Problem Solving and Search in Artificial Intelligence

Local Search, Stochastic Hill Climbing, Simulated Annealing

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Local Search

1. Pick a solution from the search space and evaluate its merit. Define this as current solution
2. Apply a transformation to the current solution to generate a new solution and evaluate its merit
3. If the new solution is better than the current solution then exchange it with the current solution
4. Repeat steps 2 and three until no transformation in the given set improves the current solution

Local search for SAT

- **GSAT algorithm is based on flip of variable that results in the largest decrease number of unsatisfied clauses**

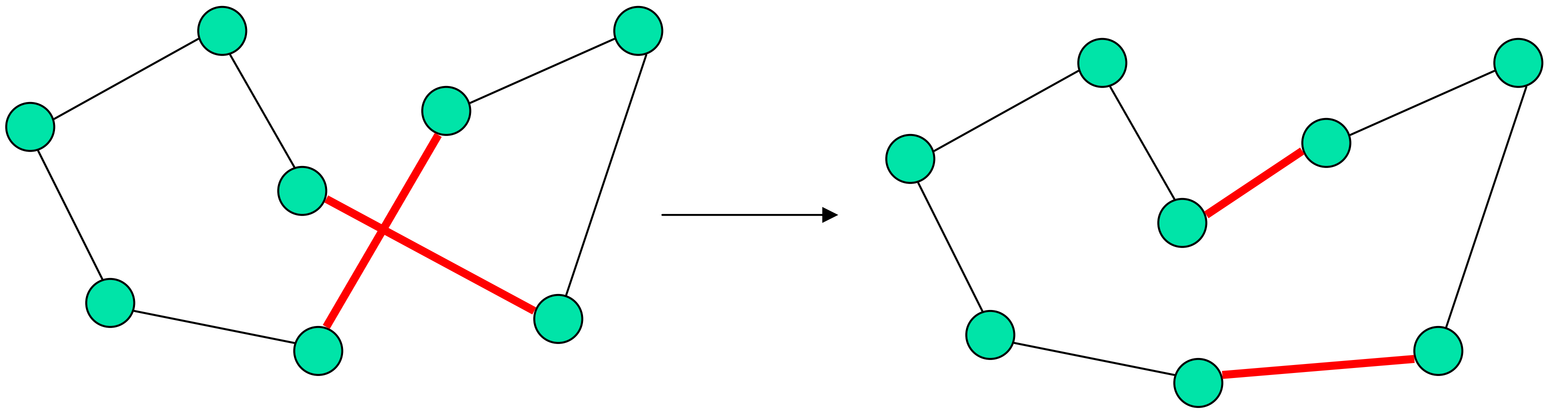
```
Procedure GSAT
begin
   for i=1 step 1 until MAX-TRIES do
   begin
     T<- a randomly generated truth assignment
     for j=1 step 1 until MAX-FLIPS do
              if T satisfies the formula then return(T)
              else make a flip of variable in T that results in the
                   largest decrease in the number of unsatisfied clauses
   end
   return(“no satisfying assignment found“)
end
```

Local Search and TSP

- One of simplest algorithm is 2-opt algorithm
  - Start with the random permutation of the cities (call this tour T)
  - Tries to improve T based in its neighbourhood
  - Neighbourhood of T is defined as the set of all tours that can be reached by changing two nonadjacent edges in T
  - Move is called 2-interchange

Local Search and TSP

2-interchange move



2-Opt Algorithm

- A new tour T`after the 2-interchange move replaces T if it is better
- If non of the tours in neighbourhood is better than the tour T the algorithm terminates
- The algorithm should be started from several random permutations
- 2-opt algorithm can be extended to k-opt algorithm

Lin-Kernighan Algorithm

- Refines the k-opt strategy by allowing k to vary from one iteration to another
- It favors the largest improvement in neighbourhood, not the first improvement like in k-opt
- Generates near optimal solutions for TSPs with up to million cities
- Needs under one hour on a modern workstation

Greedy Algorithms

- Simple algorithms
- Assigns the values for all decisions variables one by one and at every step makes the best available decision
- Heuristic provides the best possible move at each step
- Do not always return the optimum solution

