FAKULTÄT FÜR INFORMATIK

Faculty of Informatics

# Splitting Abstract Dialectical Frameworks

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Der Wissenschaftsfonds.

# Introduction

- Abstract Dialectical Frameworks (ADFs)[Brewka and Woltran, 2010]
  - Generalization of Abstract Argumentation Frameworks [Dung, 1995]
  - Acceptance condition for each argument
- Provide high modelling power
  - Support, set attack, set support, . . .
  - Acceptance as an arbitrary Boolean function

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  - Acceptance as an arbitrary Boolean function
- ADFs can capture many AF-generalizations
  - Translations to ADFs [Polberg, 2014]
  - SETAF [Nielsen and Parsons, 2006], AFN [Nouioua and Risch, 2011], AFRA [Baroni et al., 2011], EAF [Modgil, 2009], . . .
  - Efficient solvers for ADFs beneficial for argumentation in general
- Instantiation of defeasible theory bases satisfying rationality postulates [Strass, 2013]

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- Properties of semantics which can be exploited in implementations of reasoning tasks within ADFs.

- Directional Splitting: Positive results for all standard semantics defined in [Brewka et al., 2013].
- General Splitting: Preliminary results for models and admissible interpretations.
- Completing the picture of relations between semantics.

# Motivation

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- Common technique in nonmontonic formalisms.
  - Abstract Argumentation Frameworks [Baumann, 2011].
  - Logic Programs [Lifschitz and Turner, 1994].

# Background

## Definition

An ADF is a tuple $D = (A, L, C)$ where

- $A$ is a set of arguments (statements, positions),
- $L \subseteq A \times A$ is a set of links, and
- $C = \{C_a \mid a \in A\}$ is a set of total functions $C_a : 2^{a_D^-} \mapsto \{\mathbf{t}, \mathbf{f}\}$,

with $a_D^- = \{b \in A \mid (b, a) \in L\}$. $C_a$ is called acceptance condition of $a$.

- We will use propositional (acceptance) formulas $\varphi_a$ as acceptance conditions of an argument $a$.
- Atoms in acceptance formulas determine ingoing links: ADF may be represented as $\{\langle a, \varphi_a \rangle \mid a \in A\}$.

# Background

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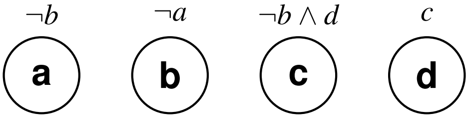

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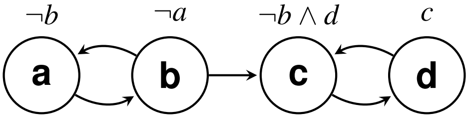

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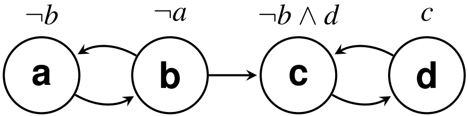


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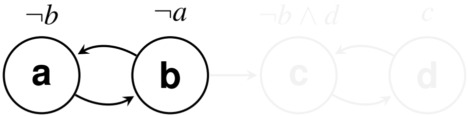


- $D|_B = \{\langle a, \varphi_a\rangle \in D \mid a \in B\}$.
- $D|_{\{a,b\}} = \{\langle a, \neg b\rangle, \langle b, \neg a\rangle\}$.

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## Definition

Given an ADF $D$, an interpretation is a mapping $v : A_D \mapsto \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. We denote $v^x = \{a \in A_D \mid v(a) = x\}$ for $x \in \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. The interpretation $v$ is two-valued if $v^{\mathbf{u}} = \emptyset$.

Partial order $\leq_i$ on truth values: $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$.

Given ADF D,

- $v_1 \leq_i v_2$ iff $v_1(a) \leq_i v_2(a)$ for all $a \in A_D$.
- $[v]_2$ denotes the set of two-valued interpretations $w$ with $v \leq_i w$.

