

# Computational Argumentation – Formal Models and Complexity Results

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## TECHNOLOGY

# IBM's debating AI battles it out with human champion, wins

June 21, 2018 | 12:04 am



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At an event held at IBM's Watson West site in San Francisco on June 18, champion debater Dan Zafrir (pictured) and IBM's AI system, Project Debater, began by preparing arguments for and against the statement: "We should subsidize space exploration." Both sides then delivered a four-minute opening statement, a four-minute rebuttal, and a two-minute summary. This video screenshot shows Zafrir as he listens to Project Debater's opening statement. -- IBM

## SPOTLIGHT



### Record-breaking €205M jackpot could be won from the Philippines

The Italian SuperEnalotto lottery is currently offering a €205.4 million jackpot prize, which is the biggest jackpot anywhere in the world at the moment...

## LATEST NEWS

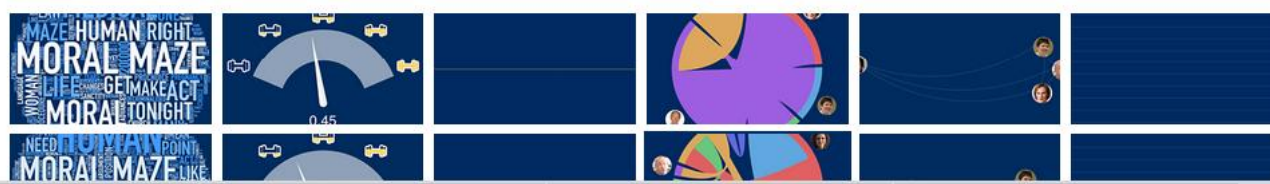
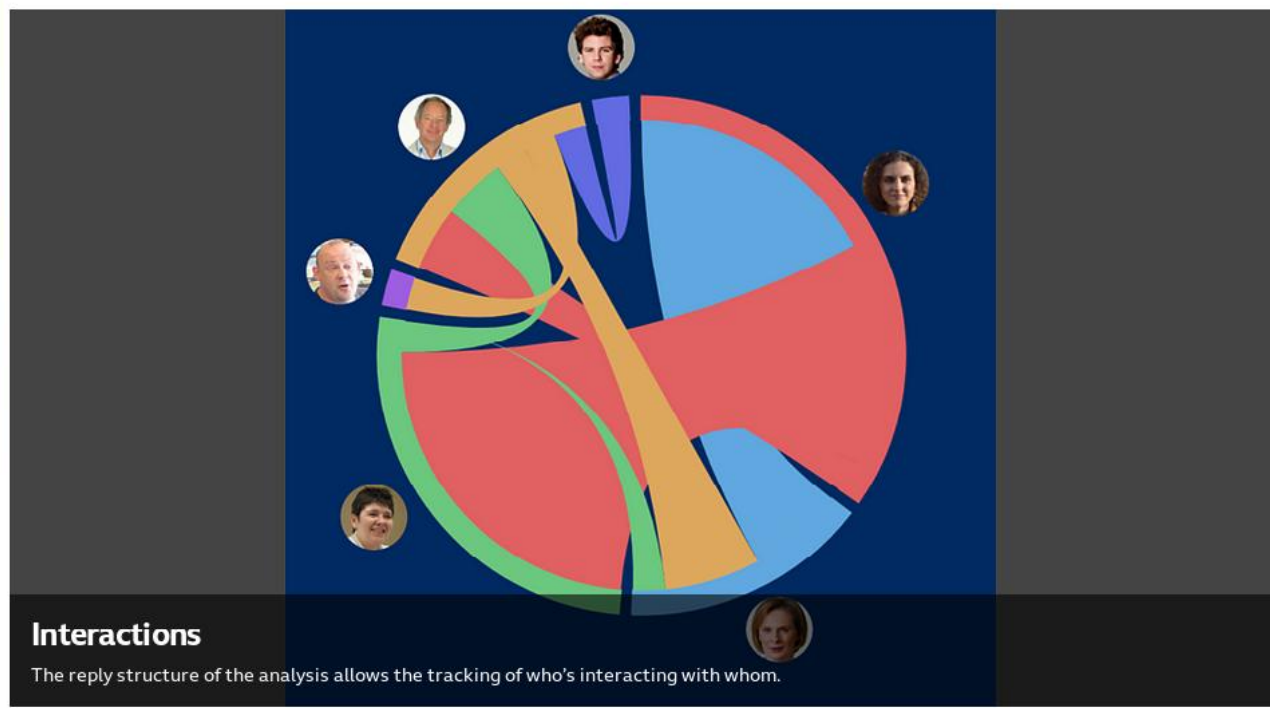
Peso plunges on geopolitical concerns

PSFist...have 2,000...aid...entire...

# 50 Years of the Abortion Act Argument Analytics

Dig into the debate with tools from the Centre for Argument Technology at the University of Dundee.

10/38



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## Projects

Christian Biemann (Hamburg)  
Matthias Hagen (Halle)

[ACQuA: Argumentation in Comparative Question Answering](#)

Michael Kohlhasse (Erlangen)

[ALMANAC: Argumentation Logics Manager and Argument Context Graph](#)

Torsten Zesch (Duisburg)  
Jürgen Ziegler (Duisburg)

[Assure: Argument-Based Decision Support for Recommender Systems](#)

Kristian Kersting (Dortmund)  
Matthias Thimm (Koblenz)

[CALM: Argumentative Machine Learning](#)

Anette Frank (Heidelberg)  
Heiner Stuckenschmidt (Mannheim)

[ExplAIN: Between the Lines - Knowledge-enhanced Argument Analysis in a Formal Argumentation Reasoning System](#)

Elisabeth Andre (Augsburg)  
Wolfgang Minker (Ulm)

[EVA: How to Win Arguments - Empowering Virtual Agents to Improve their Persuasiveness](#)

Prof. Dr. Gerhard Heyer (Leipzig)  
Ringo Baumann (Leipzig)  
Gregor Wiedemann (Hamburg)

[FAME: A Framework for Argument Mining and Evaluation](#)

Sebastian Haunss (Bremen)  
Jonas Kuhn (Stuttgart)  
Sebastian Pado (Stuttgart)

[MARDY: Modeling Argumentation Dynamics in Political Discourse](#)

Iryna Gurevych (Darmstadt)  
Christian Stab (Darmstadt)

[Open Argument Mining](#)

Stefan Evert (Erlangen)  
Lutz Schröder (Erlangen)

[RANT: Reconstructing Arguments from Noisy Text](#)

Ralph Bergmann (Trier)  
Ralf Schenkel (Trier)

[ReCAP: Information Retrieval and Case-Based Reasoning for Robust Deliberation and Synthesis of Arguments in the Political Discourse](#)

Philipp Cimiano (Bielefeld)

[RecomRatio: Rationalizing Recommendations](#)

## A First Definition

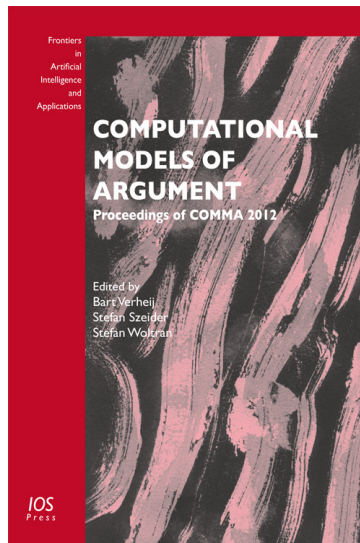
**Argumentation** is the study of processes “concerned with how assertions are **proposed**, **discussed**, and **resolved** in the context of issues upon which several **diverging opinions** may be held”.

