FAKULTÄT
FÜR INFORMATIK
Faculty of Informatics

Verifiability of Argumentation Semantics

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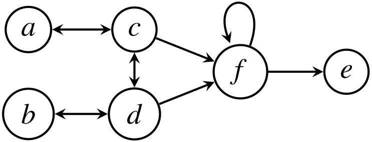
Introduction

- Abstract Argumentation Framework (AF) [Dung, 1995]:



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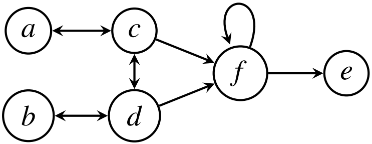

- Evaluation: argumentation semantics
- Extension: set of jointly acceptable arguments

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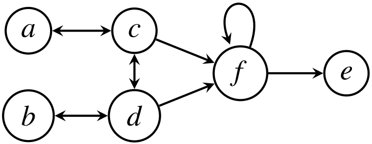

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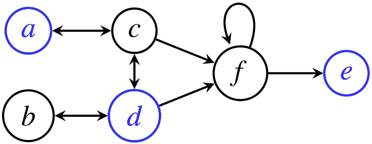

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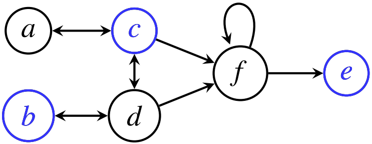

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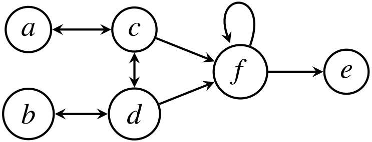

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$$\mathit{stb}(\mathcal{F}) = \{\{a, d, e\}, \{b, c, e\}\}$$

- Further semantics: preferred, complete, semi-stable, stage, ...

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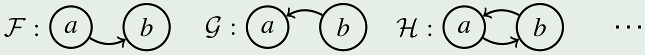

$$na(\mathcal{F}) = na(\mathcal{G}) = na(\mathcal{H}) = \cdots = \{\{a\}, \{b\}\}.$$

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$$\mathcal{F}: a \rightarrow b \qquad \mathcal{G}: a \leftarrow b \qquad \mathcal{H}: a \rightleftarrows b \qquad \cdots$$

$$na(\mathcal{F}) = na(\mathcal{G}) = na(\mathcal{H}) = \cdots = \{\{a\},\{b\}\}.$$

$\Rightarrow$ not stage semantics (range-maximal conflict-free sets)

$$stg(\mathcal{F}) = \{\{a\}\},\ stg(\mathcal{G}) = \{\{b\}\},\ stg(\mathcal{H}) = \{\{a\},\{b\}\}.$$

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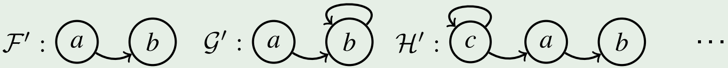

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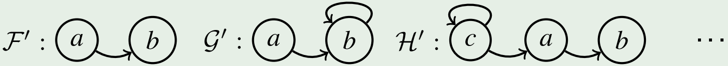

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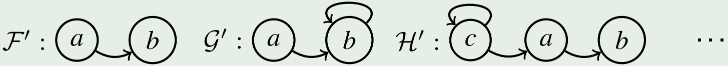

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$\Rightarrow$ not preferred semantics (maximal admissible sets)
$pr(\mathcal{F}') = pr(\mathcal{G}') = \{\{a\}\}$, $pr(\mathcal{H}') = \{\emptyset\}$.

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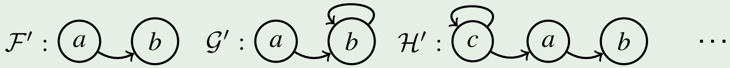

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$$pr(\mathcal{F}') = pr(\mathcal{G}') = \{\{a\}\},\ pr(\mathcal{H}') = \{\emptyset\}.$$

- Which information on top of conflict-free sets has to be added in order to compute a certain semantics?