# Background

## Definition

Given an ADF $D$ and an interpretation $v$, the characteristic function $\Gamma_D(v) = v'$ is defined as

$$v'(a) = \begin{cases} \mathbf{t} \text{ if } w(\varphi_a) = \mathbf{t} \text{ for all } w \in [v]_2 \\ \mathbf{f} \text{ if } w(\varphi_a) = \mathbf{f} \text{ for all } w \in [v]_2 \\ \mathbf{u} \text{ otherwise} \end{cases}$$

# Background

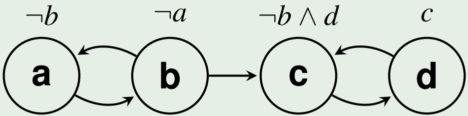
## Definition [Brewka et al., 2013]

Given an ADF $D$, a three-valued interpretation $v$ is

- a model of D iff $v^{\mathbf{u}} = \emptyset$ and for all $a \in A_D$, $v(a) = v(\varphi_a)$,
- a three-valued model of $D$ iff for all $a \in A_D$, $v(a) \neq \mathbf{u}$ implies $v(a) = v(\varphi_a)$,
- an admissible interpretation of $D$ iff $v \leq_i \Gamma_D(v)$,
- a preferred interpretation of $D$ iff $v$ is a $\leq_i$-maximal admissible interpretation of $D$,
- a complete interpretation of $D$ iff $v = \Gamma_D(v)$,
- the grounded interpretation of $D$ iff $v$ is the least fixpoint of $\Gamma_D$ wrt. $\leq_i$,
- a stable model of $D$ iff $v$ is a model of $D$ and $v^{\mathbf{t}} = w^{\mathbf{t}}$, with $w$ being the grounded interpretation of $D^{v-} = \{\langle a, \varphi_a[x/\bot : v(x) = \mathbf{f}]\rangle \mid a \in v^{\mathbf{t}}\}$.

# Background

## Example

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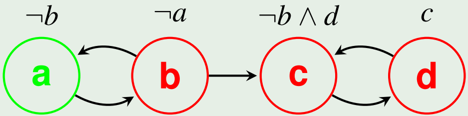


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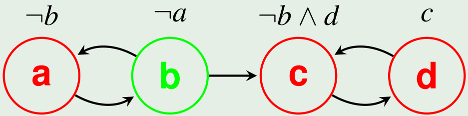


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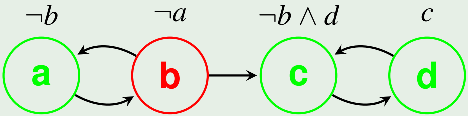


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# Directional Splitting

## Definition

Let $G_1 = (A_1, L_1)$ and $G_2 = (A_2, L_2)$ be directed graphs such that $A_1 \cap A_2 = \emptyset$ and let $L_3 \subseteq A_1 \times A_2$. The tuple $(G_1, G_2, L_3)$ is a directional splitting of an ADF $D = (A_1 \cup A_2, L_1 \cup L_2 \cup L_3, C)$.

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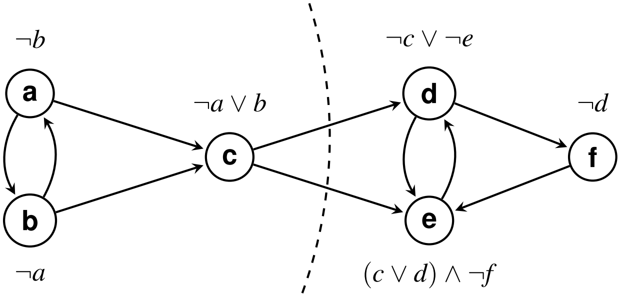

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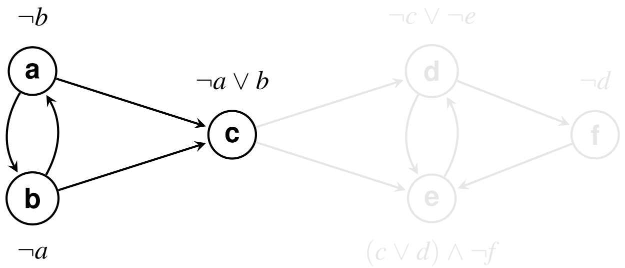