[Bench-Capon & Dunne: Argumentation in AI. Artif. Intell. 171:619-641, 2007]

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- Tasks: Decision Support/Making, Persuasion, Dialogues, Negotiation, Dialectical Reasoning, ..
- Challenges: inconsistency, inherently dynamic, empathy, strategic thinking, ...



COMMA2020 - Mozilla Firefox

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OVERVIEW CFP SUBMISSION PROGRAM ORGANIZATION REGISTRATION WORKSHOPS LOCAL INFO SUMMER SCHOOL

# COMMA 2020

8th International Conference on Computational Models of Argument

8th - 11th September 2020

## Venue

The conference will hold in Perugia, an historical city located in the center of Italy. Perugia, considered to be one of the most famous cities of Italy due to its rich history and its artistic treasures, has preserved noble signs of beauty and charm from the beginning right through to modern times which are clear witness of the splendour of its past. It hosts one of the oldest Italian universities.



## How to reach Perugia

## Important Dates

13 July: registration deadline for full/short/demo papers

31 August: registration deadline with no proceedings paper

31 August: deadline for receiving payment and finalizing registration (be aware that bank transfer needs some days)

## News

- SEPTEMBER 11, 2020  
Change in programme:  
13:45-15:00: Abstract approaches (2)  
15:00-15:20: closing session



# Outline

- Vision: Informed Citizens in a Web of Arguments
- The Gold Standard: Dung's Argumentation Frameworks
- Beyond Dung: Acceptance Problems from a Claim-Centric View

# DIGHUM

EVENTS MANIFESTO DIGHUM CHANNEL ABOUT



Vienna, May 2019

**"The system is failing"** – stated by the founder of the Web, Tim Berners-Lee – emphasizes that while digitalization opens unprecedented opportunities, it also raises serious concerns: the monopolization of the Web, the rise of extremist opinions and behavior orchestrated by social media, the formation of filter bubbles and echo chambers as islands of disjoint truths, the loss of privacy, and the spread of digital surveillance. Digital technologies are disrupting societies and questioning our understanding of what it means to be human. The stakes are high and the challenge of building a just and democratic society with humans at the center of technological progress needs to be addressed with determination as well as scientific ingenuity. Technological innovation demands social innovation, and social innovation requires broad societal engagement.

**This manifesto is a call to deliberate and to act on current and future technological development.** We encourage our academic communities, as well as industrial leaders, politicians, policy makers, and professional societies all around the globe, to actively participate in policy formation. Our demands are the result of an emerging process that unites scientists and practitioners across fields and topics, brought together by concerns and hopes for the future. We are aware of our joint responsibility for the current situation and the future – both as professionals and citizens.

**Today, we experience the co-evolution of technology and humankind.** The flood of data, algorithms, and computational power is disrupting the very fabric of society by changing human interactions, societal institutions, economies, and political structures. Science and the humanities are not exempt. This disruption simultaneously creates and threatens jobs, produces and destroys wealth, and improves and damages our ecology. It shifts power structures, thereby blurring the human and the machine.

**The quest is for enlightenment and humanism.** The capability to automate human cognitive activities is a revolutionary aspect of computer science / informatics. For many tasks, machines surpass already what humans can accomplish in speed, precision, and even analytic deduction. The time is right to bring together humanistic ideals with critical thoughts about technological progress. We therefore link this manifesto to the intellectual tradition of humanism and similar movements striving for an enlightened humanity.

**Like all technologies, digital technologies do not emerge from nowhere.** They are shaped by implicit and explicit choices and thus incorporate a set of values, norms, economic interests, and assumptions about how the world around us is or should be. Many of these choices remain hidden in software programs implementing algorithms that remain invisible. In line with the renowned Vienna Circle and its contributions to modern thinking, we want to espouse critical rational reasoning and the interdisciplinarity needed to shape the future.

**We must shape technologies in accordance with human values and needs, instead of allowing technologies to shape humans. Our task is**



nuclear power pros and cons



Anmelden

Feedback geben

https://www.renewableresourcescoalition.org › nuc... ▾ Diese Seite übersetzen

**Nuclear Energy Pros & Cons | Renewable Resources Coalition**

19.11.2016 - Below you will find the **pros** that led to the revival of **nuclear energy**. Low Greenhouse Gas Emissions. High **Power** Output. Inexpensive **Electricity**. **Nuclear Energy** Doesn't Rely on Fossil Fuels. Economic Impact. Back end Environmental Impact. Past History of **Nuclear** Accidents. High Up-Front and End Stage Cost.

https://www.conserve-energy-future.com › pros-a... ▾ Diese Seite übersetzen

**Pros and Cons of Nuclear Energy - Conserve Energy Future**

**Pros and cons of nuclear energy:** As of today, **nuclear energy** is considered as one of the most environmentally friendly source of energy as it produces fewer ...

https://energyinformative.org › nuclear-energy-pr... ▾ Diese Seite übersetzen

**Nuclear Energy Pros and Cons - Energy Informative**

What are the most important **pros and cons** of **nuclear energy**? Read this article to find out.

https://www.power-technology.com › features › n... ▾ Diese Seite übersetzen

**Nuclear power pros and cons: What's the impact of the energy ...**

28.05.2019 - What are the **pros and cons** of **nuclear power**? Power-technology.com weighs up opinions on the controversial source of energy.

https://www.nsenergybusiness.com › features › ne... ▾ Diese Seite übersetzen

**Profiling the top nuclear power pros and cons - NS Energy**

28.02.2019 - **Nuclear power pros and cons:** Here we take a closer look at the benefits and drawbacks of the power source.

https://vittana.org › 26-important-nuclear-energy-... ▾ Diese Seite übersetzen

**26 Important Nuclear Energy Pros and Cons – Vittana.org**

Recent plans have indicated nuclear power plants may be constructed once again in the near future. This means the **nuclear energy pros and cons** are more ...

https://www.conservationinstitute.org › pros-and-c... ▾ Diese Seite übersetzen

**Pros and Cons of Nuclear Energy and Its Effect To The ...**

27 09 2018 - **Nuclear energy** is comparable to renewable energy sources, but not without risk. What are the **pros and cons** of **nuclear energy**? Learn more ...

https://timeforchange.org › Blogs ▾ Diese Seite übersetzen



nuclear power



All Discussions People

Pro vs. con view 1990 arguments retrieved in 1.0ms

## PRO

Thanks for accepting the LonelyMoutain. I will be...

► Show full argument

Thanks for accepting the LonelyMoutain. I will be presenting my arguments in this round. Good luck! This is the first time I did this topic, and I'm making this debate just to see if the

...  
<https://www.debate.org/debates/Nuclear-Power/7/> score ▼

Sorry for the incoherent nature of the opening statement...

► Show full argument

Sorry for the incoherent nature of the opening statement I should have defined it as a look at large scale **nuclear power** station, as my primary example of **nuclear power**. In response to ...

<https://www.debate.org/debates/Nuclear-Power/1/> score ▼

The primary intention of my argument, will be to convince...

