FAKULTÄT FÜR INFORMATIK
Faculty of Informatics

Characteristics of Multiple Viewpoints in Abstract Argumentation

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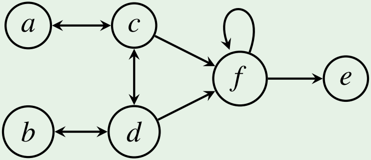
Joint work with Paul E. Dunne, Wolfgang Dvořák, and Stefan Woltran

Der Wissenschaftsfonds.

Introduction

- Argumentation has become a major topic in AI research
- Gives answers to "how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held" [Bench-Capon and Dunne, 2007]
- Dung's Abstract Argumentation Frameworks [Dung, 1995] conceal the concrete contents of arguments; only consider the relation between them
- Heavy research on argumentation semantics, i.e. rules for identifying sets of acceptable arguments
- Surprisingly, a structural analysis of their capabilities has been neglected so far

Example



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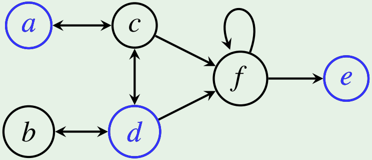

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Motivation

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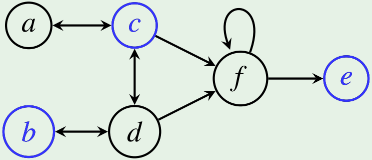

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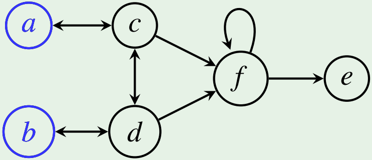

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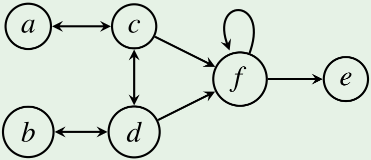

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Natural Questions

- How to adapt the AF to get $\{a, b, e\} \in \textit{pref}(F)$, but $\{a, b\} \notin \textit{pref}(F)$?
- How to adapt the AF to get $\{a, b, d\} \in \textit{pref}(F)$, but $\{a, b\} \notin \textit{pref}(F)$?

We investigate characterizations of the signatures

$$\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF}\}$$

for various important semantics σ (conflict-free, naive, stable, admissible, preferred [Dung, 1995], stage [Verheij, 1996], semi-stable [Caminada, 2006]). We approach signatures via

- necessary properties for extensions $\mathbb{S} \in \Sigma_\sigma$;
- realizability: given a set $\mathbb{S}$ of extensions, is there an AF F with $\sigma(F) = \mathbb{S}$.
  - Constructions of canonical argumentation-frameworks.

Outline

- Argumentation Frameworks, Semantics
- Results on Realizability
  - Conflict-free Sets
  - Stable Semantics
  - Preferred Semantics
- Signatures
- Relations between Signatures
- Conclusion

Background

Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

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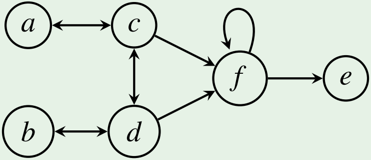

$F = \big(\{a, b, c, d, e, f\},$
$\{(a,c), (c,a), (c,d), (d,c), (d,b), (b,d), (c,f), (d,f), (f,f), (f,e)\}\big)$

Background (ctd.)

Conflict-free Sets

Given an AF $F = (A, R)$, a set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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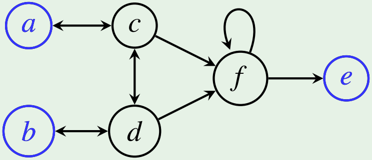

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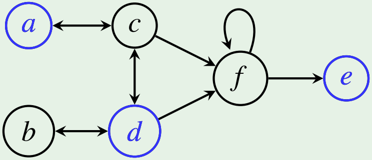

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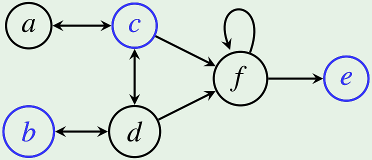