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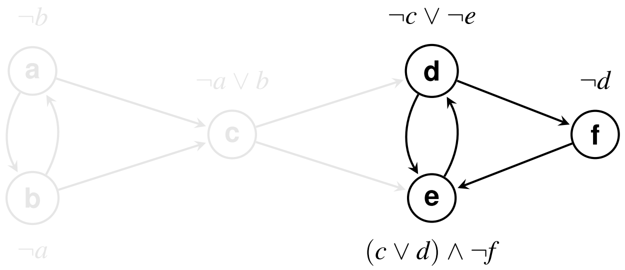

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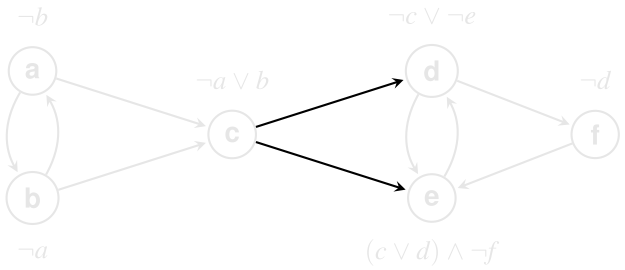

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$$D^v = \{\langle a, \varphi_a[b/v(b) : b \in (v^{\mathbf{t}} \cup v^{\mathbf{f}})][c/x_c : c \in v^{\mathbf{u}}]\rangle \mid a \in A_2\} \cup \{\langle x_c, \neg x_c\rangle \mid c \in v^{\mathbf{u}}\}.$$

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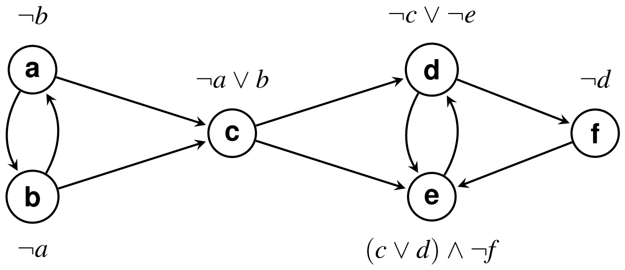
## Theorem

Let $\sigma \in \{\mathit{mod}_2, \mathit{mod}_3, \mathit{adm}, \mathit{pref}, \mathit{comp}, \mathit{grd}, \mathit{stb}\}$, and $G_1 = (A_1, L_1)$ and $G_2 = (A_2, L_2)$ be directed graphs such that $(G_1, G_2, L_3)$ is a directional splitting of the ADF $D$. Further let $D_1 = D|_{A_1}$. It holds that

1. $v_1 \in \sigma(D_1) \wedge v_2 \in \sigma(D^{v_1}) \Rightarrow (v_1 \cup v_2)|_A \in \sigma(D)$,
2. $v \in \sigma(D) \Rightarrow v|_{A_1} \in \sigma(D_1) \wedge \exists v_2 \in \sigma(D^{v|_{A_1}}) : v_2|_{A_2} = v|_{A_2}$.

# Directional Splitting – Procedure

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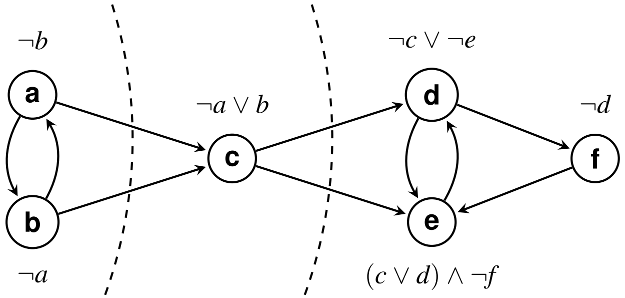


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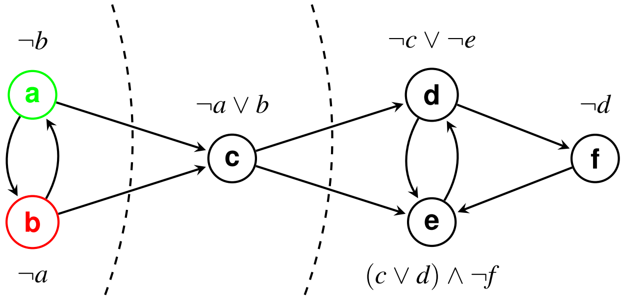