► Show full argument

The primary intention of my argument, will be to convince the reader that **nuclear** energy is becoming obsolete. I hope to put **nuclear** energy in the same category as fossil fuels, and perhaps ...

<https://www.debate.org/debates/Nuclear-Power/8/> score ▼

Thank You Mr President for an insightful look into the...

► Show full argument

Thank You Mr President for an insightful look into the "benefits" of **nuclear power**. Now onto my rebuttal! **Nuclear power** is safe? I would have to consider the claim by my opponent that ...

<https://www.debate.org/debates/Nuclear-Power/8/> score ▼

I would like to argue that Nuclear power as a technology...

► Show full argument

I would like to argue that **Nuclear power** as a technology, which have been largely underated due to events such as Chernobyl, is detrimental to the environment more so than the danger radioactive ...

<https://www.debate.org/debates/Nuclear-Power/1/> score ▼

For once, this is not impossible to accept. If you want...

► Show full argument

For once, this is not impossible to accept. If you want to accept, you can right now. I'm

## CON

I will agree to my opponents wishes and confine my...

► Show full argument

I will agree to my opponents wishes and confine my arguments to the subject of large scale **nuclear power** facilities, but am still unclear as to his overall resolution. Even taking this ...

<https://www.debate.org/debates/Nuclear-Power/1/> score ▼

Thanks to Smooosh for this debate. I will now present my...

► Show full argument

Thanks to Smooosh for this debate. I will now present my case. I. Intro Pro is implicitly using a cost-benefit analysis framework to analyze the arguments in this debate by appealing to the ...

<https://www.debate.org/debates/Nuclear-Power/8/> score ▼

Thanks, bsh1! I apologize in advance for my delay. I have...

► Show full argument

Thanks, bsh1! I apologize in advance for my delay. I have been extremely busy over this past weekend and I like to put a lot of time and effort into researching and writing my arguments. ...

<https://www.debate.org/debates/Nuclear-Power/5/> score ▼

Unfortunately, my opponent accidentally forfeited his...

► Show full argument

Unfortunately, my opponent accidentally forfeited his final round. Yes, there's still Round 4, but TBR and I agreed before the debate that he'd pass in Round 4, since he started the debate ...

<https://www.debate.org/debates/Nuclear-power/1/> score ▼

Thanks for starting the debate off, TBR. I agree that we...

► Show full argument

Thanks for starting the debate off, TBR. I agree that we face some difficult energy challenges, especially given the problem of climate change and looming carbon constraints. So this is ...

<https://www.debate.org/debates/Nuclear-power/1/> score ▼

Thank you to my opponent spaceman for this debate. I will...

► Show full argument

Thank you to my opponent spaceman for this debate. I will be debating the Con position of....well, I'm not quite sure. I'm going to use this round to untangle Pro's opening statement as



Found 129 arguments (68 pro; 61 con) in 14 documents (classified 327 sentences in 4.99 s)

PRO

thefraserdomain.typepad.com / Feb 13, 2016

And when we've proven, for example, really cheap and safe reprocessing, we can start implementing it on a large scale. 99.65%

PRO

greenanswers.com / Feb 14, 2016

Nuclear power is perhaps one of the cleaner sources of energy. 99.64%

PRO

thefraserdomain.typepad.com / Feb 13, 2016

Revived interest in nuclear power in the 21st Century, as a clean air solution which contributes to world sustainable development, is encouraging the development of new materials and technologies. 99.62%

PRO

thefraserdomain.typepad.com / Feb 13, 2016

This will enable "extending by a hundred-fold the amount of energy extracted from the same amount of mined uranium." 99.62%

PRO

thefraserdomain.typepad.com / Feb 13, 2016

thefraserdomain.typepad.com/energy/2008/03/an-update-on-ur.html y and waste IS reduced. 99.61%

CON

wcupa.edu / Feb 6, 2016

On the one hand there are the worries that its use may contribute to large scale accidents and nuclear weapons proliferation. 99.39%

CON

thefraserdomain.typepad.com / Feb 13, 2016

Cyril: you are right that cleanup of a reprocessing facility is expensive. 99.39%

CON

thefraserdomain.typepad.com / Feb 13, 2016

Both reprocessing as well as fast breeders have proven to be very expensive, and the latter have proven to not work very well commercially at all. 99.38%

CON

thefraserdomain.typepad.com / Feb 13, 2016

There were other safety issues as well with sodium breeders, for example it is difficult to control fast neutrons compared to slow ones. 99.36%

CON

thefraserdomain.typepad.com / Feb 13, 2016

Convincing? Not yet ...

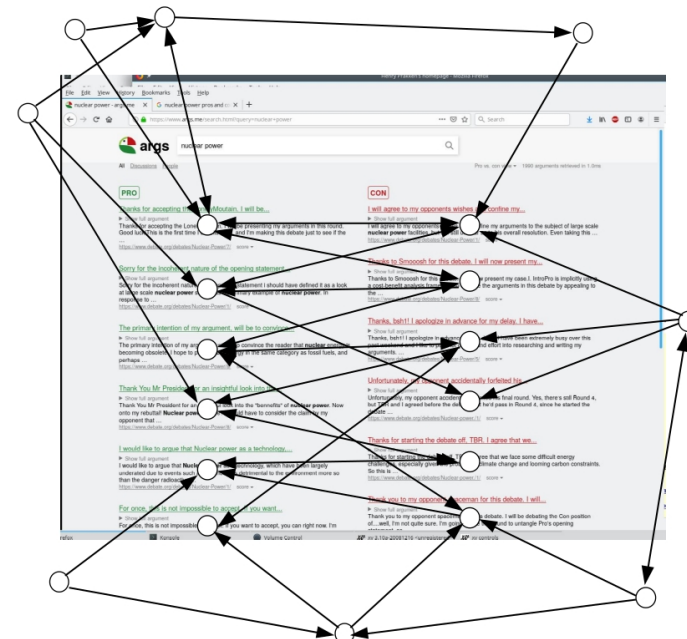
- Further arguments might be needed to obtain a full picture
- Relation between arguments needs to be drawn on solid logical grounds
- Ultimately, this leads to a network of arguments instead of a simple list of pro and cons.



## Convincing? Not yet ...

- Further arguments might be needed to obtain a full picture
- Relation between arguments needs to be drawn on solid logical grounds
- Ultimately, this leads to a network of arguments instead of a simple list of pro and cons.
- Desiderata:

- Evaluation: which arguments are jointly acceptable?
- Short response times
- Good visualisation required (avoid bias)



# Different AI Challenges

Computational Statistics

Computational Logic

Computational Complexity

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Computational Statistics

Computational Logic

Computational Complexity

Mine arguments from text

# Different AI Challenges

Computational Statistics

Mine arguments from text

Computational Logic

Determine relations between arguments

Computational Complexity

# Different AI Challenges

Computational Statistics

Mine arguments from text

Computational Logic

Determine relations between arguments

Computational Complexity

Devise algorithms for acceptance problems



**Seminal Paper** by Phan Minh Dung:

On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.* 77(2):321–358, 1995.





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- “The purpose of this paper is to study the fundamental mechanism, humans use in argumentation, and to explore ways to implement this mechanism on computers.”



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- “The purpose of this paper is to study the fundamental mechanism, humans use in argumentation, and to explore ways to implement this mechanism on computers.”
- “The idea of argumentational reasoning is that a statement is believable if it can be argued successfully against attacking arguments.”



