Comparing the Expressiveness of Argumentation Semantics[◇]

COMMA 2012

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Motivation

"Plethora" of Argumentation Semantics

Comparison of semantics still relates to

- basic properties,
- computational aspects,

but do not provide satisfying answers about expressiveness.

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Intertranslatability

A translation function transforms Argumentation Frameworks s.t. one can switch from one semantics to another.

- Intertranslatability w.r.t. efficiency has been studied for several semantics and gives a clear hierarchy [Dvořák and Woltran, 2011].
- Considering expressiveness we no longer care about efficiency.

Outlook

- We consider 9 semantics: conflict-free, naive, grounded, admissible, stable, complete, preferred, semi-stable and stage.
- We present consider two kinds of translations (faithful and exact), and provide full hierarchies of expressiveness.
- Semi-stable and preferred are of same expressiveness (although they have different complexity).

Argumentation Frameworks

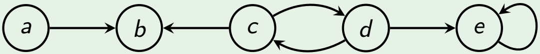
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a non-empty set of arguments
- $R \subseteq A \times A$ is a relation representing "attacks" ("defeats")

Example

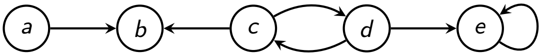
F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})



Translations

Definition

A *Translation Tr* is a function mapping (finite) AFs to (finite) AFs.



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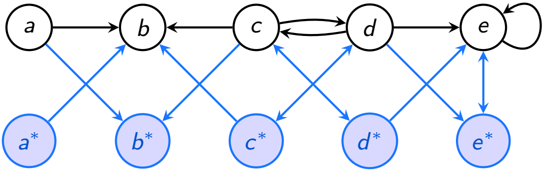

Translations

"Levels of Faithfulness" (for semantics σ, σ')

- exact: for every AF F, $\sigma(F) = \sigma'(Tr(F))$
- faithful: for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.

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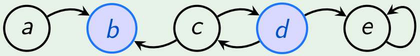
Example (An exact translation: $cf \Rightarrow adm$)



$\{b, d\} \in cf(\mathcal{F})$

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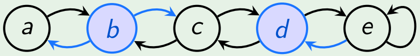

$\{b, d\} \in cf(\mathcal{F})$ $\{b, d\} \in adm(Tr(\mathcal{F}))$

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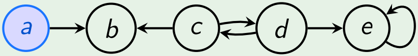
Example (A faithful translation: *comp* $\Rightarrow$ *stable*)



$\{a\} \in comp(\mathcal{F})$

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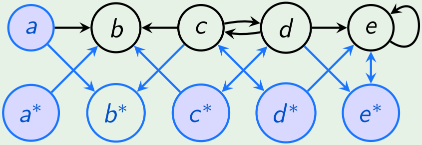

$\{a\} \in comp(\mathcal{F})$ $\quad\quad$ $\{a, a^*, c^*, d^*, e^*\} \in stable(Tr(\mathcal{F}))$

Translations

"Levels of Faithfulness" (for semantics σ, σ')

- exact: for every AF F, $\sigma(F) = \sigma'(Tr(F))$
- faithful: for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.
- weakly exact: there is a fixed $\mathcal{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathcal{S}$;
- weakly faithful: there is a fixed $\mathcal{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(\mathcal{F})) \setminus \mathcal{S}\}$ and $|\sigma(\mathcal{F})| = |\sigma'(\mathcal{F}) \setminus \mathcal{S}|$.
- We further consider translations w.r.t. the properties efficient, covering, embedding, monotone, and modular.

State of the Art

Table: Faithful / exact intertranslatability (efficient).

	cf	*naive*	*ground*	*adm*	*stable*	*comp*	*pref*	*semi*	*stage*
cf	✓								
naive		✓							
ground			✓	✓ / -	✓ / -	✓ / -	✓ / ?	✓ / ?	✓ / ?
adm			–	✓	✓ / -	✓	✓ / -	✓ / -	✓ / -
stable			–	✓	✓	✓	✓	✓	✓
comp			–	✓ / -	✓ / -	✓	✓ / -	✓ / -	✓ / -
pref			–	–	–	–	✓	✓	? / -
semi			–	–	–	–	–	✓	? / -
stage			–	–	–	–	–	✓	✓

State of the Art

Table: Faithful / exact intertranslatability (inefficient).

