarrow B$.*

Not every conflict-free set of arguments is considered to be an argument extension. Several additional requirements have been formulated by Dung (1995), resulting in different semantic definitions.

Definition 4 *Let $AF = \langle \mathcal{A}, \longrightarrow \rangle$ be an argumentation framework and let $\mathcal{E} \subseteq \mathcal{A}$.*

- *$\mathcal{E}$ is a* stable extension *iff $\mathcal{E}$ is conflict-free and for every argument $A \in (\mathcal{A} \setminus \mathcal{E})$, $\mathcal{E} \longrightarrow A$.*
- *$\mathcal{E}$ is a* preferred extension *iff $\mathcal{E}$ is maximal (w.r.t. $\subseteq$) admissible set of arguments.*
- *$\mathcal{E}$ is a* complete extension *iff (i) $\mathcal{E}$ is an admissible set of arguments, and (ii) every argument $A \in \mathcal{A}$ that is acceptable w.r.t. $\mathcal{E}$ belongs to $\mathcal{E}$.*
- *$\mathcal{E}$ is a* grounded extension *iff $\mathcal{E}$ is the minimal (w.r.t. $\subseteq$) complete extension.*

Since the publication of Dung's paper a number of new semantics have been proposed. Baroni et al. (2005) propose a different way of handling odd loops. Dung et al. (2007) have proposed the *ideal semantics*. Caminada studies argumentation semantics in terms of labelings and describes the *semi-stable semantics* (Caminada 2006), which was introduced by Verheij (1996). For an overview of the new argumentation semantics that have been proposed, see (Baroni and Giacomin 2007; Bench-Capon and Dunne 2007).

Preferential model semantics Preferential model semantics was introduced by Shoham (1987) as a generalization of circumscription (McCarthy 1980), and has subsequently been extended to a general semantic theory by Makinson (1988) and Kraus et al. (1990). The definitions given below are based on the formalization given by Makinson in (1994).[2]

[1] These points were raised in discussion about the here reported research.

[2] The preferential model semantics should not be confused with the handling of conflicting arguments using a preference rela-

We start with a propositional language $\mathcal{L}$ for which we define the preferential model semantics. The preferential model semantics uses *preferential models* to define an agent's beliefs given its knowledge about the world.

Definition 5 *A preferential model $P = (S, \models, <)$ is a triple where:*

- *S is a set of states,*
- *$\models \subseteq (S \times \mathcal{L})$ is an arbitrary relation between states and propositions, called the* entailment relation[3]*,*
- *$< \subseteq (S \times S)$ is an arbitrary relation between states, called the* preference relation*.*

Note that a preferential model does not specify what the states and the entailment relation exactly are. In general, one can view a state as an interpretation or a set of interpretations of propositional or first order logic. The entailment relation can then be viewed as a specification of the semantics of a proposition with respect to a state. For the moment, however, we do not consider such a restricted view on what the states and the entailment relation represent.
Note that the preference relation denotes that we *prefer* a state s to a state s' if $s < s'$.[4]

A preferential model $P = (S, \models, <)$ can be used to specify that a state preferentially satisfies a proposition $\varphi \in \mathcal{L}$. Preferential entailment focusses on the preferred states among the states satisfying the proposition.

Definition 6 *Let $P = (S, \models, <)$ be a preferential model, $s \in S$ be a state and let $\varphi \in \mathcal{L}$ be a proposition.*

s preferentially entails *φ, denoted by $s \models_< \varphi$, iff $s \models \varphi$ and for no $s' \in S$: $s' < s$ and $s' \models \varphi$.*

We extend the notion of entailment of a proposition to a set of propositions: $s \models \Sigma$ iff for every $\sigma \in \Sigma$, $s \models \sigma$. This immediately gives us the preferential entailment of a set of propositions: $s \models_< \Sigma$.[5] The latter we need to define the preferential consequences of a set of propositions Σ. Preferential consequences are those propositions that are entailed (satisfied) by all states that preferentially entail the set of propositions Σ. We will use the *preferential entailment operator $C_<$* to denote this set of consequences.

Definition 7 *Let $P = (S, \models, <)$ be a preferential model, and let $\Sigma \subseteq \mathcal{L}$ be a set of propositions.*

The preferential entailment operator $C_<$ is defined as:

$$C_<(\Sigma) = \{\varphi \in \mathcal{L} \mid \textit{for all } s \in S, \textit{ if } s \models_< \Sigma, \textit{ then } s \models \varphi\}$$

Closure properties One of the main advantages of the preferential model semantics is the relations with closure properties. The closure properties describe desirable properties of the preferential entailment operator $C_<(\cdot)$, which can be related to restrictions on the entailment and the preference relation.

The above defined preferential entailment operator possesses two important properties, namely, *Inclusion* and *Cut*. *Inclusion* says that an agent should be able to conclude its initial beliefs:

$$\Sigma \subseteq C_<(\Sigma)$$

Knowing that *Inclusion* holds, *Cut* says that an agent should not be able to conclude more after adding some conclusions to the initial set of beliefs:

$$\text{if } \Sigma \subseteq \Gamma \subseteq C_<(\Sigma), \text{ then } C_<(\Gamma) \subseteq C_<(\Sigma)$$

Beside that an agent should not be able to conclude more it should also not be able to conclude less after adding some conclusions to the initial set of beliefs. This property is called *Cautious Monotony*:

$$\text{if } \Sigma \subseteq \Gamma \subseteq C_<(\Sigma), \text{ then } C_<(\Sigma) \subseteq C_<(\Gamma)$$

In order to guarantee that a preferential model possesses the property *Cautious Monotony*, we have to place a restriction on the preference relation. A sufficient condition for Cautious Monotony is a preference relation that is *smooth*[6].