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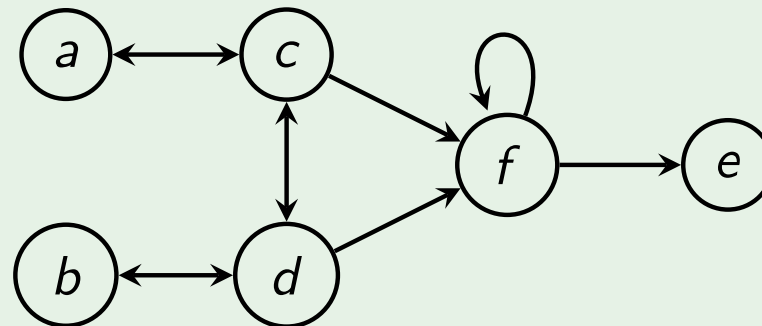
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- “The purpose of this paper is to study the fundamental mechanism, humans use in argumentation, and to explore ways to implement this mechanism on computers.”
- “The idea of argumentational reasoning is that a statement is believable if it can be argued successfully against attacking arguments.”
- “[...] a formal, abstract but simple theory of argumentation is developed to capture the notion of acceptability of arguments.”

## Argumentation Frameworks

...thus abstract away from everything but attacks

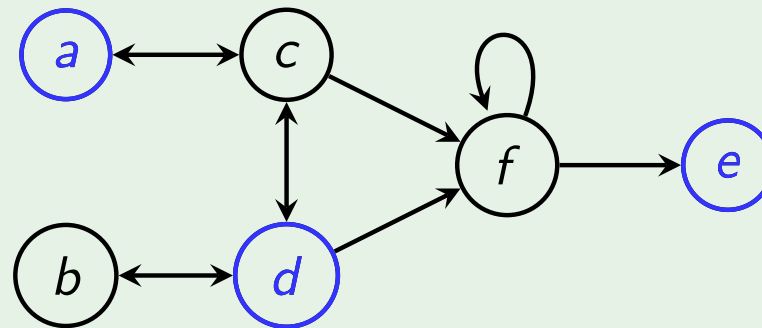
### Example



## Argumentation Frameworks

...thus abstract away from everything but attacks

### Example

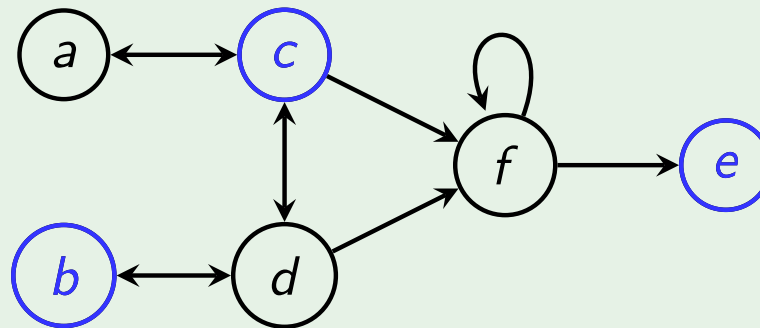


$$stb(F) = \{\{a, d, e\},$$

## Argumentation Frameworks

...thus abstract away from everything but attacks

### Example



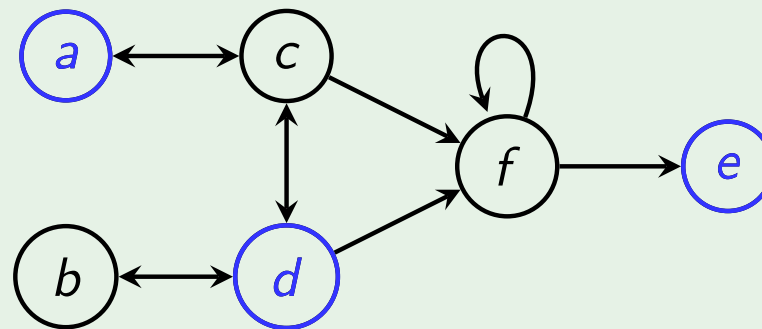
$$stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$$



## Argumentation Frameworks

...thus abstract away from everything but attacks

### Example



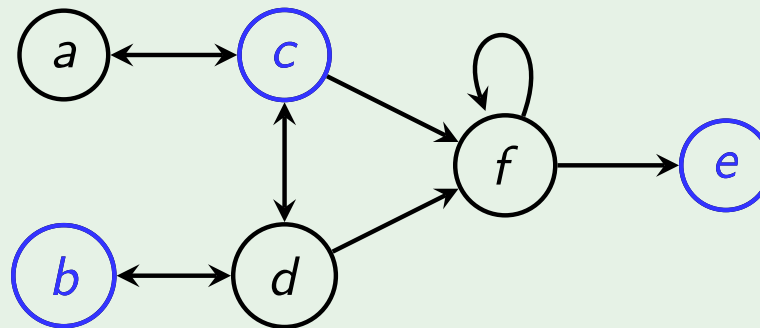
$$stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$$

$$pref(F) = \{\{a, d, e\},$$

## Argumentation Frameworks

...thus abstract away from everything but attacks

### Example



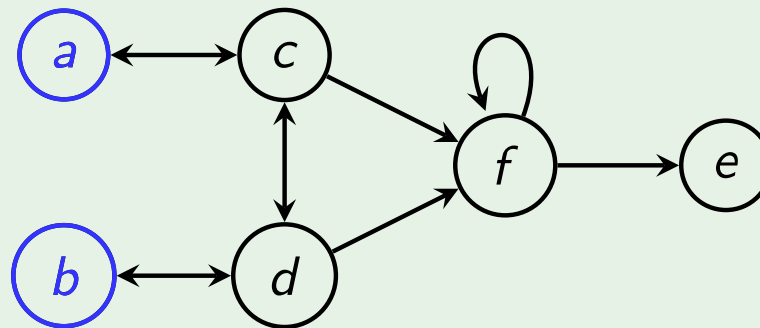
$$stb(F) = \{\{a, d, e\}, \{b, c, e\}\}$$

$$pref(F) = \{\{a, d, e\}, \{b, c, e\},$$

## Argumentation Frameworks

...thus abstract away from everything but attacks

### Example



$$\begin{aligned} stb(F) &= \{\{a, d, e\}, \{b, c, e\}\} \\ pref(F) &= \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \end{aligned}$$

# Complexity Results – Dung AFs

Basic Decision Problems:

- $Cred_\sigma$ : is an argument contained in some  $\sigma$ -extension?
- $Skept_\sigma$ : is an argument contained in all  $\sigma$ -extensions?
- $Ver_\sigma$ : is a set of arguments a  $\sigma$ -extension?

# Complexity Results – Dung AFs

Basic Decision Problems:

- $Cred_\sigma$ : is an argument contained in some  $\sigma$ -extension?
- $Skept_\sigma$ : is an argument contained in all  $\sigma$ -extensions?
- $Ver_\sigma$ : is a set of arguments a  $\sigma$ -extension?