Introduction

- Systematic comparison of argumentation semantics
  - Computational complexity [Dunne and Bench-Capon, 2002, Dvořák and Woltran, 2010]
  - Principle-based evaluation [Baroni and Giacomin, 2007]

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$\Rightarrow$ Hierarchy of verification classes

$\Rightarrow$ Classification of semantics into these classes

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$\Rightarrow$ Each “rational” semantics is exactly verifiable by one of these classes

- Strong equivalence
  - Central notion in non-monotonic reasoning [Lifschitz et al., 2001, Turner, 2004, Truszczynski, 2006, Baumann and Strass, 2016]
  - Studied for most argumentation semantics
    [Oikarinen and Woltran, 2011, Baumann, 2016]

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⇒ Missing results for naive and strong admissible semantics
⇒ Characterization theorems for intermediate semantics

Background

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathcal{U}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Definition

Given an AF $\mathcal{F} = (A, R)$ and $S \subseteq A$,

- S is conflict-free ($S \in$ *cf*$(\mathcal{F})$) if $\forall a, b \in S : (a, b) \notin R$.
- $a \in A$ is defended by S if $\forall b \in A : (b, a) \in R \Rightarrow \exists c \in S : (c, b) \in R$
- $S^+_\mathcal{F} = S \cup \{a \mid \exists b \in S : (b, a) \in R\}$ (the range of S)
- $S^-_\mathcal{F} = S \cup \{a \mid \exists b \in S : (a, b) \in R\}$ (the anti-range of S)

Background

Semantics

Given an AF $\mathcal{F} = (A, R)$, a set $S \subseteq A$ is

- admissible set if $S \in cf(\mathcal{F})$ and each $a \in S$ is defended by S,
- complete extension if $S \in ad(\mathcal{F})$ and $a \in S$ if a is defended by S,
- naive extension if $S \in cf(\mathcal{F})$ and $\nexists T \in cf(\mathcal{F}) : T \supset S$,
- stable extension if $S \in cf(\mathcal{F})$ and $S^{+}_{\mathcal{F}} = A$,
- stage extension if $S \in cf(\mathcal{F})$ and $\nexists T \in cf(\mathcal{F}) : T^{+}_{\mathcal{F}} \supset S^{+}_{\mathcal{F}}$,
- preferred, grounded, semi-stable, ideal, eager, strongly admissible extensions

Verifiability

Definition

We call a function $\mathfrak{r}^x : 2^{\mathcal{U}} \times 2^{\mathcal{U}} \rightarrow \left(2^{\mathcal{U}}\right)^n$ which is expressible via basic set operations only[a] neighborhood function.

[a] $\mathfrak{r}^x(A, B)$ is in the language $X ::= A \mid B \mid (X \cup X) \mid (X \cap X) \mid (X \setminus X)$

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The verification class induced by $\mathfrak{r}^x$ maps each AF $\mathcal{F}$ to

$$\widetilde{\mathcal{F}}^x = \left\{ \left(S, \mathfrak{r}^x(S^+_{\mathcal{F}}, S^-_{\mathcal{F}})\right) \mid S \in \mathit{cf}(\mathcal{F}) \right\}.$$

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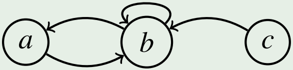
$\mathcal{F}$:


$\mathfrak{r}^+ : \mathfrak{r}^x(A, B) = A$
$\widetilde{\mathcal{F}}^+ = \{(\emptyset, \emptyset), (\{a\}, \{a, b\}), (\{c\}, \{b, c\}), (\{a, c\}, \{a, b, c\})\}$

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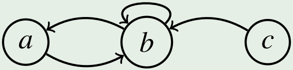

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$\mathfrak{r}^{-\pm} : \mathfrak{r}^x(A, B) = (B, A \setminus B)$

$\widetilde{\mathcal{F}}^{-\pm} = \{(\emptyset, \emptyset, \emptyset), (\{a\}, \{a, b\}, \emptyset), (\{c\}, \{c\}, \{b\}), (\{a, c\}, \{a, b, c\}, \emptyset)\}$

Verifiability

- Neighborhood functions for $n = 1$:

$$\begin{aligned}
\mathfrak{r}^{\epsilon}(A, B) &= \emptyset \\
\mathfrak{r}^{+}(A, B) &= A \\
\mathfrak{r}^{-}(A, B) &= B \\
\mathfrak{r}^{\mp}(A, B) &= B \setminus A \\
\mathfrak{r}^{\pm}(A, B) &= A \setminus B \\
\mathfrak{r}^{\cap}(A, B) &= A \cap B \\
\mathfrak{r}^{\cup}(A, B) &= A \cup B \\
\mathfrak{r}^{\Delta}(A, B) &= (A \cup B) \setminus (A \cap B)
\end{aligned}$$