> A preferential model is called *smooth* iff for every set of propositions Σ and for every state $s \in S$, if $s \models \Sigma$, then there exists a state $s' \in S$ such that $s' \leq s$ and $s' \models_< \Sigma$. ($\leq$ denotes: $<$ or $=$)

Cautious Monotony together with *Cut* gives us the property *Cumulativity*:

$$\text{if } \Sigma \subseteq \Gamma \subseteq C_<(\Sigma), \text{ then } C_<(\Sigma) = C_<(\Gamma)$$

Many forms of knowledge representation including argumentation systems do not possess the property *Cumulativity*. Although they possess the property *Cut*, they do not process the property *Cautious Monotony*.

Different formulations of knowledge and beliefs that are in some sense equivalent should not lead to different sets of conclusions. The property *Loop* expresses this idea:

$$\text{if } \Sigma_2 \subseteq C_<(\Sigma_1), \ldots, \Sigma_n \subseteq C_<(\Sigma_{n-1}), \Sigma_1 \subseteq C_<(\Sigma_n), \\ \text{then } C_<(\Sigma_1) = C_<(\Sigma_i) \text{ for all } 1 \leq i \leq n$$

A smooth preferential model possesses the property *Loop* if the preference relation is *transitive*.

Suppositional argumentation frameworks

An important aspect of the preferential model semantics is its relation with closure properties of the preferential entailment operator $C_<(\cdot)$. We cannot investigate the closure properties of an argumentation framework because the facts on which arguments are based are integrated in the arguments. Moreover, we also abstracted from the propositions supported by the arguments.

To study the closure properties of argumentation frameworks, we must be able to identify the arguments that can

tion defined over the arguments; see for instance (Dimopoulos, Moraitis, and Amgoud 2008). A preference relation over arguments expresses in some way the strength of an argument while a preference relation over states expresses that the world should correspond to one of the preferred states.

[3]A state s is said to entail or satisfy a proposition φ iff $s \models \varphi$.

[4]For historical reasons, namely minimizing exceptions, preference is associated with minimality.

[5]$s \models_< \Sigma$ iff $s \in min_<|\Sigma|$ where $|\Sigma| = \{s \in S \mid s \models \Sigma\}$.

[6]The term 'smooth' was introduced by Kraus et al. (Kraus, Lehmann, and Magidor 1990). Makinson uses the term 'stoppered'.

be constructed *supposing* that a set of facts Σ holds in the world. We therefore propose a *suppositional argumentation framework*. In this argumentation framework, the set of premises of an argument is made explicit. Since we also need to know the conclusions supported by the arguments, also the conclusion of an argument is made explicit. A suppositional argumentation framework is therefore less abstract than Dung's argumentation framework. Note that we still abstract from the structure of an argument and the way the argument is constructed. So, we do not consider any specific argumentation system.

The definition of a suppositional argumentation framework extends the definition of Dung's argumentation framework with operators for specifying the premises and the conclusion of an argument. The language of propositions that is used to describe the premises and the conclusion of an argument is denoted by $\mathcal{L}^B$.

Definition 8 *A* suppositional argumentation framework *is a tuple $SAF = \langle \mathcal{A}, \longrightarrow, ^{-}, \hat{} \rangle$ where $\mathcal{A}$ is a finite set of suppositional arguments, $\longrightarrow \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation over the arguments, $\bar{A} \subseteq \mathcal{L}^B$ denotes the set of premises of the suppositional argument $A \in \mathcal{A}$, and $\hat{A} \in \mathcal{L}^B$ denotes the conclusion supported by A.*

An argument A with a single premise σ and a conclusion φ will sometimes be denoted by: A^{φ}_{σ}.

The following example gives an illustration of a suppositional argumentation framework.

Example 1 *Suppose that we have three defeasible rules that are used to construct suppositional arguments: $p \rightsquigarrow q$, $q \rightsquigarrow r$, and $s \rightsquigarrow \neg r$. Let $p \rightsquigarrow q$ have the lowest preference and let $q \rightsquigarrow r$ have the highest preference. Using these rules the following arguments might be constructed: $A^q_p = [p \rightsquigarrow q]$, $B^r_p = [p \rightsquigarrow q, q \rightsquigarrow r]$, $C^{\neg r}_s = [s \rightsquigarrow \neg r]$, $D^r_q = [q \rightsquigarrow r]$ and $E^{\neg q}_s = [s \rightsquigarrow \neg r, q \rightsquigarrow r]$.*[7]

Now suppose that the least preferred rule must be invalid if a contradiction is derived. This gives us the attack relation: $\longrightarrow = \{(C,B),(D,C),(D,E),(E,A),(E,B)\}$.

We can now identify argument extensions for different sets of facts that may hold in the world.

- *Given the facts p and s, an argument extension will contain the arguments: $C^{\neg r}_s$ and $E^{\neg q}_s$.*
- *Given the facts q and s, an argument extension will contain the argument: D^r_q.*
- *Finally, given the facts p, q and s, an argument extension will contain the arguments: A^q_p, B^r_p and D^r_q.*

A preferential model for SAF

A state of a preferential model is often interpreted as giving an abstract representation of the world or the agent's beliefs about the world. An agent's beliefs about the world consists of propositions supported by arguments the agent accepts. Therefore, a state should also specify the arguments that are accepted. These arguments are entailed by the state. The conclusions of the accepted / entailed arguments specify propositions entailed by the state.

[7] We use square brackets to emphasize that the rules between the brackets are somehow used in the construction of argument.