$\sigma$	$Cred_\sigma$	$Skept_\sigma$	$Ver_\sigma$
<i>cf</i>	in P	trivial	in P
<i>naive</i>	in P	in P	in P
<i>grd</i>	P-c	P-c	P-c
<i>stb</i>	NP-c	coNP-c	in P
<i>adm</i>	NP-c	trivial	in P
<i>comp</i>	NP-c	P-c	in P
<i>pref</i>	NP-c	$\Pi_2^P$ -c	coNP-c

We observe a certain gap:

- Due to the abstraction, reasoning is solely based on argument names, rather than on their claims
- in fact, several arguments might have the same claim
- thus, checking whether a claim is supported by every possible extension is a different problem compared to checking whether an argument is contained in every possible extension
- we propose a shift from an argument-centric view to a claim-centric view
- **how does this affect complexity of the basic decision problems?**<sup>1</sup>

---

<sup>1</sup>W. Dvořák and S. Woltran. Complexity of Abstract Argumentation under a Claim-Centric View. *Artif. Intell.* 285, 2020.



# Argumentation Frameworks with Claims

## Definition

A **Claim-augmented Argumentation Framework (CAF)** is a triple  $(A, R, \gamma)$  where  $(A, R)$  is an AF and  $\gamma : A \rightarrow C$  maps arguments to claims.

# Argumentation Frameworks with Claims

## Definition

A **Claim-augmented Argumentation Framework (CAF)** is a triple  $(A, R, \gamma)$  where  $(A, R)$  is an AF and  $\gamma : A \rightarrow C$  maps arguments to claims.

## Definition

For a semantics  $\sigma$ , we define its claim-based variant as follows:

$$\sigma_c((A, R, \gamma)) = \{\gamma(S) \mid S \in \sigma((A, R))\}.$$

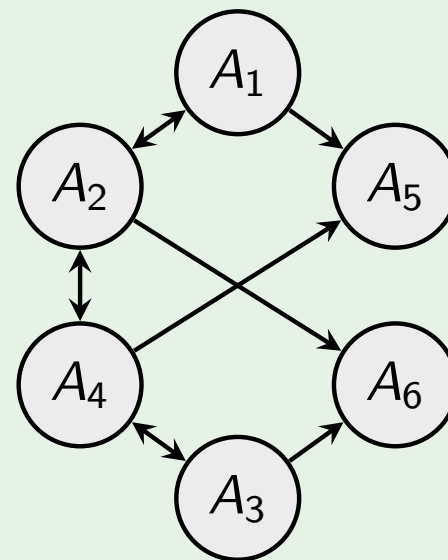
(Given a set  $S \subseteq A$  of arguments and  $\gamma : A \rightarrow C$ , let  $\gamma(S) = \{\gamma(a) \mid a \in S\}$ .)

## Example

Consider an ASPIC<sup>+</sup> knowledge base with premises  $\mathcal{K}_p = \{b, \bar{b}, c, \bar{c}\}$  and strict rules  $\mathcal{K}_s = \{\bar{b} \rightarrow a, \bar{c} \rightarrow a\}$ .

Pairs  $(b, \bar{b})$ ,  $(c, \bar{c})$ ,  $(\bar{b}, \bar{c})$  are contradictory.

Arg.	support	claim
$A_1$	$b$	$b$
$A_2$	$\bar{b}$	$\bar{b}$
$A_3$	$c$	$c$
$A_4$	$\bar{c}$	$\bar{c}$
$A_5$	$\bar{b}; \bar{b} \rightarrow a$	$a$
$A_6$	$\bar{c}; \bar{c} \rightarrow a$	$a$

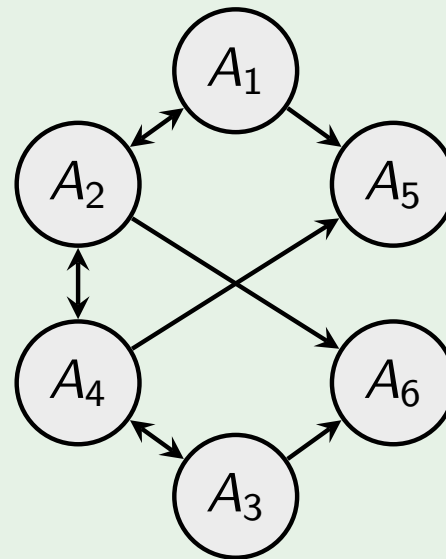


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$A_2$	$\bar{b}$	$\bar{b}$
$A_3$	$c$	$c$
$A_4$	$\bar{c}$	$\bar{c}$
$A_5$	$\bar{b}; \bar{b} \rightarrow a$	$a$
$A_6$	$\bar{c}; \bar{c} \rightarrow a$	$a$



Stable extensions of AF:  $\{A_1, A_3\}$ ,  $\{A_2, A_3, A_5\}$  and  $\{A_1, A_4, A_6\}$ .

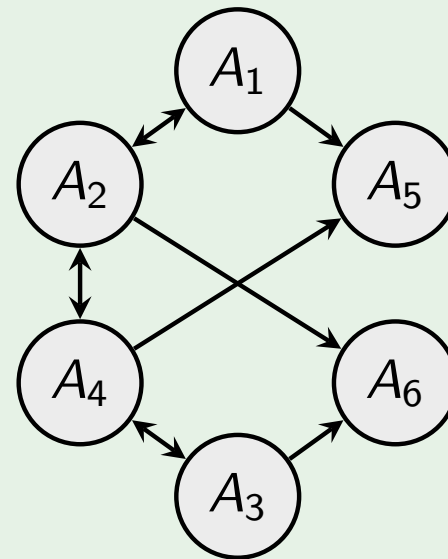
Re-interpretation in terms of claims:  $\{b, c\}$ ,  $\{a, \bar{b}, c\}$  and  $\{a, b, \bar{c}\}$ .

## Example

Consider an ASPIC<sup>+</sup> knowledge base with premises  $\mathcal{K}_p = \{b, \bar{b}, c, \bar{c}\}$  and strict rules  $\mathcal{K}_s = \{\bar{b} \rightarrow a, \bar{c} \rightarrow a\}$ .

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Arg.	support	claim
$A_1$	$b$	$b$
$A_2$	$\bar{b}$	$\bar{b}$
$A_3$	$c$	$c$
$A_4$	$\bar{c}$	$\bar{c}$
$A_5$	$\bar{b}; \bar{b} \rightarrow a$	$a$
$A_6$	$\bar{c}; \bar{c} \rightarrow a$	$a$



Stable extensions of AF:  $\{A_1, A_3\}$ ,  $\{A_2, A_3, A_5\}$  and  $\{A_1, A_4, A_6\}$ .

Re-interpretation in terms of claims:  $\{b, c\}$ ,  $\{a, \bar{b}, c\}$  and  $\{a, b, \bar{c}\}$ .

Observation: claim  $a$  appears in two extensions but different arguments are responsible for this.

# Complexity Results – General CAFs

Decision Problems Reformulated:

- $Cred_{\sigma}$ : is a claim contained in some  $\sigma$ -extension?
- $Skept_{\sigma}$ : is a claim contained in all  $\sigma$ -extensions?
- $Ver_{\sigma}$ : is a set of claims a  $\sigma$ -extension?

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- $Ver_\sigma$ : is a set of claims a  $\sigma$ -extension?