Greedy Algorithm for the SAT

- Possible greedy heuristic for SAT
  - For each variable from 1 to n, in any order, assign the truth value that result in satisfying the greatest number of currently unsatisfied clauses
- Performance of such greedy algorithm is quite poor
  - For example:

$$\bar{x}_1 \wedge (x_1 \vee x_2) \wedge (x_1 \vee x_4)$$

Greedy Algorithm for the SAT

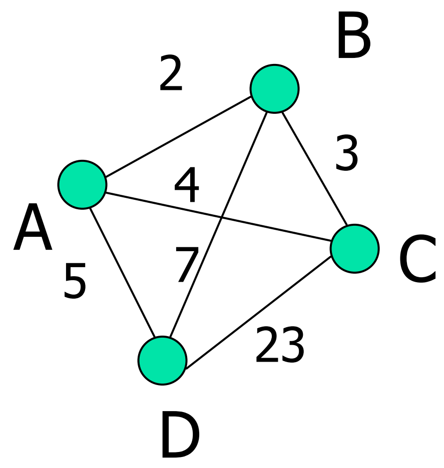
- Possible improve of previous greedy algorithm
  - Sort all variables on basis of their frequency, from the smallest to the largest
  - For each variable in order, assign a value that would satisfy the greatest number of currently unsatisfied clauses
- Further improves to the greedy algorithm can be done
- There is no good greedy algorithm for the SAT

Greedy Algorithm for the TSP

- Nearest neighbourhood heuristic
  - Start from random city
  - Proceed to the nearest unvisited city
  - Continue with step 2 until every city has been visited
- The tour with this algorithms can be far from perfect

Greedy Algorithm for the TSP

- For example with this heuristic if we start from A the following tour will be generated: A-B-C-D-A (cost=33)
- There exist much better tour A-C-B-D-A (cost=19)



Local search

- +: Ease of implementation
- +: Guarantee of local optimality usually in small computational time
- +: No need for exact model of the problem
- -: Poor quality of solution due to getting stuck in poor local optima

Modern Heuristics (Metaheuristics)

- These algorithms guide an underlying heuristic/local search to escape from being trapped in a local optima and to explore better areas of the solution space
- Examples:
  - Single solution approaches: Simulated Annealing, Tabu Search, etc.
  - Population based approaches: Genetic algorithm, Memetic algorithm, ACO, etc.
- +: Able to cope with inaccuracies of data and model, large sizes of the problem and real-time problem solving
- +: Including mechanisms to escape from local optima of their embedded local search algorithms
- +: Ease of implementation
- +: No need for exact model of the problem
- -: Usually no guarantee of optimality

Elements of Local Search

- Representation of the solution
- Evaluation function
- Neighbourhood function: to define solutions which can be considered close to a given solution. For example:
  - For optimisation of real-valued functions in elementary calculus, for a current solution x_0, neighbourhood is defined as an interval $(x_0 - r, x_0 + r)$
  - In clustering problem, all the solutions which can be derived from a given solution by moving one customer from one cluster to another

Elements of Local Search

- The larger the neighbourhood, the harder it is to explore and the better the quality of its local optimum
- Finding an efficient neighbourhood:
  - balance between the quality of the solution and the complexity of the search
- Neighbourhood search strategy
  - random
  - systematic search
- Acceptance criterion:
  - first improvement
  - best improvement,
  - best of non-improving solutions,
  - random criteria

Hill Climbing Algorithm

1. Pick a random point in the search space
2. Consider all the neighbours of the current state
3. Choose the neighbour with the best quality and move to that state
4. Repeat 2 through 4 until all the neighbouring states are of lower quality
5. Return the current state as the solution state

The Problem with Hill Climbing

- Gets stuck at local minima
- Possible solutions:
  - Try several runs, starting at different positions

  - Increase the size of the neighbourhood (e.g. in TSP try 3-opt rather than 2-opt)

  - Stochastic Hill-Climbing
    - Only one solution from neighbourhood is selected
    - This solution will be accepted for the next iteration with some probability, which depends from the difference between current solution and selected solution

Stochastic Hill-Climbing

```
Procedure stochastic hill-climber
 begin
   t=0
   select a current string v_c at random
   evaluate v_c
   repeat
    select the string v_n from the neighborhood
    of v_c
    select v_n with probability
    t=t+1
   until t=MAX
 end
```

$$\frac{1}{1+e^{\frac{eval(v_c)-eval(v_n)}{T}}}$$

Stochastic Hill Climbing

- The neighborhood of a current solution v_c consist from only one solution v_n
- The probability of acceptance of the solution v_n depends on:
  - Difference in merit between v_c and v_n
  - Parameter T

$$p = \frac{1}{1 + e^{\frac{eval(v_c) - eval(v_n)}{T}}}$$

- T remains constant during the execution of algorithm

Role of parameter T

- Example:
  - $eval(v_c)=107$, $eval(v_n)=120$
  - maximization problem

$$p = \frac{1}{1+e^{\frac{-13}{T}}}$$

T	p
1	1.00
5	0.93
10	0.78
20	0.66
50	0.56
10^{10}	0.5…

Role of parameter T

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The greater the parameter T, the smaller the importance of the relative merit of the competing points v_c and v_n