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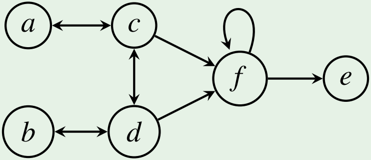

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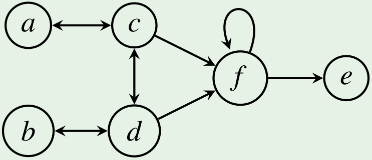

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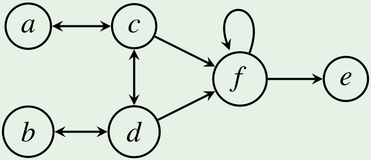

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Naive Extensions

Given an AF $F = (A, R)$, a set $S \subseteq A$ is a naive extension in F, if

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$\Rightarrow$ Maximal conflict-free sets (w.r.t. set-inclusion).

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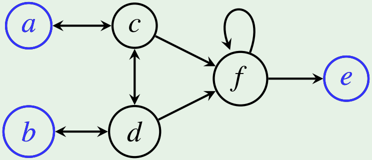

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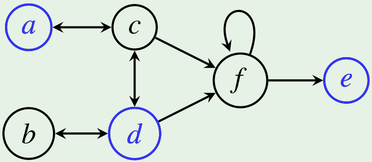

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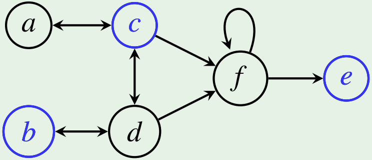

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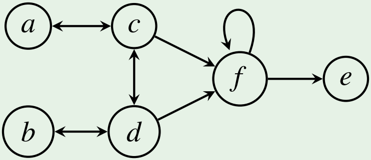

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Given an AF $F = (A, R)$, a set $S \subseteq A$ is a stable extension in F, if

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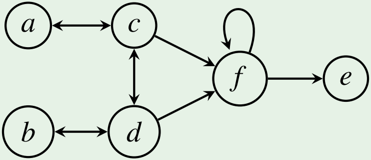

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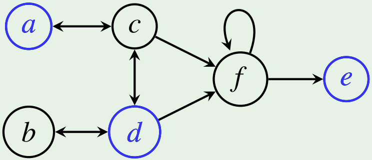

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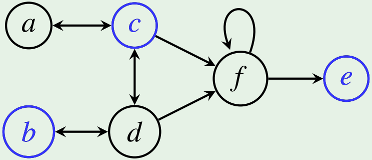

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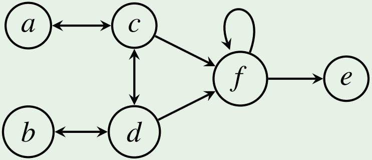

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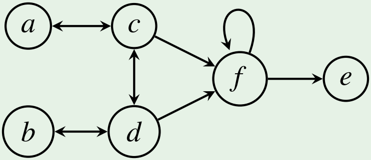

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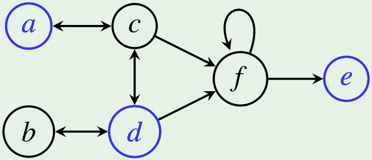

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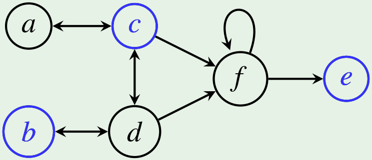

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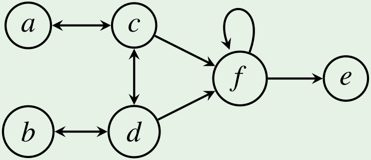

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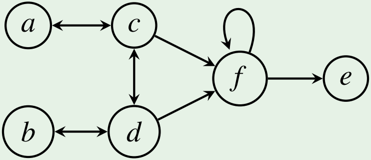

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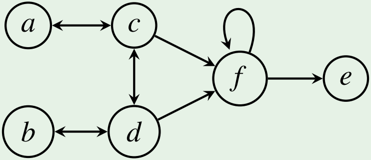

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Preferred Extensions

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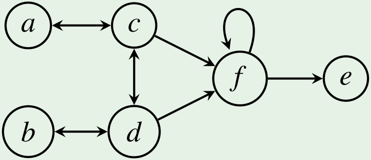

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Definition

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Definition

Given an extension-set $\mathbb{S}$,

- $\mathit{Args}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, and
- $\mathit{Pairs}_{\mathbb{S}} = \{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$.