$\sigma$	$Cred_\sigma$	$Skept_\sigma$	$Ver_\sigma$
<i>cf</i>	in P	trivial	<b>NP-c</b>
<i>naive</i>	in P	<b>coNP-c</b>	<b>NP-c</b>
<i>grd</i>	P-c	P-c	P-c
<i>stb</i>	NP-c	coNP-c	<b>NP-c</b>
<i>adm</i>	NP-c	trivial	<b>NP-c</b>
<i>comp</i>	NP-c	P-c	<b>NP-c</b>
<i>pref</i>	NP-c	$\Pi_2^P$ -c	$\Sigma_2^P$ -c

# Complexity Results

## Theorem

*Ver<sub>stb</sub> is NP-complete.*



# Complexity Results

## Theorem

$Ver_{stb}$  is NP-complete.

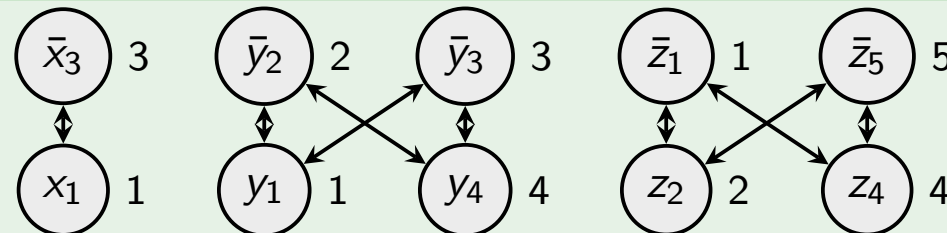
Proof Sketch (Hardness). We reduce from 3-SAT. Let  $\varphi$  be given as set  $Cl = \{cl_1, \dots, cl_m\}$  of clauses over atoms  $X$ . We construct a CAF  $CAF = (A, R, \gamma)$  with the arguments given by the two sets  $V = \{x_i \mid x \in X, x \in cl_i\}$  and  $\bar{V} = \{\bar{x}_i \mid x \in X, \neg x \in cl_i\}$ :

$$A = V \cup \bar{V} \quad R = \{(x_i, \bar{x}_j), (\bar{x}_j, x_i) \mid x_i \in V, \bar{x}_j \in \bar{V}\}$$

$$\gamma(x_i) = i \text{ for } x_i \in V \text{ and } \gamma(\bar{x}_i) = i \text{ for } \bar{x}_i \in \bar{V}.$$

It holds that  $\varphi$  is satisfiable iff  $\{1, \dots, m\}$  is stable.

Example:  $\varphi = \{\{x, y, \neg z\}, \{\neg y, z\}, \{\neg x, \neg y\}, \{y, z\}, \{\neg z\}\}$ .



# Well-Formed Argumentation Frameworks with Claims

## Definition

A CAF  $(A, R, \gamma)$  is called **well-formed** if, for any  $a, b$  with  $\gamma(a) = \gamma(b)$ ,  $\{c \mid (a, c) \in R\} = \{c \mid (b, c) \in R\}$ .

# Well-Formed Argumentation Frameworks with Claims

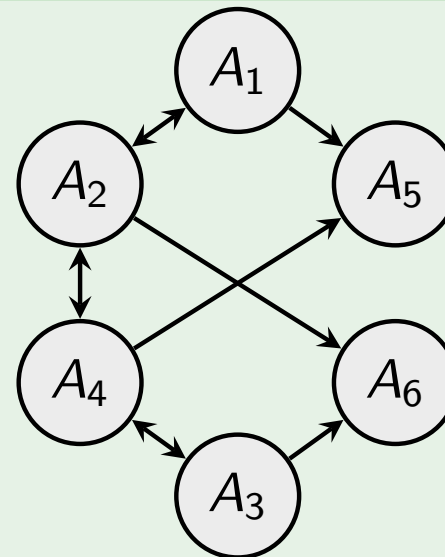
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Instantiating ASPIC<sup>+</sup> knowledge bases always yields well-formed CAFs.

## Example

Arg.	support	claim
$A_1$	$b$	$b$
$A_2$	$\bar{b}$	$\bar{b}$
$A_3$	$c$	$c$
$A_4$	$\bar{c}$	$\bar{c}$
$A_5$	$\bar{b}; \bar{b} \rightarrow a$	$a$
$A_6$	$\bar{c}; \bar{c} \rightarrow a$	$a$



# Complexity Results – Well-formed CAFs

Decision Problems Reformulated:

- $Cred_{\sigma}^{wf}$ : is a claim contained in some  $\sigma$ -extension?
- $Skept_{\sigma}^{wf}$ : is a claim contained in all  $\sigma$ -extensions?
- $Ver_{\sigma}^{wf}$ : is a set of claims a  $\sigma$ -extension?

$\sigma$	$Cred_{\sigma}^{wf}$	$Skept_{\sigma}^{wf}$	$Ver_{\sigma}^{wf}$
<i>cf</i>	in P	trivial	in P
<i>naive</i>	in P	<b>coNP-c</b>	in P
<i>grd</i>	P-c	P-c	P-c
<i>stb</i>	NP-c	coNP-c	in P
<i>adm</i>	NP-c	trivial	in P
<i>comp</i>	NP-c	P-c	in P
<i>pref</i>	NP-c	$\Pi_2^P$ -c	coNP-c

# Complexity Results – Well-formed CAFs

graph class	task	<i>naive</i>	<i>stb</i>	<i>adm</i>	<i>comp</i>	<i>pref</i>
acyclic	$Cred_{\sigma}^{wf}$	in P	P-c	P-c	P-c	P-c
	$Skept_{\sigma}^{wf}$	<b>coNP-c</b>	P-c	trivial	P-c	P-c
	$Ver_{\sigma}^{wf}$	in P	in P	in P	in P	in P
noeven	$Cred_{\sigma}^{wf}$	in P	P-c	P-c	P-c	P-c
	$Skept_{\sigma}^{wf}$	<b>coNP-c</b>	P-c	trivial	P-c	P-c
	$Ver_{\sigma}^{wf}$	in P	in P	in P	in P	in P
symmetric & irreflexive	$Cred_{\sigma}^{wf}$	in P	in P	in P	in P	in P
	$Skept_{\sigma}^{wf}$	in P	in P	trivial	in P	in P
	$Ver_{\sigma}^{wf}$	in P	in P	in P	in P	in P
symmetric	$Cred_{\sigma}^{wf}$	in P	NP-c	in P	in P	in P
	$Skept_{\sigma}^{wf}$	in P	<b>coNP-c</b>	trivial	in P	in P
	$Ver_{\sigma}^{wf}$	in P	in P	in P	in P	in P
bipartite	$Cred_{\sigma}^{wf}$	in P	P-c	P-c	P-c	P-c
	$Skept_{\sigma}^{wf}$	<b>coNP-c</b>	<b>coNP-c</b>	trivial	P-c	<b>coNP-c</b>
	$Ver_{\sigma}^{wf}$	in P	in P	in P	in P	in P

# Parameterized Complexity Results (1)

## Theorem

*$Cred_\sigma$ ,  $Skept_\sigma$ ,  $Ver_\sigma$  maintain their full complexity for CAFs with only two claims (exception  $Ver_{cf}$ ).*

## Theorem

*$Cred_\sigma^{wf}$ ,  $Skept_\sigma^{wf}$ , and  $Ver_\sigma^{wf}$  can be solved in time  $O(2^{|k|} \cdot \text{poly}(n))$  and CAFs  $(A, R, \gamma)$  with  $n = |A|$  and  $|\gamma(A)| \leq k$ .*