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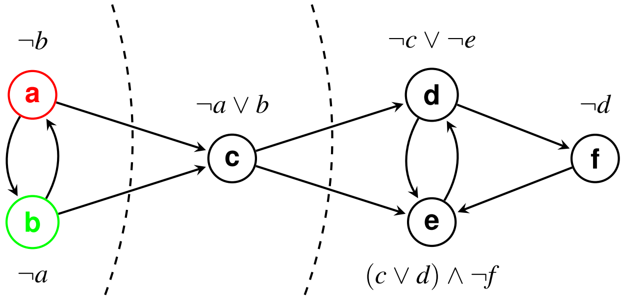


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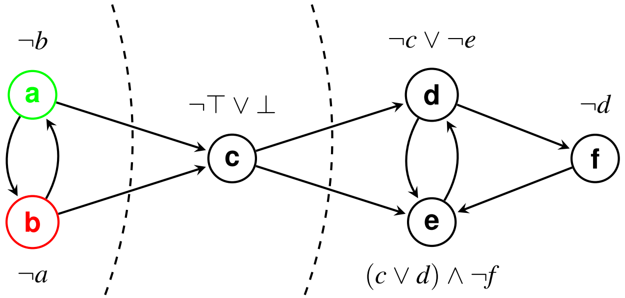


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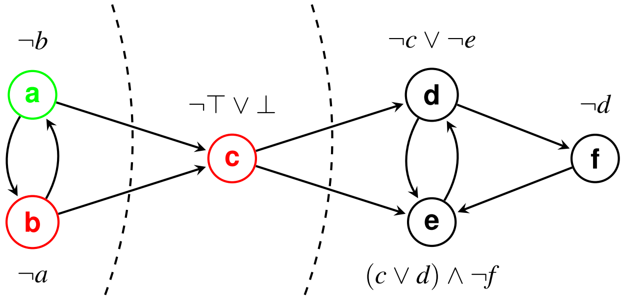


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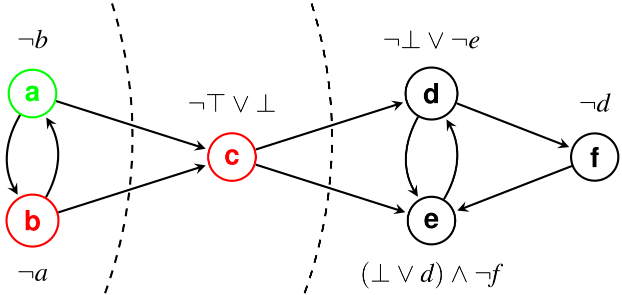


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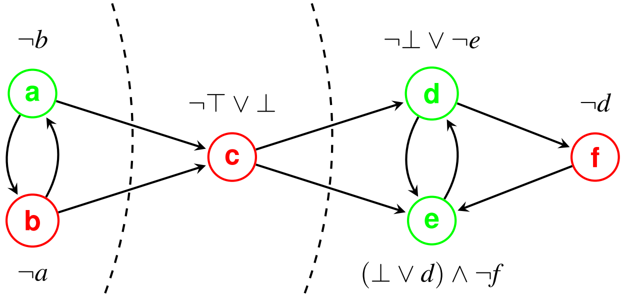


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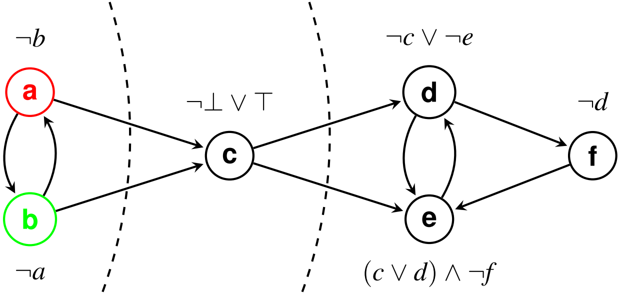


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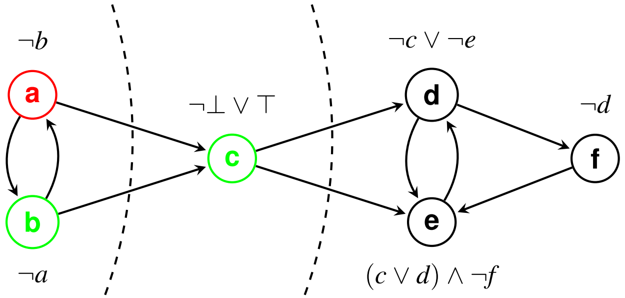