	cf	*naive*	*ground*	*adm*	*stable*	*comp*	*pref*	*semi*	*stage*
cf	✓								
naive		✓							
ground			✓	✓ / ?	✓ / ?	✓ / ?	✓ / ?	✓ / ?	✓ / ?
adm			–	✓	✓ / -	✓	✓ / -	✓ / -	✓ / -
stable			–	✓	✓	✓	✓	✓	✓
comp			–	✓ / -	✓ / -	✓	✓ / -	✓ / -	✓ / -
pref			–				✓	✓	? / -
semi			–					✓	? / -
stage			–					✓	✓

Summarized Results

Table: Faithful / exact intertranslatability

	cf	*naive*	*ground*	*adm*	*stable*	*comp*	*pref*	*semi*	*stage*
cf	✓	✓ / -	–	✓	✓ / -	✓	✓ / -	✓ / -	✓ / -
naive	–	✓	–	✓ / -	✓ / -	✓ / -	✓	✓	✓
ground	–	✓	✓	✓ / -	✓ / -	✓	✓	✓	✓
adm	–	–	–	✓	✓ / -	✓	✓ / -	✓ / -	✓ / -
stable	–	–	–	✓	✓	✓	✓	✓	✓
comp	–	–	–	✓ / -	✓ / -	✓	✓ / -	✓ / -	✓ / -
pref	–	–	–	✓ / -	✓ / -	✓ / -	✓	✓	✓ / -
semi	–	–	–	✓ / -	✓ / -	✓ / -	✓	✓	✓ / -
stage	–	–	–	✓ / -	✓ / -	✓ / -	✓	✓	✓

Main Contributions

The Paper

For the 9 Semantics under our considerations we

- provide exact / faithful translations whenever possible, and
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The Talk

In the following we give examples for both kind of results.

- Translation 8: exact for semi-stable to stage semantics.
- Theorem 3: There is no weakly faithful translation for preferred to naive semantics.

Definition

For $\mathcal{F} = (A, R)$ an Argumentation Framework and a set $S \subseteq A$ we call

$$S^+ = S \cup \{a \in A \mid \exists b \in A, b \rightarrowtail a\}$$

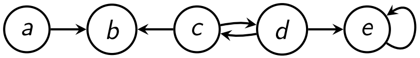
the range of S.

Definition

Let $\mathcal{F} = (A, R)$ be an Argumentation Framework. For $S \subseteq A$ it holds that

- $S \in cf(\mathcal{F})$ if there are no $a, b \in S$, such that $(a, b) \in R$;
- $S \in adm(\mathcal{F})$, if each $a \in S$ is defended by S;
- $S \in pref(\mathcal{F})$, if $S \in adm(\mathcal{F})$ and there is no $T \in adm(\mathcal{F})$ with $T \supset S$;
- $S \in semi(\mathcal{F})$, if $S \in adm(\mathcal{F})$ and there is no $T \in adm(\mathcal{F})$ with $T_R^+ \supset S_R^+$.

Translation 8, *semi* $\Rightarrow$ *pref*



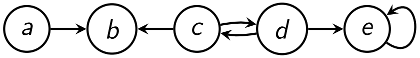
Example

- $pref(\mathcal{F}) = \{\{a, c\}, \{a, d\}\}$
- $semi(\mathcal{F}) = \{\{a, d\}\}$

Translation 8, *semi* $\Rightarrow$ *pref*

Definition

- $Tr(A, R) = (A', R')$
- $A' = A \cup \{E \mid E \in pref(\mathcal{F}) \setminus semi(\mathcal{F})\}$
- $R' = R \cup \{(a, E), (E, E), (E, b) \mid a \in A \setminus E, b \in E\}$



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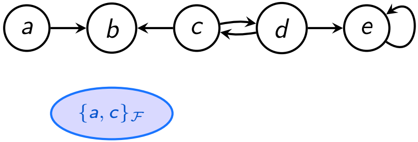

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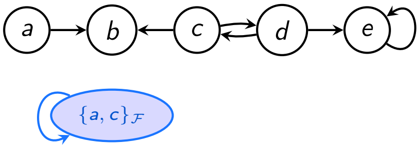

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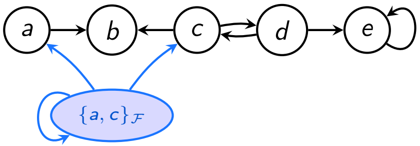

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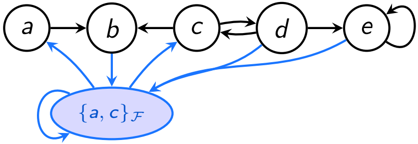

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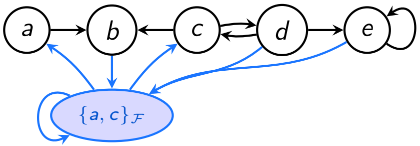