A state cannot cannot specify any set of arguments. First, the set of arguments must be conflict-free w.r.t. the attack relation $\longrightarrow$. Second, a suppositional argument can only be entailed by a state if the state also entails the premises of the argument. Premises are either facts about the world or propositions supported by arguments. To create a uniform approach, we will represent facts about the world by a special type of arguments, which we call *hypotheses*.

Definition 9 *The set of arguments representing all possible hypotheses is defined as:*

$$\mathcal{H} = \{H \mid \hat{H} = \sigma \in \mathcal{L}^B, \bar{H} = \varnothing\}$$

A hypothesis H supporting a fact σ may be denoted by: H^{σ}.

Note that we distinguish between a hypothesis $H^{\sigma} \in \mathcal{H}$ and an argument $A^{\sigma}_{\varnothing} \in \mathcal{A}$ with an empty set of premisses.

The introduction of hypotheses makes it possible to specify a state using a subset of the arguments $\mathcal{H} \cup \mathcal{A}$.

Definition 10 *A state s is defined by a subset of $\mathcal{H} \cup \mathcal{A}$, which we denote by $[s]$. So, $[s] \subseteq (\mathcal{H} \cup \mathcal{A})$.*

The entailment relation w.r.t. a state s is defined as:

- *$s \models A$ with $A \in (\mathcal{H} \cup \mathcal{A})$ iff $A \in [s]$.*
- *$s \models \varphi$ with $\varphi \in \mathcal{L}^B$ iff $\varphi = \hat{A}$ for some $A \in [s]$.*

Finally, a state s must satisfy the following two requirements:

- *$[s] \not\longrightarrow [s]$*
- *For every $A \in [s]$ and for every $\sigma \in \bar{A}$, $s \models \sigma$.*

The set of all states satisfying these requirements is denoted by S^*.

The choice for states represented by set of arguments is similar to (Roos 2010). In the latter paper, states are represented by set of arguments because we do not know the premises and the conclusion of an argument in Dung's argumentation framework. Here states also represent sets of arguments. Thereofore, the preference relation of the preferential model can be defined in a similar way as is done in (Roos 2010). The main difference is that we have to take the applicability of arguments into account. The following definition defines the applicable arguments of a state and other useful functions.

Definition 11 *Let S^* be a set of states, $\mathcal{A}$ be a set of suppositional arguments and $\mathcal{H}$ be a set of hypotheses.*

- *$\mathcal{A}p(s) = \{A \in \mathcal{A} \mid s \models \bar{A}\}$ denotes the applicable arguments of $s \in S^*$.*
- *$\mathcal{A}p(R) = \bigcap_{s \in R} \mathcal{A}p(s)$ denotes the applicable arguments of a set of states $R \subseteq S^*$.*
- *$\mathcal{A}(s) = \{A \in \mathcal{A} \mid s \models A\}$ denotes the arguments entailed by $s \in S^*$*
- *$\mathcal{H}(s) = \{H \in \mathcal{H} \mid s \models H\}$ denotes the hypotheses entailed by $s \in S^*$*

An attack relation $B \longrightarrow A$ does not only express that the two arguments A and B cannot be entailed by the same state. The attack relation also expresses a preference between states. We prefer a state entailing the attacking arguments to a state entailing the attacked arguments.

Requirement 1 *A state s is preferred to a state s' if every argument entailed by the state s' that is no longer entailed by s but is applicable in s is attacked by an argument entailed by s that is also applicable in s'.*

Example 2 *Consider two states, s and s' where $[s] = \{H^\sigma, H^\mu, A^\alpha_\sigma\}$ and $[s'] = \{H^\sigma, H^\mu, B^\beta_\mu\}$. Moreover, let $B \longrightarrow A$. Since the arguments A and B are applicable in both states, we should prefer the state s' entailing the attacking argument B to the state s entailing the attacked argument A.*

Now consider two additional states s'' and s''' where $[s''] = \{H^\sigma, H^\mu, A^\alpha_\sigma, C^\gamma_\alpha\}$ and $[s'] = \{H^\sigma, H^\mu, B^\beta_\mu, D^\delta_\beta\}$. Moreover, let $C \longrightarrow D$. Since the arguments A and B are still the only applicable arguments in both states, we should prefer the state s''' to the state s''. Moreover, we prefer s'' to s, s''' to s', s' to s'' and s''' to s.

The requirement enable us to define a weak preference relation $\lesssim$ over the states S of a preferential model that share the same set of hypotheses. Preference between states cannot change the set of facts that hold in the world. Therefore the set of hypotheses may not change.

Definition 12 *Let $SAF = \langle \mathcal{A}, \longrightarrow, \bar{\ }, \hat{\ } \rangle$ be a suppositional argumentation framework. Moreover, let S^* be a set of states and let $\models \subset (S^* \times (\mathcal{A} \cup \mathcal{H} \cup \mathcal{L}^B))$ be an entailment relations over states, arguments and propositions.*
The weak preference relation $\lesssim \subset (S^* \times S^*)$ is defined as: $s \lesssim s'$ iff

- *for every $B \in \mathcal{A}p(\{s, s'\})$ such that $s' \models B$ and $s \not\models B$ there is an $A \in \mathcal{A}p(\{s, s'\})$ such that $s \models A$, $A \longrightarrow B$;*
- *$\mathcal{H}(s) = \mathcal{H}(s')$.*

Note that the weak preference relation $\lesssim$ is not strict. Consider for example the well known Nixon diamond in which we have an argument for Nixon being a pacifist and an argument for Nixon being a non-pacifist. Clearly the two arguments attack each other and without additional information we have no reason to prefer either one of them. Hence, the preferences generated by mutual attacks cannot be strict.