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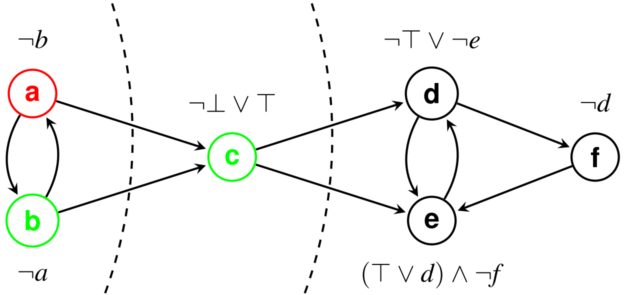


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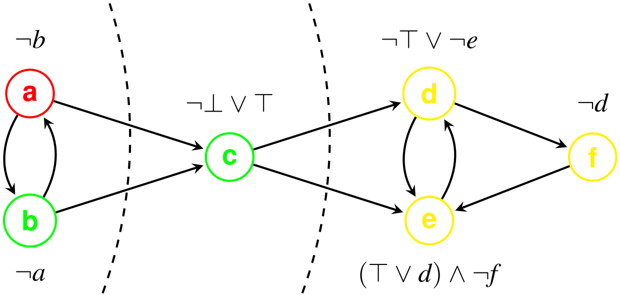


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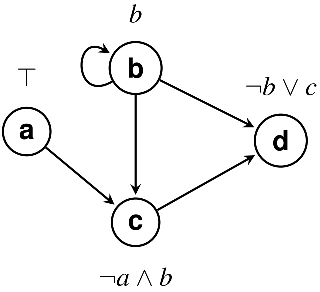


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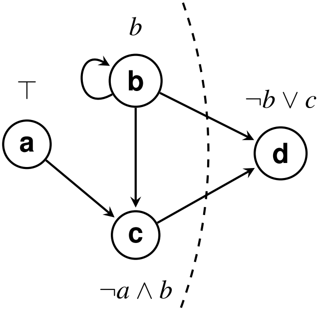


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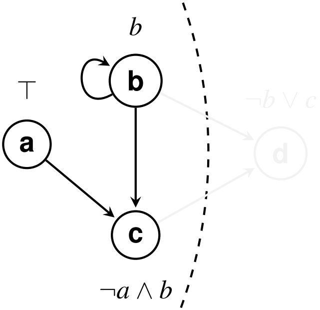


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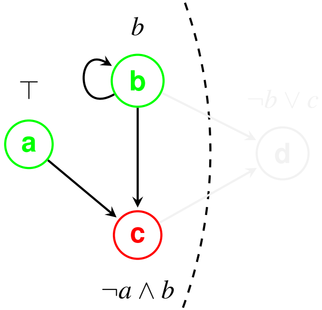


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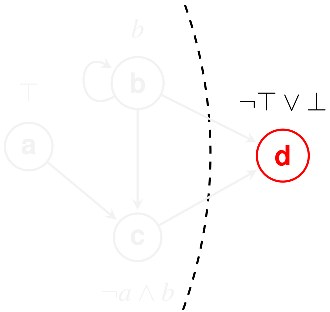


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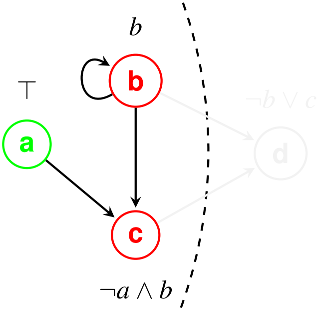


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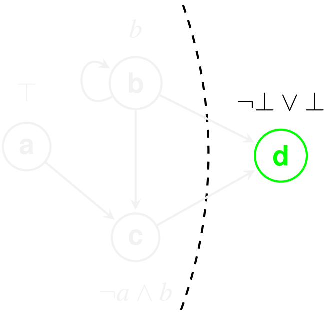


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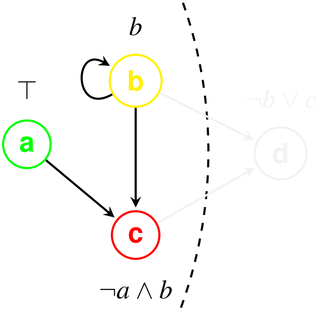
$$\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\},$$