- $2^7 + 1$ syntactically different neighborhood functions
- $r^{x_1,\ldots,x_n}(A, B) ::= (r^{x_1}(A, B), \ldots, r^{x_n}(A, B))$

Verifiability

Definition

$\mathfrak{r}^x$ is more informative than $\mathfrak{r}^y$ ($\mathfrak{r}^x \succeq \mathfrak{r}^y$): there is a function $\delta : \left(2^{\mathcal{U}}\right)^n \rightarrow \left(2^{\mathcal{U}}\right)^m$ such that $\delta\left(\mathfrak{r}^x(A,B)\right) = \mathfrak{r}^y\left(A,B\right)$ for any $A, B \subseteq \mathcal{U}$.

In case $\mathfrak{r}^x \approx \mathfrak{r}^y$ ($\mathfrak{r}^x \succeq \mathfrak{r}^y$ and $\mathfrak{r}^y \succeq \mathfrak{r}^x$), we say that $\mathfrak{r}^x$ represents $\mathfrak{r}^y$.

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Example

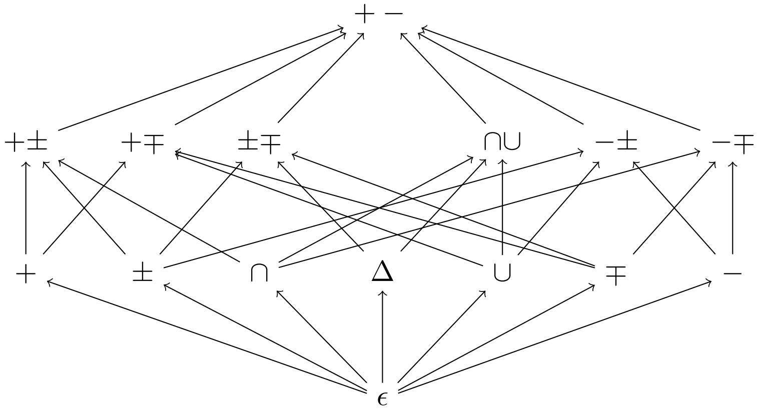
- $\delta_1(\mathfrak{r}^{+\pm}(A,B)) = \delta_1(A, A \setminus B) =_{def} (A, A \setminus (A \setminus B)) = (A, A \cap B) = \mathfrak{r}^{+\cap}(A,B)$;
- $\delta_2(\mathfrak{r}^{+\cap}(A,B)) = \delta_2(A, A \cap B) =_{def} (A \setminus (A \cap B), A \cap B) = (A \setminus B, A \cap B) = \mathfrak{r}^{\pm\cap}(A,B)$;
- $\delta_3(\mathfrak{r}^{\pm\cap}(A,B)) = \delta_3(A \setminus B, A \cap B) =_{def} ((A \setminus B) \cup (A \cap B), A \setminus B) = (A, A \setminus B) = \mathfrak{r}^{+\pm}(A,B)$.

$\Rightarrow$ $\mathfrak{r}^{+\pm} \approx \mathfrak{r}^{+\cap} \approx \mathfrak{r}^{\pm\cap}$

Verifiability

Lemma

All neighborhood functions are represented by the ones depicted below and the $\prec$-relation represented by arcs holds.



Verifiability

Definition

A semantics σ is verifiable by the verification class induced by the neighborhood function $\mathfrak{r}^x$ (x-verifiable) iff there is a function $\gamma_\sigma : \left(2^{\mathcal{U}}\right)^n \to 2^{2^{\mathcal{U}}}$ s.t.

$$\forall \mathcal{F} : \quad \gamma_\sigma\left(\widetilde{\mathcal{F}}^x\right) = \sigma(\mathcal{F}).$$

Moreover, σ is exactly x-verifiable iff σ is x-verifiable and there is no $\mathfrak{r}^y$ with $\mathfrak{r}^y \prec \mathfrak{r}^x$ such that σ is y-verifiable.

Verifiability

Proposition

Complete semantics is exactly $+-$-verifiable.

Verifiability

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Complete semantics is exactly $+-$-verifiable.

