On the Complexity of Computing the Justification Status of an Argument◇

dbai Research Seminar, Vienna

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Motivation

We adress the problem of:

Determining the acceptance status of an argument in abstract argumentation (Given a semantics for computing the extensions).

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- Traditional: Skeptical and/or Credulous Acceptance.
- Wu and Caminada recently proposed a new approach: The Justification Status of an Argument.
- Their original approach is stated in terms of complete semantics. $\hookrightarrow$ We generalize it to arbitrary semantics
- Computational issues where neglected. $\hookrightarrow$ We provide an comprehensive complexity analysis.

Outline

Dung's Abstract Argumentation Frameworks

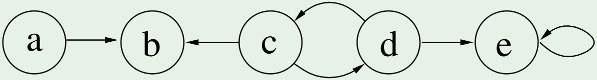
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example

F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})



Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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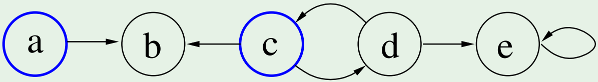

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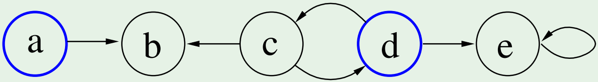

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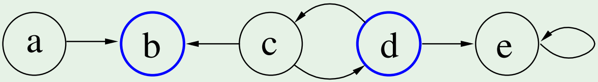

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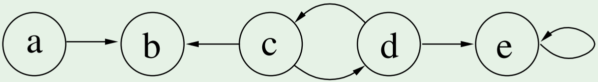

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Basic Properties (ctd.)

Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

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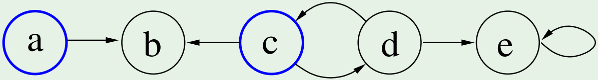

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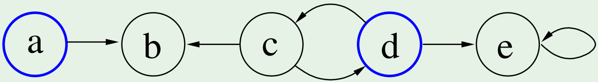

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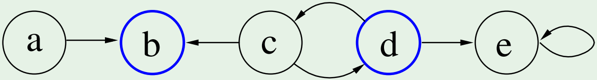

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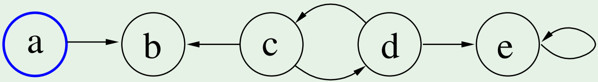


$adm(F) = \{\{a, c\}, \{a, d\}, \sout{\{b, d\}}, \{a\}, \sout{\{b\}}, \{c\}, \{d\}, \emptyset\}$

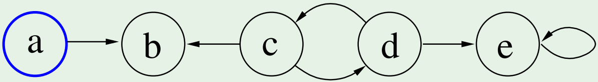
Semantics (ctd.)

Grounded Extension [Dung, 1995]

Given an AF (A, R). The unique grounded extension is defined as the smallest set S such that:

- each argument $a \in A$ which is not attacked in F belongs to S
- each $a \in A$ defended by S in F is contained in S

Example



$ground(F) = \{\{a\}\}$

Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF (A, R). A set $S \subseteq A$ is complete in F, if

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- each $a \in A$ defended by S in F is contained in S
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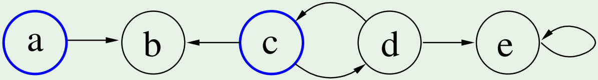

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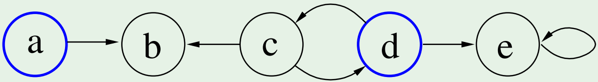

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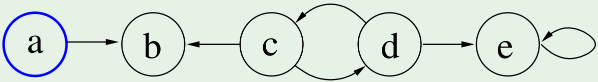

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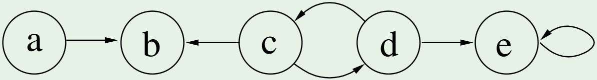

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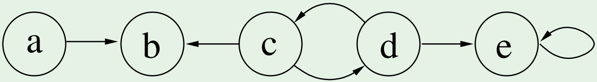
Semantics (ctd.)

Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



$\mathit{pref}(F) = \{\{a, c\}, \{a, d\},$ ~~$\{a\}, \{c\}, \{d\}, \emptyset$~~ $\}$

Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

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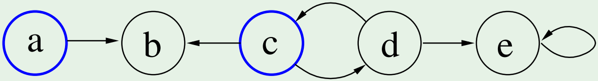

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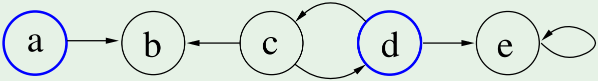

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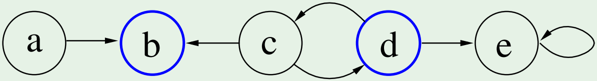

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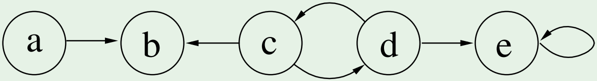

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Argumentation Labelings

Let $F = (A, R)$ be an AF.

Definition

A labeling for F is a function $\mathcal{L} : A \rightarrow \{in, out, undec\}$. We denote labelings by triples $(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec})$, with $\mathcal{L}_l = \{a \in A \mid \mathcal{L}(a) = l\}$.

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The range of a set $S \subseteq A$ is defined as $S_R^+ = S \cup \{b \mid \exists a \in S : (a, b) \in R\}$.
We define the induced labeling $\mathrm{Ext2Lab}_F(E)$ of an extension $E \subseteq A$:

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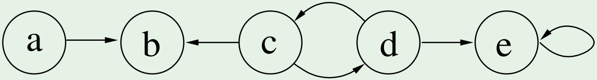
Definition

Let σ be an extension-based semantics. The corresponding labeling-based semantics $\sigma_{\mathcal{L}}$ is defined as $\sigma_{\mathcal{L}}(F) = \{\text{Ext2Lab}(E) \mid E \in \sigma(F)\}$.

Argumentation Labelings - Example

Example



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The *complete labelings* are:

- $(\{a\}, \{b\}, \{c, d, e\})$,
- $(\{a, c\}, \{b, d\}, \{e\})$,
- $(\{a, d\}, \{b, c, e\}, \{\})$

Justification Status of an Argument

Definition

Let $F = (A, R)$ be an AF and σ a semantic. The justification status of an $a \in A$ wrt σ is defined as $\mathcal{JS}_\sigma(F, a) = \{\mathcal{L}(a) \mid \mathcal{L} \in \sigma_\mathcal{L}(F)\}$.

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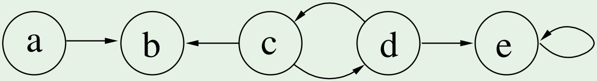

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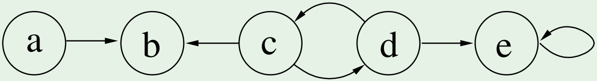

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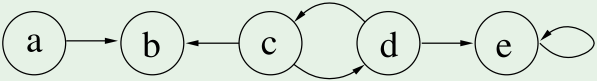

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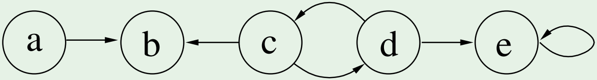

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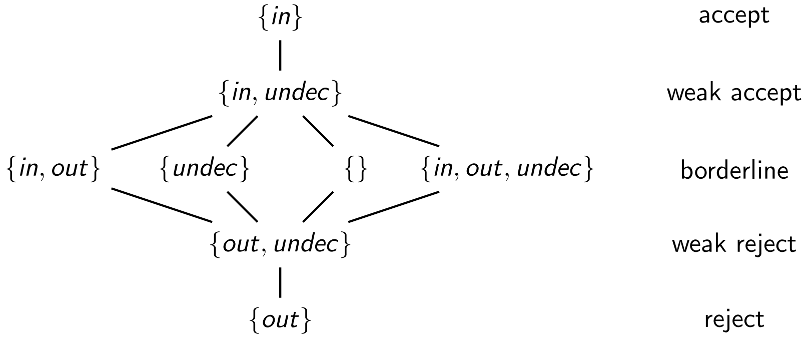
Possible Justification Statuses