Role of parameter T

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T	p
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- If T is huge -> search becomes random
- T is very small -> stochastic hill-climber reverts into ordinary hill climber

- Example:
  - $eval(v_c)=107$, $T=10$

$eval(v_n)$	$eval(v_c)-eval(v_n)$	p
80	27	0.06
100	7	0.33
107	0	0.50
120	-13	0.78
150	-43	0.99

- Example:
  - $eval(v_c)=107$, $T=10$

$eval(v_n)$	$eval(v_c)-eval(v_n)$	p
80	27	0.06
100	7	0.33
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If $eval(v_c)=eval(v_n)$, the probability of acceptance is 0.5

Simulated Annealing

- Changes the parameter T during the search
- Starts with high value for T – random search
- The value of T gradually decreases
- To the end T is very small and the SA behaves like an ordinary Hill-climber

Simulated Annealing

- Is based on the analogy from the thermodynamics
- To grow a crystal, the row material is heated to a molten state
- The temperature of the crystal melt is reduced until the crystal structure is frozen in
- Cooling should not be done two quickly, otherwise some irregularities are locked in the crystal structure

Simulated Annealing

```
Prozedure simulated annealing
 begin
    t=0
    Intialize T
    select a current string v_c at random
    evaluate v_c
    repeat
     repeat
          select a new point v_n in the neighborhood of v_c
          if eval(v_c) < eval(v_n) then  v_c =v_n
          else if random[0,1) < e^((eval(v_n)-eval(v_c))/T) then v_c=v_n
     until (termination-condition)
     T=g(T,t)
     t=t+1
    until (halting-criterion)
 end
```

SA – problem specific questions

- What is a solution?
- What are the neighbors of a solution?
- What is a cost of a solution
- How do we determine the initial solution

SA – specific questions

- How do we determine the intial “temperature” T”
- How do we determine the cooling ration g(T,t)?
- How do we determine the termination condition?
- How do we determine the halting criterion?

- STEP 1: $T=T_{max}$
  select v_c at random
- STEP 2: pick a point v_n from the neighborhood of v_c

  if *eval(v_n) is better than the val(v_c)*
  then select it ($v_c=v_n$)
  else select it with probability $e^{\frac{-\Delta eval}{T}}$
  repeat this step k_T times
- STEP 3: set $T=rT$
  if $T \geq T_{\min}$
  then goto STEP 2
  else goto STEP 1

Simulated Annealing for SAT problem

Procedure SA-SAT
begin
  tries=0
  repeat
    v <- random truth assignment
    j=0
    repeat
      If v satisfies the clauses **then** return **v**

$$T = T_{\max} e^{-jr}$$

      for k=1 **to** the number of variables **do**
      begin
        compute the increase (decreases) δ in the number of clauses made true if v_k was flipped
        flip variable v_k with the probability $(1+e^{-\frac{\delta}{T}})^{-1}$
        v <- new assignment if the flip is made
      end
      j=j+1
    until $T \leq T_{\min}$
    tries=tries+1
  until tries=MAX-TRIES
end

SA for SAT

- r represents a decay rate for the temperature
- Spears (1996) used
  - T_{max}=0.03 and T_{min}=0.01
  - r depend on the number of variables and number of tries
- SA-SAT appeared to satisfy at least as many formulas as GSAT, with less work
- Advantage of SA-SAT came from its backward moves

Other application of SA

- Traveling Salesman Problem
- VLSI design
- Production scheduling
- Timetabling problems
- Image processing
- …

References

- Z. Michalewicz and D. B. Fogel. How to Solve It: Modern Heuristics
  - Chapters 3 (sec. 3.2), 4 (sec. 4.1) , 5 (sec. 5.1)

- **Other papers**
  - **Simulated annealing** for hard satisfiability problems : W.M. **Spears**

  - Optimization by **Simulated Annealing**: An Experimental Evaluation; Part I, **Graph Partitioning** DS Johnson, CR Aragon, LA McGeoch, C Schevon