Definition 6 of preferential entailment assumes that the preference relation is strict. If the preference relation $<$ of a preferential model is not strict, we cannot be sure that s' is a non-minimum state if $s < s'$. The states s and s' could also be equally preferred, implying $s' < s$.

Since the preference relation $\lesssim$ generated by the attack relation $\longrightarrow$ is not strict, we have to transform it into a strict relation. We know that $s \lesssim s'$ does not indicate a strict preference if there exists a set of states $\{s_1, ..., s_n\}$ such that:

$$s \lesssim s' \lesssim s_1 \lesssim ... \lesssim s_n \lesssim s$$

Therefore, we may define $s < s'$ as: $s \lesssim s'$ and $s' \not\lesssim^+ s$, where $\lesssim^+$ denotes the transitive closure of $\lesssim$.

In (Roos 2010), the above proposed preference relation $<$ is used to determine the preferred states of the preferential model. In case of suppositional argumentation frameworks, we need to extend this preference relation in order to prefer the right states. To illustrate the need to extend the preference relation, consider two suppositional arguments A^α_σ and B^β_μ, and the attack relation $B \longrightarrow A$. Figure 1.a shows some of the states and the preference relation between these states of the corresponding preferential model.

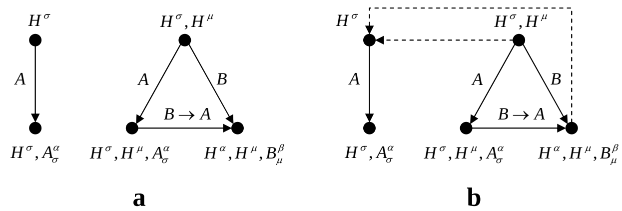

Figure 1: Unwanted preferences.

Suppose that the set of facts $\Sigma = \{\sigma\}$ is given. Then we have to restrict the set of states in the preferential model shown in Figure 1.a to those satisfying Σ, which are all the states. Within this set of states, there are two preferred states; the state entailing $H^\sigma, A^\alpha_\sigma$ and the state entailing $H^\sigma, H^\mu, B^\beta_\mu$. The preferred state entailing $H^\sigma, H^\mu, B^\beta_\mu$ is clearly unwanted because the suppositional argument B is not applicable if all we know is that Σ holds in the world.

The set of states in which the argument B is applicable overlaps with the set of states entailing the set of facts Σ. However, there is no support for the premises of the argument B given Σ. The states in which B is applicable assume more information. We should therefore prefer a less informative state entailing Σ whenever possible. The information of a state s is measured by the set of propositions $\mathcal{L}^B(s) = \{\varphi \in \mathcal{L}^B \mid s \models \varphi\}$ entailed by s. A state s entails *strictly less information than* s', denoted by $s \prec s'$, iff $\mathcal{L}^B(s) \subset \mathcal{L}^B(s')$.

We also require that a preferred less informative state consists of hypotheses only. This guarantees that preferred states are grounded in facts. The dashed lines in Figure 1.b show the preference that should be added.

Preferring less informative states may result in cycles. In the preferential model shown in Figure 3, the state entailing H^σ, H^β is less informative than the state entailing $H^\sigma, A^\alpha_\sigma, B^\beta_\alpha$. Since the less informative state assumes more facts, we should not add a preference for this state.

Requirement 2 *A state s is preferred to a state s' if s is strictly less informative than s' and s does not entail more (w.r.t. $\subseteq$) hypotheses than s'.*

This requirement does not avoid all cycles but the remaining cycles are harmless.

There is one last issues that must be addressed before we can define a preferential model for a suppositional argumentation framework. The preferences between the states reflect a part of the argumentation process. Consider for instance the preferential model in Figure 2.a. The figure shows that the argument A is attacked by the argument B resulting in preferring s_3 and s_4 to s_2. Now suppose that we consider the facts $\Sigma = \{\sigma, \mu, \alpha\}$. Then s_2 and s_4 are preferred states entailing Σ; i.e., $\{s_2, s_4\} \subseteq \min_< |\Sigma|$. Clearly, the preference for s_2 is undesirable since the argument B is applicable in s_2, and since s_4 is preferred to s_2. To avoid this problem, the preferential model should only contain states that defend themselves against attacking arguments. Figure 2.b illustrates this restriction.

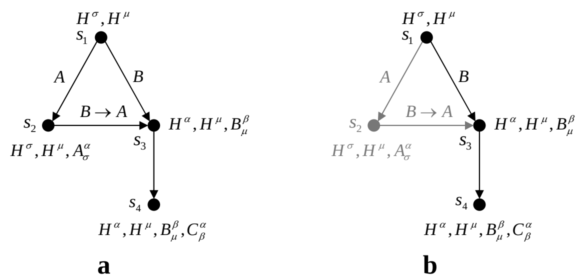

Figure 2: Valid states of the preferential model.

Requirement 3 *A state s is a valid state of the preferential model if it defends itself against attacking arguments.*

The preference relation can now be defined. Note that the second item formalizes the defense against attacking arguments.

Definition 13 *Let $s, s' \in S^*$ be two states.*

The state s is preferred *to s', denoted by $s < s'$, iff one of the two conditions holds:*

1. *$s \prec s'$, $\mathcal{A}(s) = \varnothing$ and $\mathcal{H}(s') \not\subseteq \mathcal{H}(s)$,*
2. *$s \lesssim s'$, $\mathcal{A}(s') \subseteq \mathcal{A}(s)$, and if $s'' \lesssim s$, then either $s \lesssim^+ s''$ or $\mathcal{A}(s) \subseteq \mathcal{A}(s'')$.*

Putting all requirements together leads to Definition 14.

