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“The idea of argumentational reasoning is that a statement is believable if it can be argued successfully against attacking arguments.”

“[…] a formal, abstract but simple theory of argumentation is developed to capture the notion of acceptability of arguments.”
Argumentation Frameworks

...thus abstract away from everything but attacks (calculus of opposition)
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Example

\[ stb(F) = \{a, d, e\}, \]
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\[ stb(F) = \{\{a, d, e\}, \{b, c, e\}\} \]
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Example

$$\text{stb}(F) = \{\{a, d, e\}, \{b, c, e\}\}$$
$$\text{pref}(F) = \{\{a, d, e\}\},$$
Argumentation Frameworks

...thus abstract away from everything but attacks (calculus of opposition)

Example

\[\begin{align*}
stb(F) &= \{\{a, d, e\}, \{b, c, e\}\} \\
pref(F) &= \{\{a, d, e\}, \{b, c, e\}\},
\end{align*}\]
Argumentation Frameworks

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\end{align*}
\]
Where are We now?

Multitude of semantics
- understanding their capabilities
- and relations between

Faster and better systems
ICCMA
(http://www.dbai.tuwien.ac.at/iccma17/)
so far, best systems reduction-based

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Outline

- The Role of Conflicts Revisited
- Exploiting the Babylonian Confusion
- Conclusions and Open Questions
Implicit Conflicts

**Definition**

Given AF $F = (A, R)$, $a, b \in A$, $\sigma$ a semantics. $\{a, b\}$ is a $\sigma$-implicit conflict if $(a, b) \notin R$, $(b, a) \notin R$, and there is no $E \in \sigma(F)$ s.t. $\{a, b\} \subseteq E$. 

Example

Recall $\text{pref}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ and $\text{stb}(F) = \{\{a, d, e\}, \{b, c, e\}\}$.

Implicit conflicts for $\text{pref}$: $\{a, f\}$, $\{b, f\}$

Implicit conflicts for $\text{stb}$: $\{a, b\}$, $\{a, f\}$, $\{b, f\}$
Implicit Conflicts

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Recall $\text{pref}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ and $\text{stb}(F) = \{\{a, d, e\}, \{b, c, e\}\}$.

- Implicit conflicts for $\text{pref}$: $\{a, f\}$, $\{b, f\}$
- Implicit conflicts for $\text{stb}$: $\{a, b\}$, $\{a, f\}$, $\{b, f\}$
Implicit Conflicts (ctd.)

Making Implicit Conflicts Explicit

Diagram:
- Nodes: a, c, f, e, b, d
- Edges: a → c, c → f, f → e, b → d, d → f, f → a

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We have $\text{stb}(F) = \text{stb}(F') = \{\{a, d, e\}, \{b, c, e\}\}$. 

\[ 
\begin{array}{c}
\text{a} \\
\text{b} \\
\text{c} \\
\text{d} \\
\text{f} \\
\text{e} \\
\end{array}
\]
**Conflict-Explicit Conjecture (Stable Case)**

For each AF $F = (A, R)$ there exists an AF $F' = (A', R')$ such that $stb(F) = stb(F')$ and $F'$ is free of $stb$-implicit conflicts.
Implicit Conflicts (ctd.)

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Why important?
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Why important?

- CDCL huge success story in SAT solving
- Experiments indicate that making conflicts explicit is supportive for argumentation systems
Implicit conflicts on instances from the ICCMA 2015 stable generator. Average of 17 implicit conflicts per (non-rejected) argument.
Implicit Conflicts (ctd.)

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We show that the conjecture does not hold for stable semantics (in case $A = A'$).
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For each AF $F = (A, R)$ there exists an AF $F' = (A', R')$ such that $\text{stb}(F) = \text{stb}(F')$ and $F'$ is free of $\text{stb}$-implicit conflicts.

We show that the conjecture does not hold for stable semantics (in case $A = A'$).

For preferred and semi-stable semantics, it does not hold in general.
Implicit Conflicts (ctd.)