# Directional Splitting – Procedure




$$
\mathit{comp}(D) = \{\{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}, d \mapsto \mathbf{f}\},\\
\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\},\\
\{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{f},
$$

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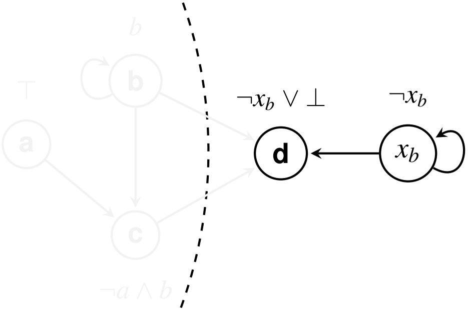


$$
\begin{aligned}
\mathit{comp}(D) = \{ & \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}, d \mapsto \mathbf{f}\},\\
& \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\},\\
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$$

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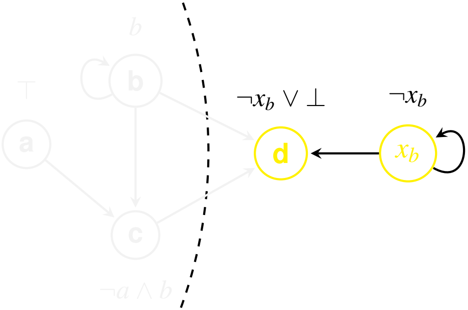
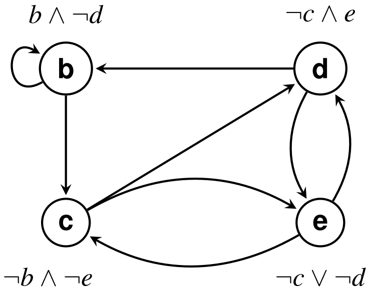



$$
\begin{aligned}
\mathit{comp}(D) = \{&\{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}, d \mapsto \mathbf{f}\},\\
&\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\},\\
&\{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{f}, d \mapsto \mathbf{u}, x_b \mapsto \mathbf{u}\}\}
\end{aligned}
$$

# General Splitting



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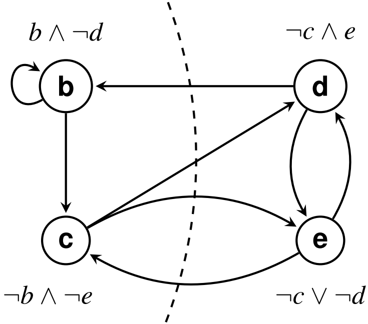

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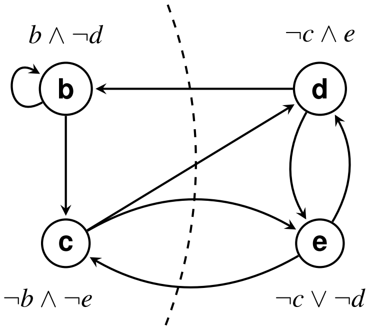
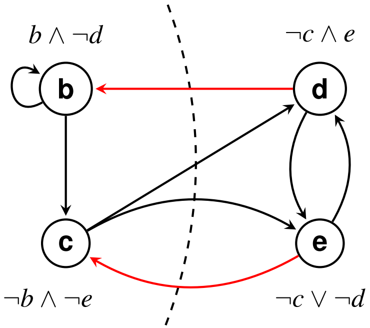



**Definition**

Given an ADF $D$ we call a set $S \subseteq A_D$ a general splitting of $D$.

# General Splitting




**Definition**

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# General Splitting

## Definition

Given an ADF $D$, let $L \subseteq L_D$ be a set of links in $D$ and $L^- = \{b \mid (b, a) \in L\}$. The $L$-elimination of $D$ is defined as

$$D^L = \{\langle a, \varphi_a[b/x_b : b \in L^-]\rangle \mid a \in A_D\} \cup \\ \{\langle x_b, x_b\rangle \mid b \in L^-\} \cup \{\langle \omega(D^L), \neg \left( \bigwedge_{b \in L^-} b \leftrightarrow x_b \right) \wedge \neg\omega(D^L)\rangle\}.$$

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## Proposition

Given an ADF $D$, for any $L \subseteq L_D$ it holds that

1. $v \in \mathit{mod}_2(D^L) \Rightarrow v|_{A_D} \in \mathit{mod}_2(D)$,
2. $v \in \mathit{mod}_2(D) \Rightarrow \exists v' \in \mathit{val}_2(D^L) : v = v'|_{A_D}$.