Definition 14 *Let $SAF = \langle \mathcal{A}, \longrightarrow, ^-, \hat{} \rangle$ be a suppositional argumentation framework, let $\mathcal{L}$ be a language and let S^* be a set of states. Moreover, let $<$ be the preference relation defined in Definition 13.*

The preferential model $P = (S, \models, <)$ for the suppositional argumentation framework SAF is defined by the restriction of the states S^* to the states S meeting the following conditions:

- *$s \in S$ if $\mathcal{A}(s) = \varnothing$,*
- *$s \in S$ if $s < s'$ and $s' \in S$,*
- *nothing else belongs to S.*

Closure properties An important advantage of preferential models is their relation with closure properties of the consequence operator $C_< : 2^{\mathcal{L}^B} \rightarrow 2^{\mathcal{L}^B}$:

$$C_<(\Sigma) = \{\varphi \in \mathcal{L}^B \mid \text{ for all } s \in S, s \models_< \Sigma \text{ implies } s \models \varphi\}$$

where $\models_<$ is the entailment relation defined in Definition 6. In the next section we will study the conditions under which an argumentation system will possess the properties Cumulativity and Loop.

Conditions for Cumulativity and Loop

To study the conditions under which a suppositional argumentation framework is Cumulative, we will use the following well known example showing the absence of Cumulative.

Example 3 *Let $\mathcal{A} = \{A^\alpha_\sigma, B^\beta_\alpha, C^{\neg\alpha}_\beta\}$ be the set of suppositional arguments. Moreover, let there be no preference between the arguments A^α_σ and $C^{\neg\alpha}_\beta$, resulting in a mutual attack between the arguments: : $A \longrightarrow C$ and $C \longrightarrow A$.*

Figure 3 shows part of the preferential model generated by the suppositional argumentation framework described in the example. Suppose that the set of facts $\Sigma = \{\sigma\}$ is given. All states in Figure 3 entail this set of facts. The state entailing $H^\sigma, A^\alpha_\sigma, B^\beta_\alpha$ is the preferred state, which entails the propositions σ, α and β.

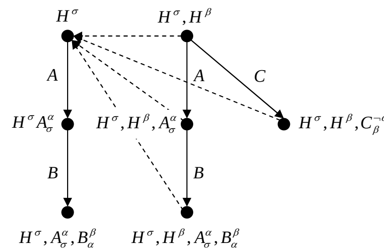

Figure 3: Unsound suppositional argumentation framework.

Next suppose that the set of facts $\Gamma = \{\sigma, \beta\}$ is given. All states in Figure 3 except for the states entailing H^σ and $H^\sigma, A^\alpha_\sigma$ entail Γ. Now, there are three preferred states because the chain of preferences is broken without the state entailing H^σ. As a result, we have two states entailing the propositions σ, α and β, and one state entailing the propositions σ, $\neg\alpha$ and β. So, we do not have Cumulativity.

We can solve the above described problem by introducing the attack relation $A \longrightarrow C$. Figure 4 shows a part of the resulting preferential model. Given the set of facts $\Gamma = \{\sigma, \beta\}$ we have two preferred states, which both entail the propositions σ, α and β. So, in this example Cumulativity holds.

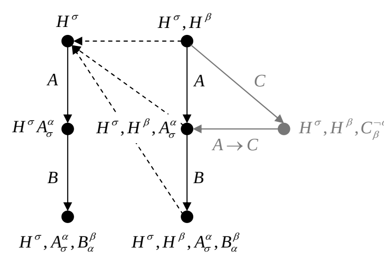

Figure 4: Sound suppositional argumentation framework.

A question that we may ask is whether there is a general reason for adding the attack relation $A \longrightarrow C$. In the example, we have a chain of arguments where the conclusion of one argument enables the next argument; i.e., is the premise of the next argument. The first argument in the chain is A^α_σ and the last is $C^{\neg\alpha}_\beta$. Since A indirectly enables C, C should not attack A.

Caminada (2004) calls arguments where A enables C and C attack A, *hang yourself* arguments. In his view these arguments are a form of *reductio ad absurdum*.

A suppositional argumentation framework without hang yourself arguments will be called *sound*.

Definition 15 *Let $SAF = \langle \mathcal{A}, \longrightarrow, \hat{}, ^- \rangle$ be a suppositional argumentation framework.*

The argumentation framework is called sound *iff for every pair of arguments $A, C \in \mathcal{A}$ and for every sequence of* *arguments* $A = B_1, B_2, \ldots, B_k = C$ *such that* $\hat{B}_i \in \bar{B}_{i+1}$, $C \not\longrightarrow A$.

The soundness of a suppositional argumentation framework turns out to be a sufficient condition for the closure properties Cumulativity and Loop.

Theorem 1 *Let* $SAF = \langle \mathcal{A}, \longrightarrow, \hat{\ }, \bar{\ } \rangle$ *be a* sound *suppositional argumentation framework. Then* SAF *satisfies the closure properties* Cumulativity *and* Loop.

The relation with argumentation frameworks

Suppositional argumentation frameworks have been introduced in order to study the closure properties and to refine the relation with preferential models. Till now we did not address the relation with Dung's argumentation frameworks.

Given a set of facts Σ, a suppositional argumentation framework specifies a set of applicable arguments; i.e., arguments of which the premisses are included in Σ. So, using the facts Σ we can transform a suppositional argument into an abstract argument of Dung's argumentation framework.