Stable extensions:
\[
\{ 
\{c\}\cup M_1^i \cup M_2^j \mid i, j \in \{1, 2, 3\}, M_1^i = \{v_i\}, M_2^i = \{u_i\}, M_3^i = \{x_i, y_i\} \\
\} \cup \\
\{ 
\{a, v_1, v_2\}, \{b, u_1, u_2\}, \{a, v_1, y_2\}, \{a, y_1, v_2\}, \{b, u_1, x_2\}, \{b, x_1, u_2\}\}
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\{ \{a, v_1, v_2\},
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\]
A general way of adding conflicts (as soon as detected) to the AF thus is not possible. Still,

- for stable semantics results show that it can be done by adding arguments
- it might work in certain situations
- relaxation: extensions need not to be fully retained
Exploiting the Babylonian Confusion

Many things have already been done:

- compute grounded extension first and use reduced AF
- compute preferred extensions via smart maximization of admissible (or complete) sets
Exploiting the Babylonian Confusion

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- compute grounded extension first and use reduced AF
- compute preferred extensions via smart maximization of admissible (or complete) sets

Can we do better?
Signatures

**Definition**

The signature of a semantics $\sigma$ is defined as

$$\Sigma_\sigma = \{ \sigma(F) \mid F \text{ is an AF} \}.$$ 

Thus signatures capture all what a semantics can express.

Example

a b 

c d e f 

$S = \{ \{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{\text{pref}}$

Questions:

- $S \in \Sigma_{\text{sem}}$?
- $S \in \Sigma_{\text{stb}}$?
Signatures

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Thus signatures capture all what a semantics can express.

Example

- $S = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{\text{pref}}$
- Questions: $S \in \Sigma_{\text{sem}}$? $S \in \Sigma_{\text{stb}}$?
Signatures (ctd.)

**Definition**

Given a collection $S$ of sets of arguments, define

$\text{Confs}_S = \{(a, b) \in \bigcup S \times \bigcup S \mid \nexists S \in S : a, b \in S\}$, and

$bd(S) = \{T \subseteq \bigcup S \mid b \in \bigcup S \setminus T \iff \exists a \in T : (a, b) \in \text{Confs}_S\}$.

**Example**

For $S = \{\{a, b\}, \{a, c, e\}, \{b, d, e\}\}$, we have

$\text{Confs}_S = \{(a, d), (d, a), (b, c), (c, b), (c, d), (d, c)\}$

$bd(S) = \{\{a, b, e\}, \{a, c, e\}, \{b, d, e\}\}$
Theorem

\[ \Sigma_{naive} = \{ S \neq \emptyset \mid S = bd(S) \} \]
\[ \Sigma_{stb} = \{ S \mid S \subseteq bd(S) \} \]
\[ \Sigma_{pref} = \{ S \neq \emptyset \mid S \text{ incomparable and conflict-sensitive} \} \]
\[ \Sigma_{sem} = \{ S \neq \emptyset \mid S \text{ incomparable and conflict-sensitive} \} \]
Signatures (ctd.)

**Theorem**

\[
\Sigma_{naive} = \{ S \neq \emptyset \mid S = bd(S) \}
\]
\[
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\[
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\]
\[
\Sigma_{sem} = \{ S \neq \emptyset \mid S \text{ incomparable and conflict-sensitive} \}
\]

**Example**

\[
S = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \in \Sigma_{pref}. \ S \in \Sigma_{sem}? \ \text{Yes.}
\]
\[
S \in \Sigma_{stb}? \ \text{No!} \ (bd(S) = \{\{a, b, e\}, \{a, c, e\}, \{b, d, e\}\})
\]
Theorem

\[\Sigma_{naive} = \{ S \neq \emptyset \mid S = bd(S) \}\]
\[\Sigma_{stb} = \{ S \mid S \subseteq bd(S) \}\]
\[\Sigma_{pref} = \{ S \neq \emptyset \mid S \text{ incomparable and conflict-sensitive} \}\]
\[\Sigma_{sem} = \{ S \neq \emptyset \mid S \text{ incomparable and conflict-sensitive} \}\]

\[\Sigma_{naive} \subset \Sigma_{stb} \setminus \{ \emptyset \} \subset \Sigma_{pref} = \Sigma_{sem}\]
Two-dimensional Signatures

Definition

Given semantics $\sigma, \tau$, their 2-dimensional signature is defined as

$$\Sigma_{\sigma, \tau} = \{ \langle \sigma(F), \tau(F) \rangle \mid F \text{ is an AF} \}.$$  

Clearly, $\langle S, T \rangle \in \Sigma_{\sigma, \tau}$ only if $S \in \Sigma_{\sigma}$ and $T \in \Sigma_{\tau}$. 
Definition