Results on Conflict-free Sets

Theorem

For each AF $F = (A, R)$ it holds that $\mathit{cf}(F)$ is a non-empty, downward-closed and tight extension-set.

An extension-set $\mathbb{S}$ is

- downward-closed, if $\mathbb{S} = \mathit{dcl}(\mathbb{S}) := \{S' \subseteq S \mid S \in \mathbb{S}\}$ and
- tight, if $\forall S \in \mathbb{S}\, \forall a \in \mathit{Args}_{\mathbb{S}}\, (S \cup \{a\}) \notin \mathbb{S} \Rightarrow (\exists s \in S :\ (a, s) \notin \mathit{Pairs}_{\mathbb{S}})$.

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Example

- $\mathbb{S} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ is tight.
- $\mathbb{T} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ is not tight, as $(\{a, b\} \cup \{c\}) \notin \mathbb{T}$, but $(a, c), (b, c) \in \mathit{Pairs}_{\mathbb{T}}$.

Intuition behind tight: Limitation of the multitude of incomparable extensions.

Results on Conflict-free Sets

Canonical Argumentation Framework

Given an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$, we define

$$F^{cf}_{\mathbb{S}} = \big(\mathit{Args}_{\mathbb{S}}, (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}}\big).$$

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For each non-empty, downward-closed, and tight extension-set $\mathbb{S}$, it holds that $\mathit{cf}(F_{\mathbb{S}}^{cf}) = \mathbb{S}$.

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Canonical Argumentation Framework

Given an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$, we define

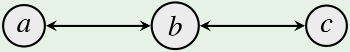
$$F^{\mathit{cf}}_{\mathbb{S}} = \big(\mathit{Args}_{\mathbb{S}}, (\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}}\big).$$

Theorem

For each non-empty, downward-closed, and tight extension-set $\mathbb{S}$, it holds that $\mathit{cf}(F^{\mathit{cf}}_{\mathbb{S}}) = \mathbb{S}$.

Example

$F^{\mathit{cf}}_{\mathbb{S}}$ with $\mathbb{S} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}\}$:



Results on Stable Semantics

Theorem

For each AF $F = (A, R)$, *stb*(F) is an incomparable and tight extension-set.

Results on Stable Semantics

Theorem

For each AF $F = (A, R)$, $\mathit{stb}(F)$ is an incomparable and tight extension-set.

Theorem

For each incomparable and tight extension-set $\mathbb{S}$, there exists an AF F such that $\mathit{stb}(F) = \mathbb{S}$.

Idea: Adapt the canonical argumentation framework (for $\mathbb{S} \neq \emptyset$) to:

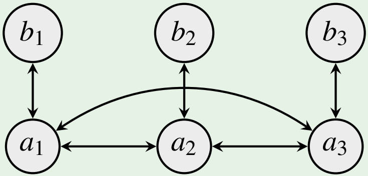
$$F^{\mathit{st}}_{\mathbb{S}} = \left(\mathit{Args}_{\mathbb{S}} \cup \{\bar{E} \mid E \in \mathbb{X}\}, R^{\mathit{st}}_{\mathbb{S}}\right), \quad \text{where}$$

$$\begin{aligned}
\mathbb{X} &= \mathit{stb}(F^{\mathit{cf}}_{\mathbb{S}}) \setminus \mathbb{S} \\
R^{\mathit{st}}_{\mathbb{S}} &= ((\mathit{Args}_{\mathbb{S}} \times \mathit{Args}_{\mathbb{S}}) \setminus \mathit{Pairs}_{\mathbb{S}}) \cup \\
&\quad \{(\bar{E}, \bar{E}), (a, \bar{E}) \mid E \in \mathbb{X}, a \in \mathit{Args}_{\mathbb{S}} \setminus E\}
\end{aligned}$$

Then $\mathit{stb}(F^{\mathit{st}}_{\mathbb{S}}) = \mathbb{S}$.