Proof

- Verifiability:

$$\gamma_{co}(\widetilde{\mathcal{F}}^{+-}) = \{S \mid (S, S^{+}, S^{-}) \in \widetilde{\mathcal{F}}^{+-}, (S^{-} \setminus S^{+}) = \emptyset,$$
$$\forall(\bar{S}, \bar{S}^{+}, \bar{S}^{-}) \in \widetilde{\mathcal{F}}^{+-} : \bar{S} \supset S \Rightarrow (\bar{S}^{-} \setminus S^{+}) \neq \emptyset)\}$$

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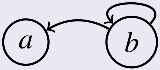
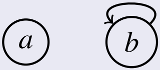
$$\forall(\bar{S}, \bar{S}^+, \bar{S}^-) \in \widetilde{\mathcal{F}}^{+-} : \bar{S} \supset S \Rightarrow (\bar{S}^- \setminus S^+) \neq \emptyset)\}$$

- Exactness:

$+\pm :$ $\mathcal{F}_1 :$  $\mathcal{F}_1' :$ 

  - $\widetilde{\mathcal{F}_1}^{+\pm} = \{(\emptyset, \emptyset, \emptyset), (\{a\}, \{a\}, \emptyset)\} = \widetilde{\mathcal{F}_1'}^{+\pm}$
  - $co(\mathcal{F}_1) = \{\emptyset\} \neq \{\{a\}\} = co(\mathcal{F}_1')$

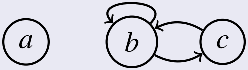
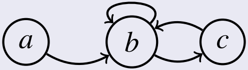
$\Rightarrow$ *co* is not $+\pm$-verifiable

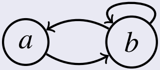
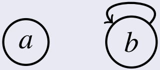
Verifiability

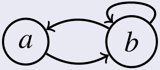
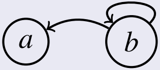
Proposition

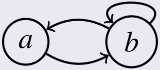
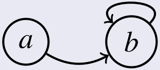
Complete semantics is exactly $+-$-verifiable.

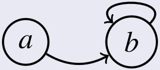
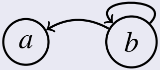
Proof (ctd.)

$-\mp$: $\mathcal{F}_2$:  $\mathcal{F}'_2$: 

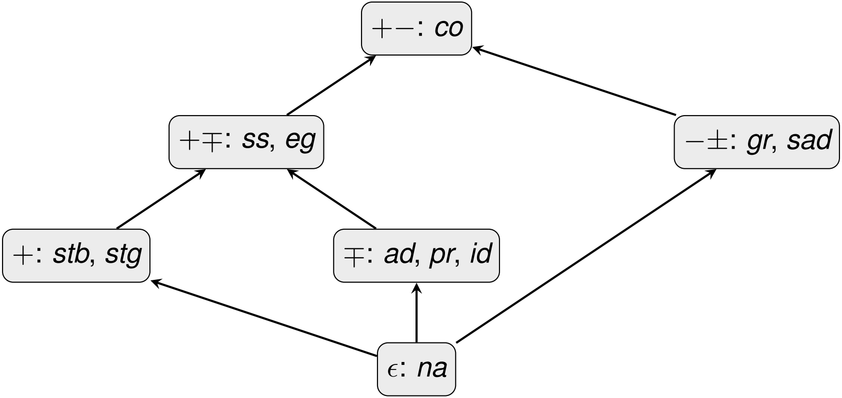
$\pm\mp$: $\mathcal{F}_3$:  $\mathcal{F}'_3$: 

$-\pm$: $\mathcal{F}_4$:  $\mathcal{F}'_4$: 

$+\mp$: $\mathcal{F}_5$:  $\mathcal{F}'_5$: 

$\cap\cup$: $\mathcal{F}_6$:  $\mathcal{F}'_6$: 

Verifiability



Verifiability

Definition

We call a semantics σ rational if self-loop-chains are irrelevant.
That is, for every AF $\mathcal{F}$ it holds that $\sigma(\mathcal{F}) = \sigma(\mathcal{F}^l)$, where
$\mathcal{F}^l = (A_\mathcal{F}, R_\mathcal{F} \setminus \{(a,b) \in R_\mathcal{F} \mid (a,a),(b,b) \in R_\mathcal{F}, a \neq b\})$.