Given semantics \( \sigma, \tau \), their **2-dimensional signature** is defined as

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\Sigma_{\sigma, \tau} = \{ \langle \sigma(F), \tau(F) \rangle \mid F \text{ is an AF} \}.
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- Clearly, \( \langle S, T \rangle \in \Sigma_{\sigma, \tau} \) only if \( S \in \Sigma_{\sigma} \) and \( T \in \Sigma_{\tau} \).
- Well-known semantics relations need to be satisfied
  - \( stb \subseteq sem \subseteq pref \quad stb \subseteq naive \)
Two-dimensional Signatures

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Given semantics $\sigma, \tau$, their 2-dimensional signature is defined as

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- Other conditions?
Two-dimensional Signatures

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Given semantics $\sigma, \tau$, their 2-dimensional signature is defined as

$$\Sigma_{\sigma, \tau} = \{ \langle \sigma(F), \tau(F) \rangle | F \text{ is an AF} \}.$$ 

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- Well-known semantics relations need to be satisfied
  - $stb \subseteq sem \subseteq pref$  $stb \subseteq naive$
- Other conditions?

$\Rightarrow$ Measure of the independence of semantics.

$\Rightarrow$ Useful for the enumeration of multiple sets of extensions.
Two-dimensional Signatures (ctd.)

Theorem

\[ \Sigma_{\text{naive}, \text{stb}} = \{ \langle S, T \rangle \mid S \in \Sigma_{\text{naive}}, T \in \Sigma_{\text{stb}}, T \subseteq S \} \]

Example

\[ F_{\text{naive}, \text{stb}}(\{\{a, b\}, \{a, d\}, \{b, c\}\}, \{\{a, d\}\}): \]

```
  a  b  d
x 1 x 2
  c
```

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Theorem

\[ \Sigma_{naive, stb} = \{ \langle S, T \rangle \mid S \in \Sigma_{naive}, T \in \Sigma_{stb}, T \subseteq S \} \]

\[ F_{naive, stb}(S, T) = (A, R) \] with

- \( A = \bigcup S \cup \{ x_S \mid S \in S \setminus T \} \) and
- \( R = \text{Conf}s_S \cup \{ (x_S, x_S), (a, x_S) \mid S \in S \setminus T, a \in \bigcup S \setminus S \} \)
Two-dimensional Signatures (ctd.)

**Theorem**

\[ \Sigma_{\text{naive, stb}} = \{\langle S, T \rangle \mid S \in \Sigma_{\text{naive}}, T \in \Sigma_{\text{stb}}, T \subseteq S\} \]

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**Example**

\[ F_{\text{naive, stb}}(\{\{a, b\}, \{a, d\}, \{b, c\}\}, \{\{a, d\}\}) : \]

Diagram:

```
  a --> c
  |    |
  v    v
  b --> d
```

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Two-dimensional Signatures (ctd.)

**Theorem**

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\[F_{\text{naive, stb}}(S, T) = (A, R) \text{ with}
\]

- \[A = \bigcup S \cup \{x_S \mid S \in S \setminus T\}\] and
- \[R = \text{Conf}_{S} \cup \{(x_S, x_S), (a, x_S) \mid S \in S \setminus T, a \in \bigcup S \setminus S\}\]

**Example**

\[F_{\text{naive, stb}}(\{\{a, b\}, \{a, d\}, \{b, c\}\}, \{\{a, d\}\}):\]

![Graph diagram]
Two-dimensional Signatures (ctd.)

**Theorem**

\[ \Sigma_{naive, stb} = \{ \langle S, T \rangle \mid S \in \Sigma_{naive}, T \in \Sigma_{stb}, T \subseteq S \} \]

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**Example**

\[ F_{naive, stb}(\{ \{a, b\}, \{a, d\}, \{b, c\} \}, \{\{a, d\}\}) : \]

![Diagram with nodes and edges representing relationships between the sets {a, b}, {a, d}, {b, c}, and {a, d}.](image_url)
Two-dimensional Signatures (ctd.)

Example – Stable vs. Preferred

- \( S = \{ \{a, b\}, \{a, d, e\} \} \)
- \( T = \{ \{a, b\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\} \} \)
- \( \langle S, T \rangle \in \Sigma_{\text{stb,pref}} \) ?