# Parameterized Complexity Results (2)

## Theorem

*$Cred_\sigma$ ,  $Skept_\sigma$  and  $Ver_\sigma^{wf}$  are fixed-parameter tractable w.r.t. the tree-width of the CAF.*

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### Theorem

*$Ver_\sigma$  is NP-hard for graphs of tree-width 1.*



# Parameterized Complexity Results (2)

## Theorem

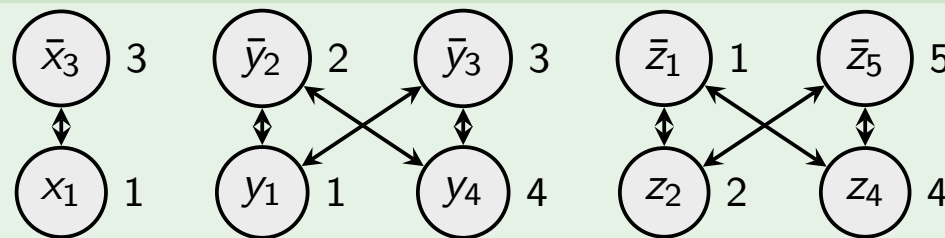
$Cred_\sigma$ ,  $Skept_\sigma$  and  $Ver_\sigma^{wf}$  are fixed-parameter tractable w.r.t. the tree-width of the CAF.

## Theorem

$Ver_\sigma$  is NP-hard for graphs of tree-width 1.

Proof Sketch: 3-SAT is NP-hard even for formulas where each variable occurs at most 3 times. Reusing our reduction yields trees. Recall:

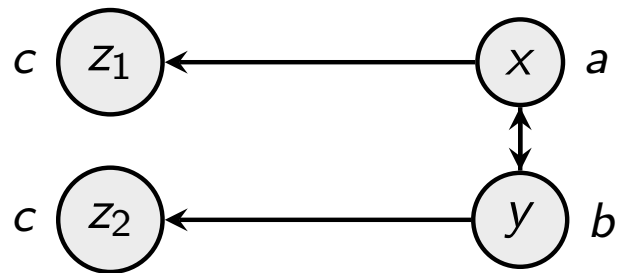
Example:  $\varphi = \{\{x, y, \neg z\}, \{\neg y, z\}, \{\neg x, \neg y\}, \{y, z\}, \{\neg z\}\}$ .



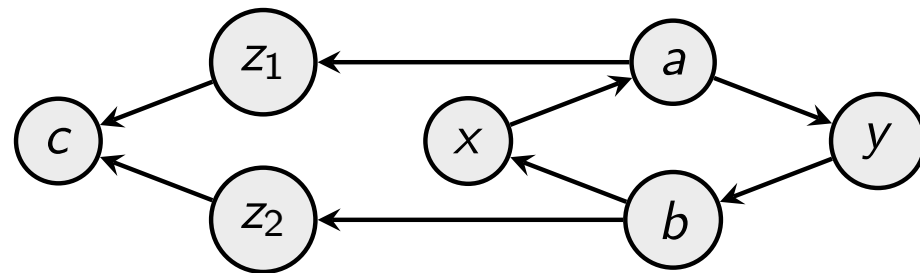
# Parameterized Complexity Results (3)

## Definition

For a well-formed  $CAF = (A, R, \gamma)$  let  $G_{CAF} = (V, E)$  with  $V = A \cup \gamma(A)$  and  $E = \{(a, \gamma(a)) \mid a \in A\} \cup \{(c, a) \mid (b, a) \in R, \gamma(b) = c\}$ .



CAF  $CAF = (A, R, \gamma)$

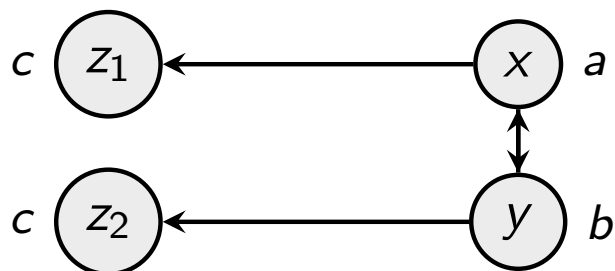


$G_{CAF} = (V, E)$

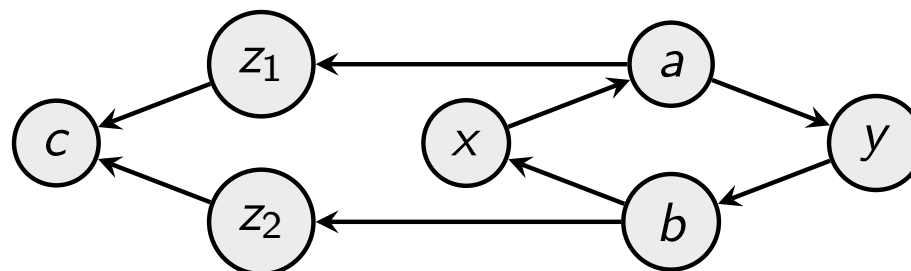
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CAF  $CAF = (A, R, \gamma)$



$G_{CAF} = (V, E)$

## Theorem

$Cred_{\sigma}^{wf}$ ,  $Skept_{\sigma}^{wf}$ , and  $Ver_{\sigma}^{wf}$  are fixed-parameter tractable w.r.t. tree-width of  $G_{CAF}$  (i.e. incidence tree-width of CAF).

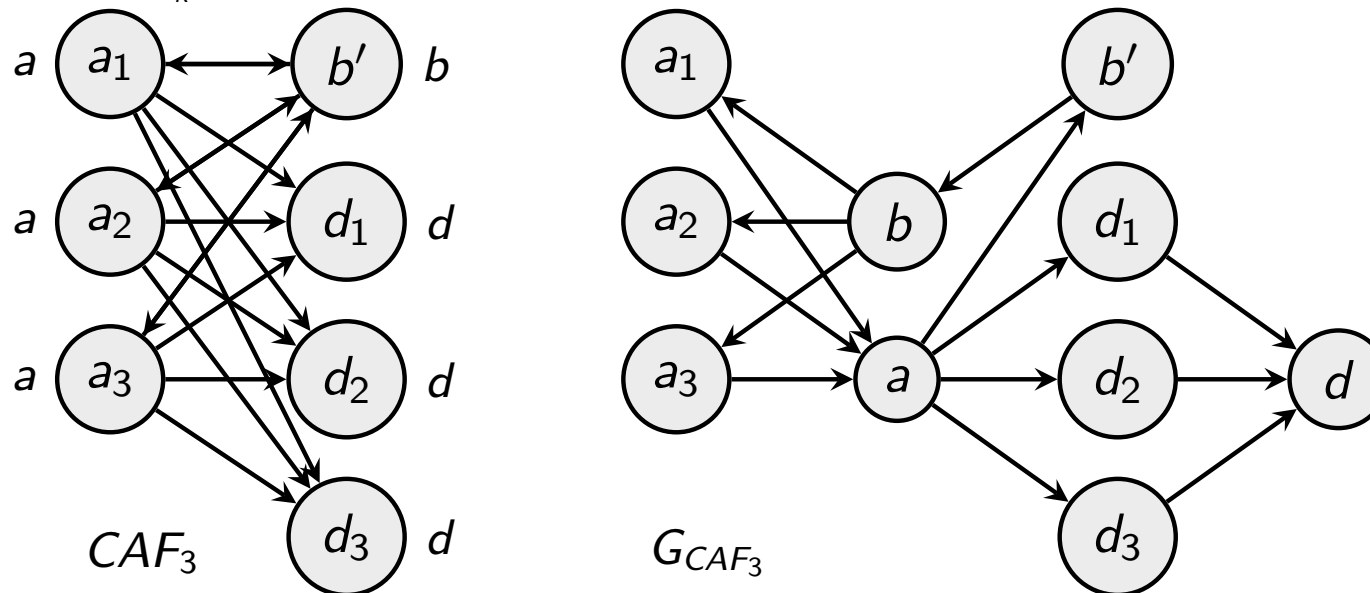
## Parameterized Complexity Results (3) ctd.