Definition 16 *Let* $SAF = \langle \mathcal{A}, \longrightarrow, \bar{\ }, \hat{\ } \rangle$ *be a suppositional argumentation framework and let* Σ *be a set of facts.*

Then the corresponding argumentation framework $AF = SAF[\Sigma] = \langle \mathcal{A}', \longrightarrow' \rangle$ *is defined as:*

- $\mathcal{A}' = \{A \in \mathcal{A} \mid \bar{A} \subseteq \Sigma\} \cup \mathcal{H}$
- $\longrightarrow' = \longrightarrow \cap (\mathcal{A}' \times \mathcal{A}')$

The preferential model $P' = (S', \models', <')$ for Dung's argumentation framework defined in (Roos 2010) can be derived from the preferential model $P = (S, \models, <)$ for SAF. S' must consist of the states that only contain arguments from $\mathcal{A}'$ and hypotheses supporting facts form Σ.

- $S' = \{s \in S \mid \{\sigma \mid s \models H^\sigma\} \subseteq \Sigma, \mathcal{A}(s) \subseteq \mathcal{A}'\}$
- $s \models' A$ iff $A \in [s]$
- $>' = > \cap (S' \times S')$

We can use the preferential model P' to define a consequence operator $C'_<[\Sigma]$ using the conclusion operator $\hat{\ }$ of SAF.

$$C'_<[\Sigma] = \{\hat{A} \mid \text{for all } s \in S', s \in \min_{<'} S' \text{ implies } s \models' A\}$$

The conclusions $C_<(\Sigma)$ supported by a suppositional argumentation framework SAF should be the same as the conclusions $C'_<[\Sigma]$ supported by the corresponding argumentation framework AF. It is not difficult to see that this is not always the case. Given the facts $\Sigma = \{\sigma\}$ the suppositional argumentation framework in Figure 4 supports the set of conclusions $\{\sigma, \alpha, \beta\}$ while the corresponding argumentation framework supports the set of conclusions $\{\sigma, \alpha\}$. The reason for the difference is that the conclusion of the argument A makes the argument B applicable while B is not applicable given Σ. These differences should be avoided.

Requirement 4 *A suppositional argument* A *that is inapplicable given a set of facts* Σ *should have no influence on the set of propositions from* $\mathcal{L}^B$ *entailed by the preferred states.*

Concatenation of arguments In order to satisfy Requirement 4, an argumentation system should generate arguments that result in preferred states satisfying the same propositions from $\mathcal{L}^B$ as a chain of suppositional arguments. The arguments that should be generated are the *concatenations* of suppositional arguments. We may concatenate two suppositional arguments to create a new suppositional argument if the conclusion of one argument is one of the premises of the other argument.

Definition 17 *Let* $A, B \in \mathcal{A}$ *be two arguments and let* $\hat{A} \in \bar{B}$. *Then we can concatenate* A *and* B *to create a new argument* $C = A \circ B$ *where* $\hat{C} = \hat{B}$ *and* $\bar{C} = \bar{A} \cup (\bar{B} - \{\hat{A}\})$.

When we create new suppositional arguments through concatenations of given arguments, we do not introduce new information. Therefore, the new set of preferred states should entail the same propositions from $\mathcal{L}^B$. Of course, the set of arguments entailed by the preferred states may change because we have created new arguments.

Requirement 5 *Extending a set of arguments with arguments that are created by concatenation of existing arguments should not change the set of propositions from* $\mathcal{L}^B$ *entailed by the new set of preferred states of a preferential model.*

This requirement places restrictions on the attack relation in which a new argument is involved in. An argument $C = A \circ B$ may not attack an argument D that is not attacked by A or B, and the argument C may not be attacked by an argument E that is not attacking A or B. These conditions are necessary but not sufficient. To meet Requirement 5, some attack relations must be specified. There are four situations that we must consider, namely, D is attacked by A, D is attacked by B, A is attacked by E, and B is attacked by E.

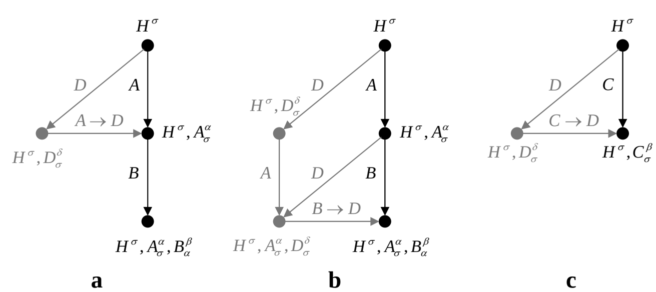

Figure 5: Attacks by concatenated arguments.

If D is attacked by A, then this results in a preference for the state entailing A to the state entailing D, as is illustrated by Figure 5.a. Since the state entailing A and B is preferred to the state entailing A, we have an indirect preference for the state entailing A and B to the state entailing D. So, since $C = A \circ B$, the state entailing C should be preferred to the state entailing D, and therefore C should attack D, as is illustrated by Figure 5.c.

If D is attacked by B, then this results in a preference for the state entailing B to the state entailing D, as is illustrated by Figure 5.b. Also in this case, the state entailing C should be preferred to the state entailing D, and therefore C should attack D, as is illustrated by Figure 5.c.

If A is attacked by E, then this results in a preference for the state entailing E to the state entailing A, as is illustrated by Figure 6.a. Similarly, if B is attacked by E, as is illustrated by Figure 6.b. In both cases, the state entailing E should be preferred to the state entailing C, and therefore E should attack C, as is illustrated by Figure 6.c.