Example

$F^{st}_{\mathbb{S}'}$ with $\mathbb{S}' = \{\{a_1, b_2, b_3\}, \{a_2, b_1, b_3\}, \{a_3, b_1, b_2\}\}$:



- $\mathbb{X} = \mathit{stb}(F^{cf}_{\mathbb{S}}) \setminus \mathbb{S} = \{\{b_1, b_2, b_3\}\}$

Results on Stable Semantics

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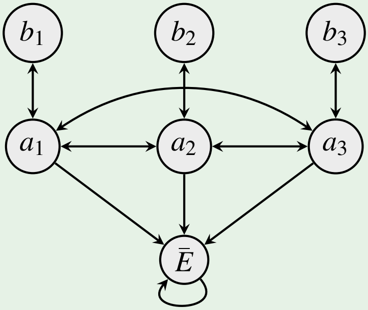

- $\mathbb{X} = stb(F^{cf}_{\mathbb{S}}) \setminus \mathbb{S} = \{\{b_1, b_2, b_3\}\}$

Results on Preferred Semantics

Definition

Given an extension-set $\mathbb{S}$, we call $\mathbb{S}$ pref-closed if for each $A, B \in \mathbb{S}$ with $A \neq B$, there exist $a, b \in (A \cup B)$ such that $(a, b) \notin \mathit{Pairs}_{\mathbb{S}}$.

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Example

- $\mathbb{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ is pref-closed.
- $\mathbb{T} = \{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$ is not pref-closed, since $\forall s_1, s_2 \in (\{a, d, e\} \cup \{a, b, d\})$ it holds that $(s_1, s_2) \in \mathit{Pairs}_{\mathbb{T}}$.

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Theorem

For each AF $F = (A, R)$, $\mathit{pref}(F)$ is a non-empty and pref-closed extension-set.

Results on Preferred Semantics

Defense Formula

Given extension-set $\mathbb{S}$ and $a \in \mathit{Args}_{\mathbb{S}}$, the defense-formula $\mathit{Def}_a^{\mathbb{S}} = \top$ if $\{a\} \in \mathbb{S}$, otherwise

$$\mathit{Def}_a^{\mathbb{S}} = \bigvee_{S \in \mathbb{S} s.t. a \in S} \quad \bigwedge_{b \in S \setminus \{a\}} b$$

$\mathit{Def}_a^{\mathbb{S}}$ converted to conjunctive normal form: CNF-defense-formula $\mathit{CDef}_a^{\mathbb{S}}$

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Example

Let $\mathbb{S} = \{\{b, c\}, \{a, c, d\}\}$.

$$\mathit{Def}_a^{\mathbb{S}} = c \wedge d \qquad \mathit{CDef}_a^{\mathbb{S}} = \{\{c\}, \{d\}\}$$

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$$\mathit{Def}_b^{\mathbb{S}} = c \qquad \mathit{CDef}_b^{\mathbb{S}} = \{\{c\}\}$$

Defense Formula

Given extension-set $\mathbb{S}$ and $a \in Args_{\mathbb{S}}$, the defense-formula $Def_a^{\mathbb{S}} = \top$ if $\{a\} \in \mathbb{S}$, otherwise

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$$\begin{aligned} Def_a^{\mathbb{S}} &= c \wedge d & CDef_a^{\mathbb{S}} &= \{\{c\}, \{d\}\} \\ Def_b^{\mathbb{S}} &= c & CDef_b^{\mathbb{S}} &= \{\{c\}\} \\ Def_c^{\mathbb{S}} &= b \vee (a \wedge d) & CDef_c^{\mathbb{S}} &= \{\{a, b\}, \{b, d\}\} \end{aligned}$$