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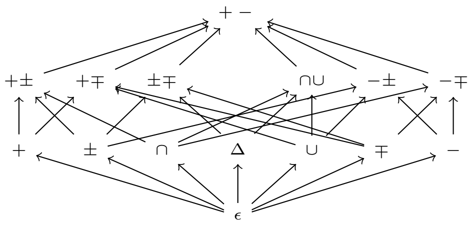
Theorem

Every semantics which is rational is exactly verifiable by a verification class induced by one of the neighborhood functions below.



Strong Equivalence

Definition

Two AFs $\mathcal{F}$ and $\mathcal{G}$ are strongly equivalent w.r.t. semanticsσ ($\mathcal{F} \equiv_E^\sigma \mathcal{G}$) iff for all AFs $\mathcal{H}$: $\sigma(\mathcal{F} \cup \mathcal{H}) = \sigma(\mathcal{G} \cup \mathcal{H})$

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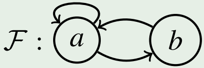
$\Rightarrow$ syntactical criteria exist

Example (stable semantics)

- *stb*-kernel: $\mathcal{F}^{k(stb)} = (A, R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\})$.
- Theorem: $\mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} \Leftrightarrow \mathcal{F}$ and $\mathcal{G}$ are strongly equivalent.



We have $\mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} = \mathcal{G}$. Thus, $\mathcal{F}$ and $\mathcal{G}$ are strong equivalent.

Strong Equivalence

Definition (σ-kernel)

Let $\mathcal{F} = (A, R)$. We define σ-kernels $\mathcal{F}^{k(\sigma)} = \left(A, R^{k(\sigma)}\right)$ whereby

1. $R^{k(stb)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\}$,
2. $R^{k(ad)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R, \{(b,a),(b,b)\} \cap R \neq \emptyset\}$,
3. $R^{k(gr)} = R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\}$,
4. $R^{k(co)} = R \setminus \{(a,b) \mid a \neq b, (a,a),(b,b) \in R\}$.
5. $R^{k(na)} = R \cup \{(a,b) \mid a \neq b, \{(a,a),(b,a),(b,b)\} \cap R \neq \emptyset\}$.

Strong Equivalence

Theorem

Strong equivalence is characterizable through kernels (see below).

$$\mathcal{F} \equiv_E^\sigma \mathcal{G} \Leftrightarrow \mathcal{F}^k = \mathcal{G}^k$$

stg	stb	ss	eg	ad	pr	id	gr	sad	co	na
$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(gr)$	$k(co)$	$k(na)$

Intermediate Semantics

- *stb* and *stg* are both characterizable through $k(stb)$.
- Does this also hold for arbitrary semantics σ with *stb*$(\mathcal{F}) \subseteq \sigma(\mathcal{F}) \subseteq$ *stg*$(\mathcal{F})$ for each AF $\mathcal{F}$?
  (e.g. when obtained from SESAME [Besnard et al., 2016])

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Example

- "Stagle semantics":
  $S \in$ *sta*$(\mathcal{F}) \Leftrightarrow S \in$ *cf*$(\mathcal{F})$, $S^+_\mathcal{F} \cup S^-_\mathcal{F} = A_\mathcal{F}$ and $\forall T \in cf(\mathcal{F}) : S^+_\mathcal{F} \not\subset T^+_\mathcal{F}$

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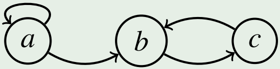
$\mathcal{F}$:



- *stb*$(\mathcal{F}) = \emptyset \subset$ *sta*$(\mathcal{F}) = \{\{b\}\} \subset$ *stg*$(\mathcal{F}) = \{\{b\}, \{c\}\}$.

Intermediate Semantics

- *stb* and *stg* are both characterizable through $k(stb)$.
- Does this also hold for arbitrary semantics σ with $\mathit{stb}(\mathcal{F}) \subseteq \sigma(\mathcal{F}) \subseteq \mathit{stg}(\mathcal{F})$ for each AF $\mathcal{F}$?
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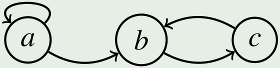
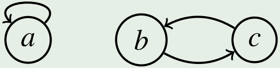
Example

- "Stagle semantics":
  $S \in \mathit{sta}(\mathcal{F}) \Leftrightarrow S \in \mathit{cf}(\mathcal{F}), S^{+}_{\mathcal{F}} \cup S^{-}_{\mathcal{F}} = A_{\mathcal{F}}$ and $\forall T \in \mathit{cf}(\mathcal{F}) : S^{+}_{\mathcal{F}} \not\subset T^{+}_{\mathcal{F}}$

$\mathcal{F}$:


- $\mathit{stb}(\mathcal{F}) = \emptyset \subset \mathit{sta}(\mathcal{F}) = \{\{b\}\} \subset \mathit{stg}(\mathcal{F}) = \{\{b\}, \{c\}\}$.