# General Splitting

## Definition

Given an ADF $D$, let $S \subseteq A_D$ be a general splitting of $D$ and $B = \{b \in (A_D \setminus S) \mid \exists a \in S : (b, a) \in L_D\}$. The primary slice of $D$ wrt. $S$ is defined as

$$D^S = \{\langle a, \varphi_a[b/x_b : b \in B]\rangle \mid a \in S\} \cup \{\langle x_b, x_b\rangle \mid b \in B\}.$$

Moreover, if $v$ is an interpretation of $D^S$, the extended $v$-reduct of $D$ wrt. $S$ is defined as

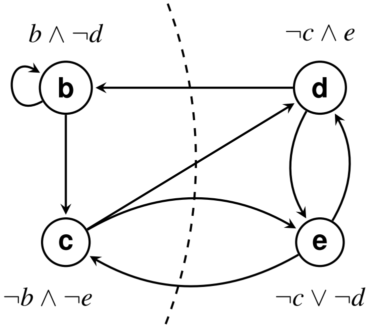
$$D^{S,v} = D^v \cup \{\langle \omega(D^{S,v}), \bigwedge_{b \in B, v(x_b) = \mathbf{t}} b \wedge \bigwedge_{b \in B, v(x_b) = \mathbf{f}} (\neg b)\rangle\}.$$

## Theorem

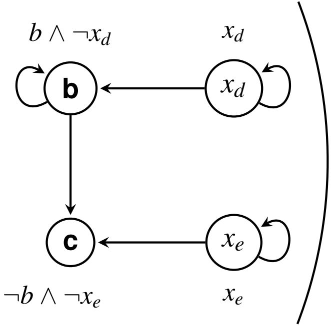
Given an ADF $D$ and a general splitting $S \subseteq A_D$ thereof, let $B = \{b \in (A_D \setminus S) \mid \exists a \in S : (b, a) \in L_D\}$. It holds that

1. $v_1 \in \mathit{adm}(D^S) \wedge v_2 \in \mathit{adm}(D^{S,v_1}) \wedge v_2(\omega(D^{S,v_1})) = \mathbf{t} \Rightarrow (v_1 \cup v_2)|_{A_D} \in \mathit{adm}(D)$
2. $v \in \mathit{adm}(D) \Rightarrow \exists v_1, v_2 : (v_1 \cup v_2)|_{A_D} = v \wedge v_1 \in \mathit{adm}(D^S) \wedge v_2 \in \mathit{adm}(D^{S,v_1}) \wedge v_2(\omega(D^{S,v_1})) = \mathbf{t}$

# General Splitting – Procedure

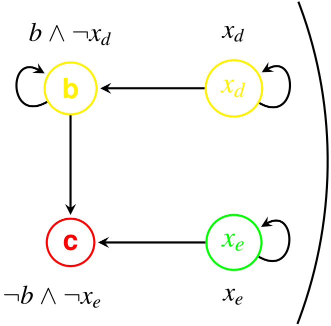


# General Splitting – Procedure




Admissible interpretations:

# General Splitting – Procedure




Admissible interpretations:

$\{b \mapsto \mathbf{u}, c \mapsto \mathbf{f}, x_d \mapsto \mathbf{u}, x_e \mapsto \mathbf{t},$

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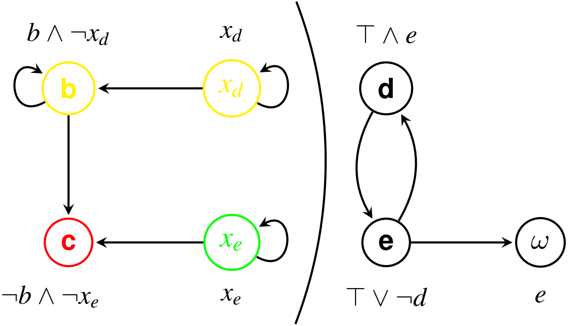