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Canonical Defense-Argumentation-Framework

Given an extension-set $\mathbb{S}$, we define $F_{\mathbb{S}}^{\mathit{def}} = (A_{\mathbb{S}}^{\mathit{def}}, R_{\mathbb{S}}^{\mathit{def}})$ with

$$A_{\mathbb{S}}^{\mathit{def}} = A_{\mathbb{S}}^{\mathit{cf}} \cup \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{\alpha_{a,\gamma} \mid \gamma \in \mathit{CDef}_a^{\mathbb{S}}\}, \text{ and}$$

$$R_{\mathbb{S}}^{\mathit{def}} = R_{\mathbb{S}}^{\mathit{cf}} \cup \bigcup_{a \in \mathit{Args}_{\mathbb{S}}} \{(b, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, a) \mid \gamma \in \mathit{CDef}_a^{\mathbb{S}}, b \in \gamma\}.$$

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Example: Let $\mathbb{S} = \{\{b, c\}, \{a, c, d\}\}$.

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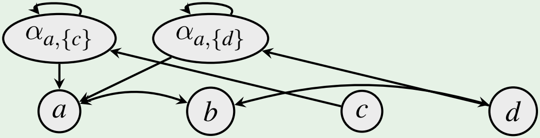

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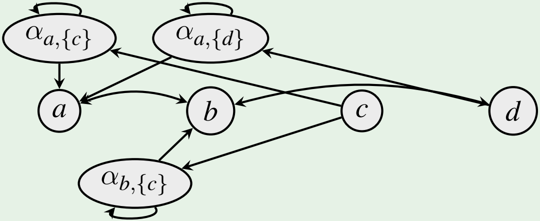
$$R^{def}_{\mathbb{S}} = R^{cf}_{\mathbb{S}} \cup \bigcup_{a \in Args_{\mathbb{S}}} \{(b, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, \alpha_{a,\gamma}), (\alpha_{a,\gamma}, a) \mid \gamma \in CDef^{\mathbb{S}}_a, b \in \gamma\}.$$

Example: Let $\mathbb{S} = \{\{b, c\}, \{a, c, d\}\}$. $CDef^{\mathbb{S}}_b = \{\{c\}\}$



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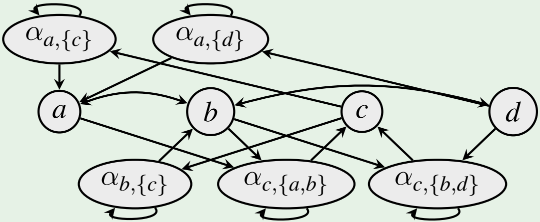

Results on Preferred Semantics

Canonical Defense-Argumentation-Framework

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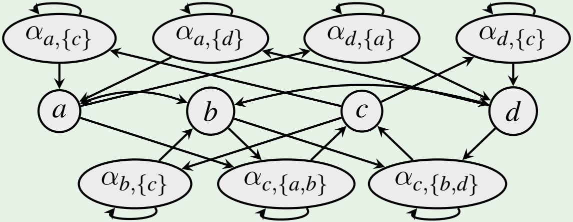
Example: Let $\mathbb{S} = \{\{b,c\},\{a,c,d\}\}$. $CDef^{\mathbb{S}}_{d} = \{\{a\},\{c\}\}$



Results on Preferred Semantics

Theorem

For each non-empty and pref-closed extension-set $\mathbb{S}$, it holds that $\mathit{pref}(F^{\mathit{def}}_{\mathbb{S}}) = \mathbb{S}$.

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For each non-empty and pref-closed extension-set $\mathbb{S}$, it holds that *pref*$(F^{\mathit{def}}_{\mathbb{S}}) = \mathbb{S}$.

Example

$\mathbb{S} = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$ is pref-closed and therefore *pref*$(F^{\mathit{def}}_{\mathbb{S}}) = \mathbb{S}$. Since $\mathbb{S}$ is not tight, $\mathbb{S}$ is not realizable under naive and stable semantics.