$\mathcal{F}^{k(\mathit{stb})}$:


- $\mathit{sta}\left(\mathcal{F}^{k(\mathit{stb})}\right) = \{\{b\}, \{c\}\} \Rightarrow \mathcal{F} \not\equiv^{\mathit{sta}}_{E} \mathcal{F}^{k(\mathit{stb})}, \mathcal{F}^{k(\mathit{stb})} = \left(\mathcal{F}^{k(\mathit{stb})}\right)^{k(\mathit{stb})}$

$\Rightarrow$ Stagle semantics is not compatible with the stable kernel.

Intermediate Semantics

Theorem

For each semantics σ which is $+$-verifiable and *stb*-*stg*-intermediate, it holds that

$$\mathcal{F}^{k(\mathit{stb})} = \mathcal{G}^{k(\mathit{stb})} \Leftrightarrow \mathcal{F} \equiv^{\sigma}_{E} \mathcal{G}.$$

Intermediate Semantics

Theorem

For each semantics σ which is $+$-verifiable and *stb*-*stg*-intermediate, it holds that

$$\mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} \Leftrightarrow \mathcal{F} \equiv_E^\sigma \mathcal{G}.$$

Theorem

For each semantics σ which is $+\mp$-verifiable and ρ-*ad*-intermediate with $\rho \in \{ss, id, eg\}$, it holds that

$$\mathcal{F}^{k(ad)} = \mathcal{G}^{k(ad)} \Leftrightarrow \mathcal{F} \equiv_E^\sigma \mathcal{G}.$$

Theorem

For each semantics σ which is $-\pm$-verifiable and *gr*-*sad*-intermediate, it holds that

$$\mathcal{F}^{k(gr)} = \mathcal{G}^{k(gr)} \Leftrightarrow \mathcal{F} \equiv_E^\sigma \mathcal{G}.$$

Conclusion

Summary:

- Hierarchy of verification classes
- Each "rational" semantics is exactly verifiable by a certain class
- Characterization of strong equivalence for intermediate semantics

Future work:

- Semantics not captured by the approach, e.g. *cf2* semantics [Baroni et al., 2005]
- Investigating labelling-based semantics [Caminada and Gabbay, 2009]
- Use classification as distance measure [Doutre and Mailly, 2016]

References I

Baroni, P. and Giacomin, M. (2007).
On principle-based evaluation of extension-based argumentation semantics.
Artif. Intell., 171(10-15):675–700.

Baroni, P., Giacomin, M., and Guida, G. (2005).
SCC-Recursiveness: A general schema for argumentation semantics.
Artif. Intell., 168(1-2):162–210.

Baumann, R. (2016).
Characterizing equivalence notions for labelling-based semantics.
In Principles of Knowledge Representation and Reasoning: Proceedings of the 15th International Conference, pages 22–32.

Baumann, R. and Strass, H. (2016).
An abstract logical approach to characterizing strong equivalence in logic-based knowledge representation formalisms.
In Principles of Knowledge Representation and Reasoning: Proceedings of the 15th International Conference, pages 525–528.

Besnard, P., Doutre, S., Ho, V. H., and Longin, D. (2016).
SESAME - A system for specifying semantics in abstract argumentation.
In Proc. SAFA@COMMA, pages 40–51.

References II

- Caminada, M. and Amgoud, L. (2007).
  On the evaluation of argumentation formalisms.
  Artif. Intell., pages 286–310.
- Caminada, M. and Gabbay, D. M. (2009).
  A logical account of formal argumentation.
  Studia Logica, 93(2):109–145.
- Doutre, S. and Mailly, J.-G. (2016).
  Quantifying the difference between argumentation semantics.
  In Computational Models of Argument - Proceedings of COMMA 2016, volume 287 of Frontiers in Artificial Intelligence and Applications, pages 255–262. IOS Press.
- Dung, P. M. (1995).
  On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
  Artif. Intell., 77(2):321–357.
- Dunne, P. E. and Bench-Capon, T. J. M. (2002).
  Coherence in finite argument systems.
  Artif. Intell., 141(1/2):187–203.
- Dvořák, W. and Woltran, S. (2010).
  Complexity of semi-stable and stage semantics in argumentation frameworks.
  Inf. Process. Lett., 110(11):425–430.