However, \( \langle S, T \rangle \not\in \Sigma_{\text{stb,pref}} \)

Theorem

\( \Sigma_{\text{stb,pref}} = \{ \langle S, T \rangle | S \in \Sigma_{\text{stb}}, T \in \Sigma_{\text{pref}}, S \subseteq T \cap \text{bd}(T) \} \)
### Example – Stable vs. Preferred

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- \( S \in \Sigma_{stb} \) ✔
- \( T \in \Sigma_{pref} \) ✔
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Theorem

$$\Sigma_{stb,pref} = \{\langle S, T \rangle \mid S \in \Sigma_{stb}, T \in \Sigma_{pref}, S \subseteq T \cap bd(T)\}$$
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$bd(T) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$
Example – Stable vs. Preferred

- $S = \{\{a, b\}, \{a, d, e\}\}$
- $\mathcal{T} = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$
- $\langle S, \mathcal{T} \rangle \in \Sigma_{stb, pref}$?

$$bd(\mathcal{T}) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$$

$\Rightarrow \langle S', \mathcal{T} \rangle \in \Sigma_{stb, pref}$ iff

$S' \subseteq \{\{a, d, e\}, \{b, c, e\}, \{c, d, e\}\} = \mathcal{T} \cap bd(\mathcal{T})$. 
Two-dimensional Signatures (ctd.)

**Example – Stable vs. Preferred**

- $S = \{\{a, b\}, \{a, d, e\}\}$
- $T = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$
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$$bd(T) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$$

$$\Rightarrow \langle S', T \rangle \in \Sigma_{\text{stb,pref}} \text{ iff } S' \subseteq \{\{a, d, e\}, \{b, c, e\}, \{c, d, e\}\} = T \cap bd(T).$$

$$\Rightarrow \langle S, T \rangle \notin \Sigma_{\text{stb,pref}}.$$
Possible Application in Practice

Consider enumerating preferred extensions:

- start with the less complex stable semantics
- Assume \( \{a, b\} \) and some \( S \cup \{a\} \) is stable
- By inspecting \( \Sigma_{\text{stb,pref}} \) we can exclude any \( S' \cup \{b\} \) with \( S \cap S' \neq \emptyset \) as preferred, even though

\[
\mathcal{T} = \{\{a, b\}, S \cup \{a\}, S' \cup \{b\}\} \in \Sigma_{\text{pref}}.
\]
Possible Application in Practice

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\[
T = \{\{a, b\}, S \cup \{a\}, S' \cup \{b\}\} \in \Sigma_{pref}.
\]

Explanation: \( bd(T) = \{(S \cap S') \cup \{a, b\}, S \cup \{a\}, S' \cup \{b\}\} \)

Recall:

\[
\Sigma_{stb, pref} = \{\langle S, T \rangle \mid S \in \Sigma_{stb}, T \in \Sigma_{pref}, S \subseteq T \cap bd(T)\}
\]
Two-dimensional signatures provide a clear picture about the relationship between semantics.

This can be exploited by solvers by first computing extensions of an “easier” semantics and – before searching for the remaining ones – prune the search space accordingly.

Open issues

- how to incorporate the information about the stable extensions into the AF such that they are not encountered twice?
- can we find properties for terminating the search for preferred extensions after having found all stable ones?
- useful in practice?
Conclusion

- ICCMA has stipulated development of abstract argumentation solvers
- So far, competitive systems rely on reductions to SAT, ASP, CSP, etc.
  - recent analysis shows the picture is not that clear

- In this talk:
  - Review of recent advancements from the theory of abstract argumentation
  - Discussion of potential advanced mechanisms to use genuine argumentation properties in solvers
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- In this talk:
  - Review of recent advancements from the theory of abstract argumentation
  - Discussion of potential advanced mechanisms to use genuine argumentation properties in solvers

- Learn from success stories in other communities:
  - clasp: combination of SAT-techniques and specific ASP features
Future Work

A ToDo-list for our community

- Continue development of native genuine argumentation systems
  - may outperform reduction-based methods on certain instances
- Enhance reduction-based methods by argumentation-specific “short-cuts”
- Preprocessing
- Better and more benchmarks!
- Understanding the shape of such “meaningful” instances and how do the presented concepts behave on such instances
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- Better and more benchmarks!

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"Dung is necessary, but not sufficient"
(H. Prakken)
Some References