$\mathbb{T} = \{\{a, d, e\}, \{b, c, e\}, \{a, b, d\}\}$ is not pref-closed, therefore $\mathbb{T}$ is not realizable under preferred semantics.

Signatures

Definition

The signature of a semantics σ is defined as $\Sigma_\sigma = \{\sigma(F) \mid F \in AF_{\mathfrak{A}}\}$.

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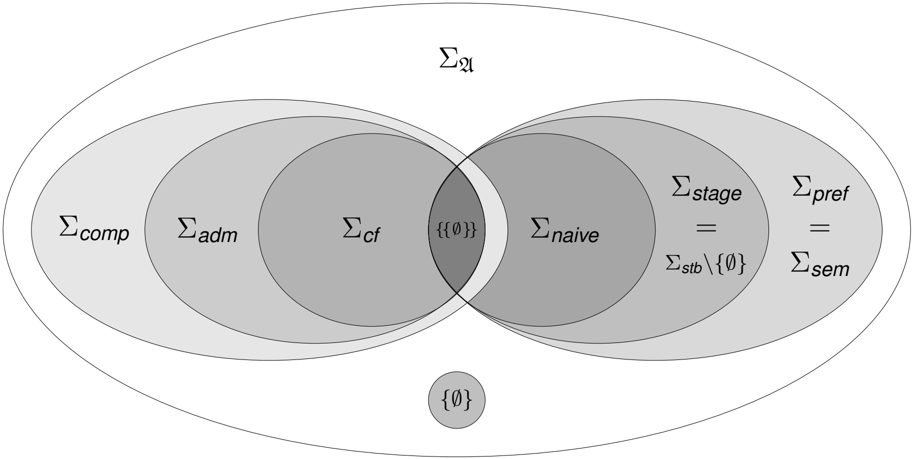
Theorem

$$\begin{aligned}
\Sigma_{cf} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is downward-closed and tight}\} \\
\Sigma_{naive} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is incomparable and } dcl(\mathbb{S}) \text{ is tight}\} \\
\Sigma_{stb} &= \{\mathbb{S} \mid \mathbb{S} \text{ is incomparable and tight}\} \\
\Sigma_{stage} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is incomparable and tight}\} \\
\Sigma_{adm} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is adm-closed and contains } \emptyset\} \\
\Sigma_{pref} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is pref-closed}\} \\
\Sigma_{sem} &= \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is pref-closed}\}
\end{aligned}$$

Relations between Signatures



$\Sigma_{\mathfrak{A}} = \{\mathbb{S} \subseteq 2^{\mathfrak{A}} \mid Args_{\mathbb{S}} \text{ is finite}\}$

Conclusions

For all main semantics we show properties, which always hold for extension-sets, and conditions for realizability. As they coincide we get exact characterizations of their signatures.

Results on realizability under the various semantics can be used for:

- Checking realizability as first step when considering dynamics.
- Constructions of canonical argumentation frameworks.

Characterizations of signatures of semantics tell us about the expressiveness of semantics.

- Comparison of expressiveness.
- Pruning of search-space possible in implementations of argumentation semantics.

Future Work

- Characterizations of strict signatures.

$$\Sigma_\sigma^s = \left\{ \sigma(F) \mid F \in AF_{\mathfrak{A}} \text{ with } A_F = Args_{\sigma(F)} \right\}.$$

- Research on realizability and signatures of
  - Signatures of other extension-based semantics, such as complete, cf2 [Baroni et al., 2005], and resolution-based grounded [Baroni et al., 2011].
  - Labelling-based semantics [Caminada and Gabbay, 2009].
  - Extensions to Dung's argumentation frameworks (ADFs [Brewka and Woltran, 2010], . . .).

Related Work: intertranslatability [Dvořák and Woltran, 2011], principle-based evaluation [Baroni and Giacomin, 2007], enforcing [Baumann and Brewka, 2010].

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