References III

Lifschitz, V., Pearce, D., and Valverde, A. (2001).
Strongly equivalent logic programs.
ACM Transactions on Computational Logic, 2(4):526–541.

Oikarinen, E. and Woltran, S. (2011).
Characterizing strong equivalence for argumentation frameworks.
Artif. Intell., 175(14-15):1985–2009.

Truszczynski, M. (2006).
Strong and uniform equivalence of nonmonotonic theories - an algebraic approach.
Annals of Mathematics and Artificial Intelligence, 48(3-4):245–265.

Turner, H. (2004).
Strong equivalence for causal theories.
In 7th International Conference on Logic Programming and Nonmonotonic Reasoning, Proceedings, volume 2923 of Lecture Notes in Computer Science, pages 289–301. Springer.

Verifiability

$$\gamma_{na}(\widetilde{\mathcal{F}}_A^{\epsilon}) = \{S \mid S \in \widetilde{\mathcal{F}}, S \textit{ is } \subseteq\textit{-maximal in } \widetilde{\mathcal{F}}\};$$

$$\gamma_{stg}(\widetilde{\mathcal{F}}_A^{+}) = \{S \mid (S, S^+) \in \widetilde{\mathcal{F}}^+, S^+ \textit{ is } \subseteq\textit{-maximal in } \{C^+ \mid (C, C^+) \in \widetilde{\mathcal{F}}^+\}\};$$

$$\gamma_{stb}(\widetilde{\mathcal{F}}_A^{+}) = \{S \mid (S, S^+) \in \widetilde{\mathcal{F}}^+, S^+ = A\};$$

$$\gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp}) = \{S \mid (S, S^{\mp}) \in \widetilde{\mathcal{F}}^{\mp}, S^{\mp} = \emptyset\};$$

$$\gamma_{pr}(\widetilde{\mathcal{F}}_A^{\mp}) = \{S \mid S \in \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp}), S \textit{ is } \subseteq\textit{-maximal in } \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp})\};$$

$$\gamma_{ss}(\widetilde{\mathcal{F}}_A^{+\mp}) = \{S \mid S \in \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp}), S^+ \textit{ is } \subseteq\textit{-maximal in } \{C^+ \mid (C, C^+, C^{\mp}) \in \widetilde{\mathcal{F}}^{+\mp}, C \in \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp})\}\};$$

$$\gamma_{id}(\widetilde{\mathcal{F}}_A^{\mp}) = \{S \mid S \textit{ is } \subseteq\textit{-maximal in } \{C \mid C \in \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp}), C \subseteq \bigcap \gamma_{pr}(\widetilde{\mathcal{F}}_A^{\mp})\}\};$$

$$\gamma_{eg}(\widetilde{\mathcal{F}}_A^{+\mp}) = \{S \mid S \textit{ is } \subseteq\textit{-maximal in } \{C \mid C \in \gamma_{ad}(\widetilde{\mathcal{F}}_A^{\mp}), C \subseteq \bigcap \gamma_{ss}(\widetilde{\mathcal{F}}_A^{+\mp})\}\};$$

$$\gamma_{sad}(\widetilde{\mathcal{F}}_A^{-\pm}) = \{S \mid (S, S^-, S^{\pm}) \in \widetilde{\mathcal{F}}^{-\pm}, \exists (S_0, S_0^-, S_0^{\pm}), \ldots, (S_n, S_n^-, S_n^{\pm}) \in \widetilde{\mathcal{F}}^{-\pm} : (\emptyset = S_0 \subset \cdots \subset S_n = S \wedge \forall i \in \{1, \ldots, n\} : S_i^- \subseteq S_{i-1}^{\pm})\};$$

$$\gamma_{gr}(\widetilde{\mathcal{F}}_A^{-\pm}) = \{S \mid S \in \gamma_{sad}(\widetilde{\mathcal{F}}_A^{-\pm}), \forall (\bar{S}, \bar{S}^-, \bar{S}^{\pm}) \in \widetilde{\mathcal{F}}^{-\pm} : \bar{S} \supset S \Rightarrow (\bar{S}^- \setminus S^{\pm}) \neq \emptyset)\}.$$