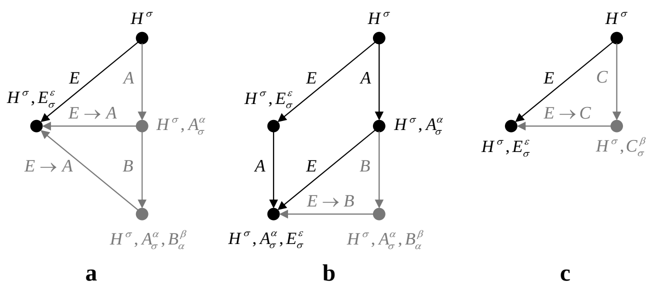

Figure 6: Attacking concatenated arguments.

The following definition formalizes the above introduced requirements with respect to the construction of suppositional arguments from other suppositional arguments.

Definition 18 *Let $P = (S, \models, <)$ be a preferential model for the argumentation framework $SAF = \langle \mathcal{A}, \longrightarrow, ^-, ^\wedge \rangle$.*

The argumentation framework SAF is called closed under the argument concatenation operator $\circ$ *iff for every $A \in \mathcal{A}$ and for every $B \in \mathcal{A}$ such that $C = A \circ B$ the following conditions hold:*

- *$C \in \mathcal{A}$;*
- *for each argument $D \in \mathcal{A}$, if $A \longrightarrow D$ or $B \longrightarrow D$, then $C \longrightarrow D$;*
- *for each argument $E \in \mathcal{A}$, if $E \longrightarrow A$ or $E \longrightarrow B$, then $E \longrightarrow C$.*

If a suppositional argumentation framework is closed under the argument concatenation operator $\circ$, we will call it a *closed* suppositional argumentation framework.

Theorem 2 *Let $C_<(\cdot)$ be the consequence operator of a suppositional argumentation framework $SAF = \langle \mathcal{A}, \longrightarrow, ^-, ^\wedge \rangle$. Moreover, let $C'_<[\Sigma]$ be the consequence operator of the corresponding argumentation framework $AF = SAF[\Sigma]$.*

If SAF is a closed *framework, then $C_<(\Sigma) = C'_<[\Sigma]$ for every $\Sigma \subseteq \mathcal{L}^B$.*

The relation with argumentation semantics Theorem 2 enables us to apply the results of (Roos 2010) to suppositional argumentation frameworks. Below the results are listed for the preferential model $P = (S, \models, <)$ of a *closed* suppositional argumentation framework $SAF = \langle \mathcal{A}, \longrightarrow, ^-, ^\wedge \rangle$.

> $\mathcal{A}(s)$ is a *stable extension* iff s is a preferred state in S, and for any state $s' \in S$, if $s' \lesssim s$, then $s \lesssim s'$.

In order to establish the relation with the preferred semantics, we have to refine the weak preference relation $\lesssim$.

Definition 19 *Let $\lesssim$ be the weak preference relation.*

The preference relation that is the result of attacking arguments only is defined as:

$$\overset{\ll}{\sim} = \{(s, s') \mid s \lesssim s', \forall A \in (\mathcal{A}(s) - \mathcal{A}(s')) \colon A \longrightarrow \mathcal{A}(s')\}$$

$s \in S$ is an admissible state *iff for every state $s' \in S$ such that $s' \overset{\ll}{\sim} s$, $s \lesssim s'$.*

A preferred state among the admissible states corresponds to a preferred extension.

> $\mathcal{A}(s)$ is a *preferred extension* iff s is a preferred admissible-state in S.

The relation with a complete extension requires the notion of an acceptable state.

Definition 20 *A state s* is acceptable with respect to *a state s' iff $\mathcal{A}(s') \subseteq \mathcal{A}(s) \subseteq \mathcal{A}p(s')$ and for every state $s'' \in S$ if $s'' \overset{\ll}{\sim} s$, then $s' \lesssim s''$.*

We can now establish the relation with the complete semantics.

> $\mathcal{A}(s)$ is a *complete extension* iff s is an admissible state in S and s is the only state that is acceptable w.r.t. s.

The grounded semantics selects the unique subset minimal complete extension.

> $\mathcal{A}(s)$ is a *grounded extension* iff s is a *least* preferred state among the states in S that are both admissible and for which s is the only state acceptable w.r.t. s.

The above four results imply that the set of preferred conclusions $C_<(\Sigma)$ of the preferential model corresponds to the set of conclusions supported by justified arguments.

Related work

A suppositional argumentation framework is closely related to a dynamic argumentation framework (DAF) introduced by Rotstein et al. (2010). A DAF aims at dealing with the dynamics of argumentation systems by considering various sets of evidence. A DAF is less abstract than a suppositional argumentation framework. In a DAF, the complement of believed propositions are defined and are used to introduce the notion of a conflict between two arguments. The attack relation is the result of resolving the conflicts using a preference relation over sets of arguments. A supposition argumentation framework stays closer to Dung's argumentation framework by following Dung in taking the attack relation as a starting point.

A suppositional argumentation framework may seem related to a assumption-based argumentation framework (ABA) (Dung, Kowalski, and Toni 2009). The assumptions in ABA play partially a similar role as the hypotheses introduced in this paper. Both play the role of making arguments applicable. However, in an ABA the assumptions are part of the argumentation framework and are subject to defeat. Hypotheses do not belong to a suppositional argumentation framework and are not subject to defeat. Hypotheses only play a role in the construction of a preferential model.

The weak preference relation $\lesssim$ introduced in this paper is related to several preference relations proposed in the literature. To the author's knowledge, such a preference relation was first used in the semantics of an instance of assumption-based argumentation described in (Roos 1988; 1992).