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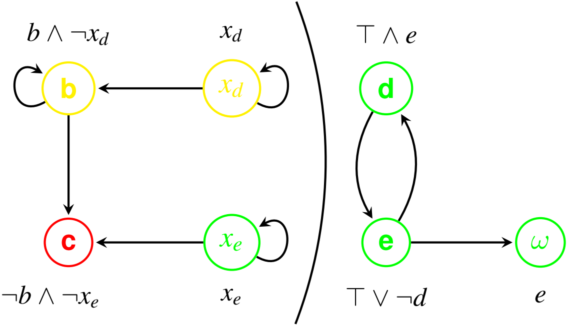
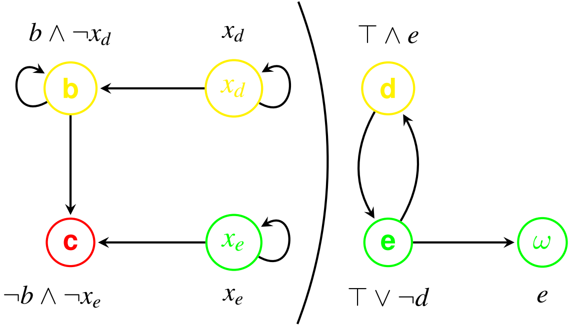



Admissible interpretations:

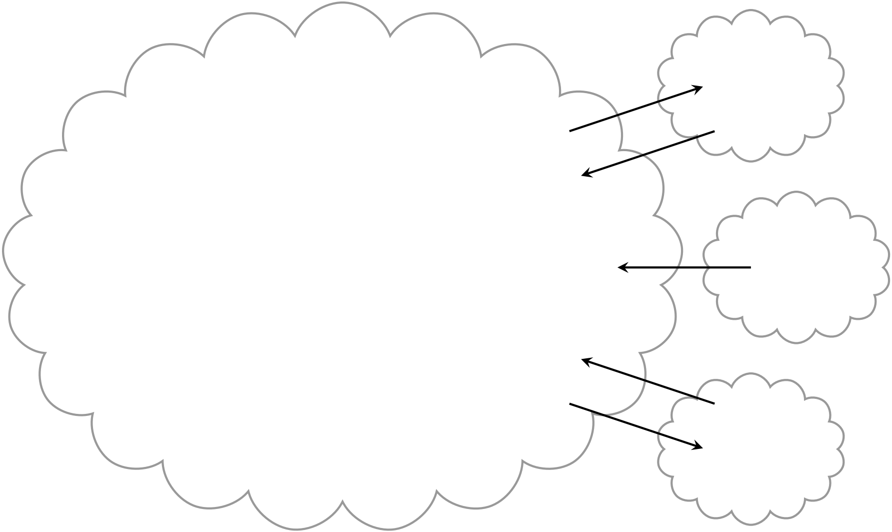
$\{b \mapsto \mathbf{u}, c \mapsto \mathbf{f}, x_d \mapsto \mathbf{u}, x_e \mapsto \mathbf{t}, d \mapsto \mathbf{t}, e \mapsto \mathbf{t}, \omega \mapsto \mathbf{t}\},$




Admissible interpretations:

$\{b \mapsto \mathbf{u}, c \mapsto \mathbf{f}, x_d \mapsto \mathbf{u}, x_e \mapsto \mathbf{t}, d \mapsto \mathbf{t}, e \mapsto \mathbf{t}, \omega \mapsto \mathbf{t}\}$,
$\{b \mapsto \mathbf{u}, c \mapsto \mathbf{f}, x_d \mapsto \mathbf{u}, x_e \mapsto \mathbf{t}, d \mapsto \mathbf{u}, e \mapsto \mathbf{t}, \omega \mapsto \mathbf{t}\}$

# General Splitting



# Conclusion

## Summary

- Directional splitting for semantics from [Brewka et al., 2013].
    - Optimization of implementations of decision problems
- Preliminary results general splitting for models and admissible interpretation.
    - To be exploited in dynamic scenarios.
- Relations between semantics.

## Future Work

- Integrate splitting techniques in implementations.
- Empirical evaluation.
- General splitting for the remaining semantics.

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