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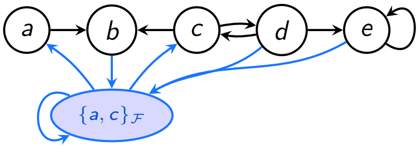

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- $S \in cf(\mathcal{F})$ if there are no $a, b \in S$, such that $(a, b) \in R$;
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Theorem 3, *pref* $\Rightarrow$ *naive*

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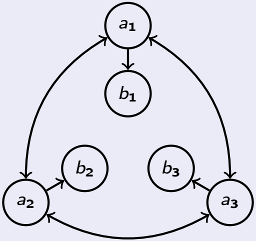
Counterexample



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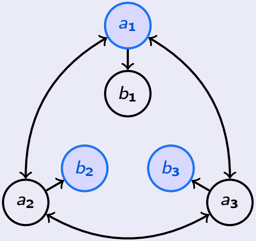

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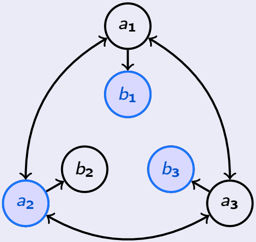

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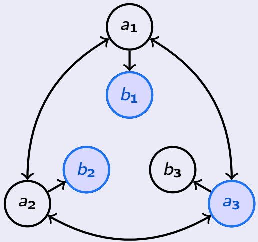

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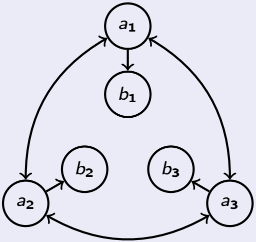

$$pref(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \{b_1, a_2, b_3\}, \{b_1, b_2, a_3\}\} \subseteq naive(Tr(\mathcal{F}))$$

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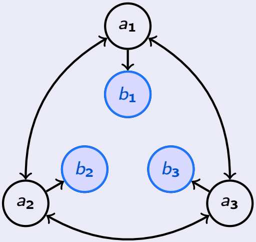


$$pref(\mathcal{F}) = \{\{a_1, b_2, b_3\}, \{b_1, a_2, b_3\}, \{b_1, b_2, a_3\}\} \subseteq naive(Tr(\mathcal{F}))$$

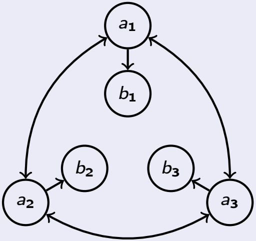
$$\Rightarrow \{b_1, b_2, b_3\} \in cf(Tr(\mathcal{F}))$$

Theorem 3, *pref* $\Rightarrow$ *naive*

Theorem

There is no weakly faithful translation for $\{stage, stable, semi, pref, comp, adm\} \Rightarrow \{cf, naive\}$.

Counterexample



$$
\begin{aligned}
pref(\mathcal{F}) = \{&\{a_1, b_2, b_3\},\\
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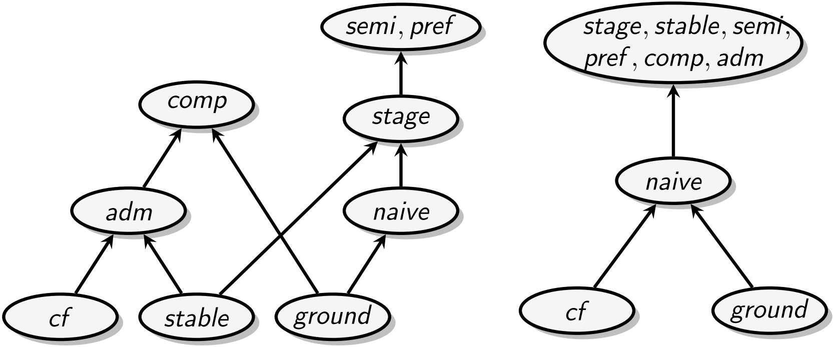
&\subseteq naive(Tr(\mathcal{F}))
\end{aligned}
$$

$$\Rightarrow \{b_1, b_2, b_3\} \in cf(Tr(\mathcal{F}))$$

Results



(weakly) exact

(weakly) faithful

Almost finished. . .

Achievments

- Full hierarchy of expressiveness for the selected semantics.
- Extended existing investigations on intertranslatability
  - to naive extensions and conflict-free sets, and
  - to the case of inefficient translations.
- Improved an existing translation w.r.t. size of transformed Argumentation Frameworks.

Open Questions

- More semantics for investigation
- Labeling-preserving translations

Finished.

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