Amgoud & Vesic (Amgoud and Vesic 2011) define a preference similar to the weak preference relation $\lesssim$. Their preference relation plays a completely different role. While $\lesssim$ is a preference relation of the preferential model semantics, Amgoud & Vesic's preference relation is a relation of the argumentation system. It is based on a preference relation over arguments and is used to choose between argument extensions. Preferences over arguments have also been used for generating or modifying the attack relation over arguments. See for instance (Dimopoulos, Moraitis, and Amgoud 2008; Modgil 2009).

Caminada & Amgoud (2007) describe rationality postulates for argumentation systems. These postulates differ from closure properties in their focus on argumentation systems instead of (non-)monotonic reasoning systems. Gorogiannis & Hunter (2011) describe rationally postulates for the attack relation and the for the content of extensions in logic-based argumentation.

Caminada (2004) addresses the use of *hang yourself* arguments in an argumentation system defined in (Prakken and Sartor 1997). He argues that a *hang yourself* argument A should only be used as counter-argument for a sub-argument B of A. Based on this view, he adapts the grounded semantics and subsequently shows that this semantics implies Cautious Monotony. This paper shows that the absence of hang yourself arguments implies Cautious Monotony in any argumentation system and in any argumentation semantics.

Conclusion

This paper introduced a suppositional argumentation framework in order to study the closure properties of argumentation systems. A preferential model semantics has been defined for the suppositional argumentation framework. The preferential models semantics has subsequently been used to prove that argumentation system satisfy the closure properties Cumulativity and Loop if self-attacking arguments of the type *hang yourself* cannot be constructed given the available knowledge and information. Closure properties such as Supra Classicality, Absorption and Distribution can easily be addressed using the here defined preferential model. They will be discussed in a future paper.

References

Amgoud, L., and Vesic, S. 2011. Two roles of preferences in argumentation frameworks. In *ECSQARU*, 86–97.

Baroni, P., and Giacomin, M. 2007. On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence* 171:675–700.

Baroni, P.; Giacomin, M.; and Guida, G. 2005. Scc-recursiveness: a general schema for argumentation semantics. *Artificial Intelligence* 168:162–210.

Bench-Capon, T., and Dunne, P. E. 2007. Argumentation in artificial intelligence. *Artificial Intelligence* 171:619–641.

Caminada, M., and Amgoud, L. 2007. On the evaluation of argumentation formalisms. *Artificial Intelligence* 171:286–310.

Caminada, M. 2004. Dialogues and hy-arguments. In *International Workshop on Nonmonotonic Reasoning (NMR 2004)*.

Caminada, M. 2006. Semi-stable semantics. In *Proceedings of the 1st Conference on Computational Models of Argument (COMMA 2006)*, volume 144 of Frontiers in Artificial Intelligence and Applications. IOS Press.

Dimopoulos, Y.; Moraitis, P.; and Amgoud, L. 2008. Theoretical and computational properties of preference-based argumentation. In *ECAI 2008*, 463–467.

Dung, P. M.; Kowalski, R.; and Toni, F. 2009. Assumption-based argumentation. In Rahwan, I., and Simari, G., eds., *Argumentation in Artificial Intelligence*. Springer. 1–20.

Dung, P.; Mancarella, P.; and Toni, F. 2007. Computing ideal sceptical argumentation. *Artificial Intelligence* 171:642–674.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and *n*-person games. *Artificial Intelligence* 77:321–357.

Gorogiannis, N., and Hunter, A. 2011. Instantiating abstract argumentation with classical logic arguments: Postulates and properties. *Artificial Intelligence* 175(9-10):1479–1497.

Kraus, S.; Lehmann, D.; and Magidor, M. 1990. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence* 44:167–207.

Makinson, D. 1988. General theory of cumulative inference. In Reinfrank, M.; de Kleer, J.; Ginsberg, M. L.; and Sandewall, E., eds., *Proceedings of the second International Workshop on Nonmonotonic Reasoning*, 1–18. Springer-Verlag.

Makinson, D. 1994. Nonmonotonic reasoning and uncertain reasoning. In Gabbay, D., ed., *Handbook of Logic in Artificial Intelligence and Logic Programming*, volume 3. Oxford University Press. 35–110.

McCarthy, J. 1980. Circumscription - a form of non-monotonic reasoning. *Artificial Intellignce* 13(1-2):27–39.

Modgil, S. 2009. Reasoning about preferences in argumentation frameworks. *Artificial Intelligence* 173(9-10):901–934.

Pollock, J. L. 1992. How to reason defeasibly. *Artificial Intelligence* 57:1–42.

Prakken, H., and Sartor, G. 1997. Argument-based extended logic programming with defeasible priorities. *Journal of Applied Non-Classical Logics* 7(1).

Roos, N. 1988. A preference logic for non-monotonic reasoning. Technical Report 88-94, Delft University of Tecnology, Faculty of Technical Mathematics and Informatics. ISSN 0922-5641.

Roos, N. 1992. A logic for reasoning with inconsistent knowledge. *Artificial Intelligence* 57:69–103.

Roos, N. 2010. Preferential model and argumentation semantics. In *Proceedings of the 13th International Workshop on Non-Monotonic Reasoning (NMR-2010)*.

Rotstein, N. D.; Moguillansky, M. O.; García, A. J.; and Simari, G. R. 2010. A dynamic argumentation framework. *Frontiers in Artificial Intelligence and Applications - COMMA 2010* 216:427–438.

Shoham, Y. 1987. A semantical approach to non-monotonic logics. In *Proceedings of the Tenth International Joint Conference on Artificial Intelligence*, 388–392.

Verheij, B. 1996. Two approaches to dialectical argumentation: Admissible sets and argumentation stages. In *In Proceedings of the biannual International Conference on Formal and Applied Practical Reasoning (FAPR) workshop*, 357–368.