The class of CAFs with bounded tree-width is incomparable with the class of CAFs with bounded incidence tree-width.

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The class of CAFs with bounded tree-width is incomparable with the class of CAFs with bounded incidence tree-width.

Consider bipartite well-formed CAFs  $CAF_k = (A, R, \gamma)$  with  $A = \{b'\} \cup \{a_i, d_i \mid 1 \leq i \leq k\}$ ,  $R = \{(a_i, b'), (a_i, d_j), (b', a_i) \mid 1 \leq i, j \leq k\}$ , and with  $\gamma(a_i) = a$ ,  $\gamma(b') = b$  and  $\gamma(d_i) = d$ . The tree-width of  $CAF_k$  increases with  $k$ , i.e.  $tw((A, R)) \geq k - 1$ , since we have a  $k$ -clique as graph minor. But as we only use 3 claims and deleting the claims leaves only isolated vertices in  $G_{CAF_k}$ , the incidence tree-width of  $CAF_k$  is  $\leq 3$ .

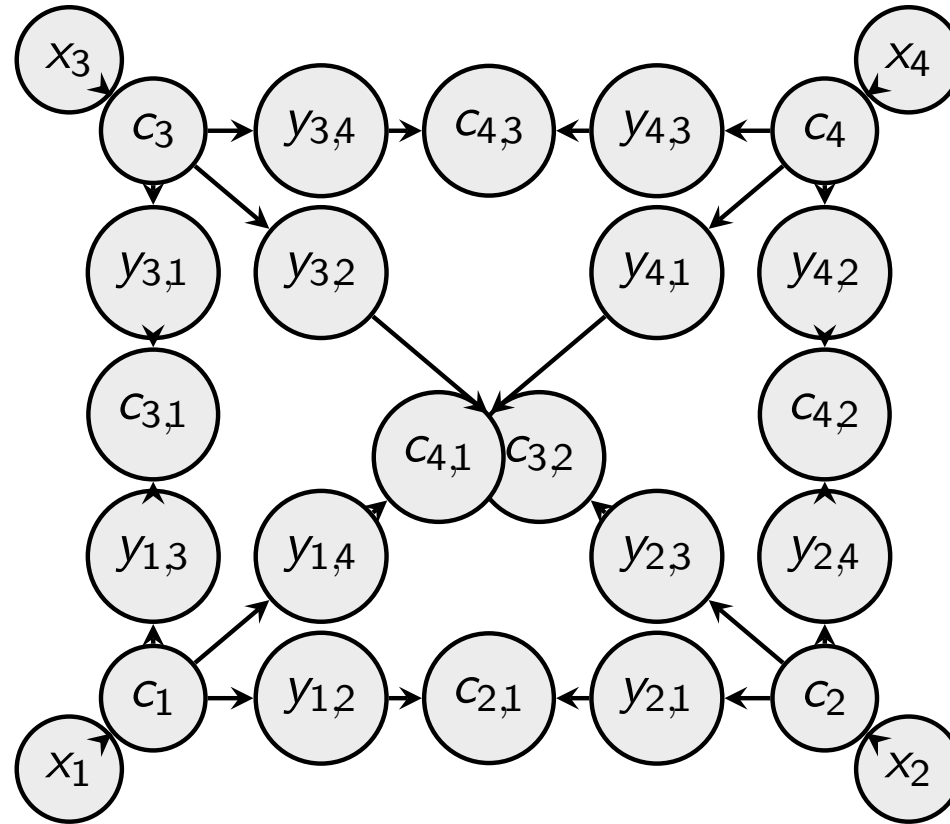


## Parameterized Complexity Results (3) ctd.

Consider the well-formed CAFs  $CAF_k = (A, R, \gamma)$  with  
 $A = \{x_i, y_{i,j} \mid 1 \leq i, j \leq k, i \neq j\}$  and  $R = \{(x_i, y_{i,j}) \mid 1 \leq i, j \leq k, i \neq j\}$ .  
 We have  $tw(CAF_k) = 1$ . Let  $\gamma(x_i) = c_i$  and  $\gamma(y_{i,j}) = \gamma(y_{j,i}) = c_{\max(i,j), \min(i,j)}$ .  
 Then,  $tw(G_{CAF_k}) \geq k - 1$ , as  $G_{CAF_k}$  has a  $k$ -clique as graph minor.

## Parameterized Complexity Results (3) ctd.

Consider the well-formed CAFs  $CAF_k = (A, R, \gamma)$  with  $A = \{x_i, y_{i,j} \mid 1 \leq i, j \leq k, i \neq j\}$  and  $R = \{(x_i, y_{i,j}) \mid 1 \leq i, j \leq k, i \neq j\}$ . We have  $tw(CAF_k) = 1$ . Let  $\gamma(x_i) = c_i$  and  $\gamma(y_{i,j}) = \gamma(y_{j,i}) = c_{\max(i,j), \min(i,j)}$ . Then,  $tw(G_{CAF_k}) \geq k - 1$ , as  $G_{CAF_k}$  has a  $k$ -clique as graph minor.



# Ongoing Work

- Translation of CAFs to AFs with collective attacks (SETAFs)<sup>2</sup>
  - Note: there is increasing interest in SETAFs within the community
- Alternative semantics for CAFs<sup>3</sup>
  - For instance, doing maximization on the claim level instead of the argument level
- Further subclasses of CAFs
- Investigation of advanced reasoning problems on CAFs (enforcement, incomplete frameworks, ...)

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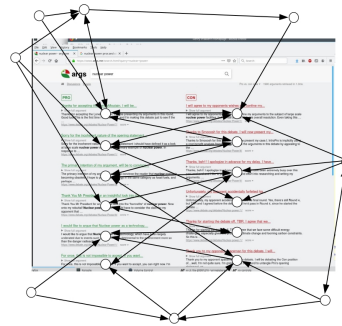
<sup>2</sup>W. Dvořák, A. Rapberger and S. Woltran: On the Relation Between Claim-Augmented Argumentation Frameworks and Collective Attacks. ECAI 2020.

<sup>3</sup>W. Dvořák, A. Rapberger and S. Woltran: Argumentation Semantics under a Claim-centric View: Properties, Expressiveness and Relation to SETAFs. KR 2020.



# Summary

- Formulated a vision towards a Web of Arguments



- Proposal for a core formalism to evaluate a network of arguments under a claim-centric view
- Thorough complexity analysis
  - Verification becomes harder for general CAFs
  - Well-formed CAFs show same complexity as Dung AFs (but there are deviations when subclasses are considered)
  - Parameterized complexity results