Each element of $2^{\{in, out, undec\}}$ is a justification status:



Possible Justification Statuses

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Theorem

Let $F = (A, R)$ be an AF and $a \in A$. Then we have that:

- $\mathcal{JS}_{ground}(F, a) \in \{\{in\}, \{out\}, \{undec\}\}$
- $\mathcal{JS}_{adm}(F, a) \in \{\{undec\}, \{in, undec\}, \{out, undec\}, \{in, out, undec\}\}$
- $\mathcal{JS}_{comp}(F, a) \in 2^{\{in, out, undec\}} \setminus \{\emptyset, \{in, out\}\}$
- $\mathcal{JS}_{stable}(F, a) \in \{\{in\}, \{out\}, \{in, out\}, \{\}\}$
- $\mathcal{JS}_{pref}(F, a) \in 2^{\{in, out, undec\}} \setminus \{\emptyset\}$
- $\mathcal{JS}_{semi}(F, a) \in 2^{\{in, out, undec\}} \setminus \{\emptyset\}$
- $\mathcal{JS}_{stage}(F, a) \in 2^{\{in, out, undec\}} \setminus \{\emptyset\}$

Computational Complexity - Problems of interest

We are interested in the following two problems:

- The justification status decision problem JS_σ
  Given: AF $F = (A, R)$, $L \subseteq \{in, out, undec\}$ and argument $a \in A$.
  Question: Does $\mathcal{JS}_\sigma(F, a) = L$ hold?
- The generalized justification status decision problem GJS_σ
  Given: AF $F = (A, R)$, $L, M \subseteq \{in, out, undec\}$ and argument $a \in A$.
  Question: Does $L \subseteq \mathcal{JS}_\sigma(F, a)$ and $\mathcal{JS}_\sigma(F, a) \cap M = \emptyset$ hold?.

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To obtain completness for both problems we show

- membership for GJS_σ and
- hardness for JS_σ

Computational Complexity - Membership

Theorem

If the problem of verifying a σ-extension is in the complexity class $\mathcal{C}$ then the problem GJS_σ is in the complexity class $\mathrm{NP}^{\mathcal{C}} \wedge \text{co-NP}^{\mathcal{C}}$.

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Proof Ideas.

We provide a $\mathrm{NP}^{\mathcal{C}}$ algorithm to decide $L \subseteq \mathcal{JS}_\sigma(F, a)$

- For each $l \in L$ guess a labeling $\mathcal{L}_l$ with $\mathcal{L}_l(a) = l$
- Test whether $\mathcal{L}_l \in \sigma(F)$ or not, using the $\mathcal{C}$-oracle.
- Accept if for each $l \in L$, $\mathcal{L}_l \in \sigma(F)$

and a $\text{co-NP}^{\mathcal{C}}$ algorithm to decide $\mathcal{JS}_\sigma(F, a) \cap M = \emptyset$,

- For each $l \in M$ guess a labeling $\mathcal{L}_l$ with $\mathcal{L}_l(a) = l$
- Test whether $\mathcal{L}_l \in \sigma(F)$ or not
- Accept if there exists an $l \in M$ such that $\mathcal{L}_l \in \sigma(F)$

□

Computational Complexity - Hardness

Theorem

The problems JS_{comp}, GJS_{comp}, JS_{adm}, GJS_{adm} *are* DP-*hard, i.e.* NP $\wedge$ co-NP-*hard.*

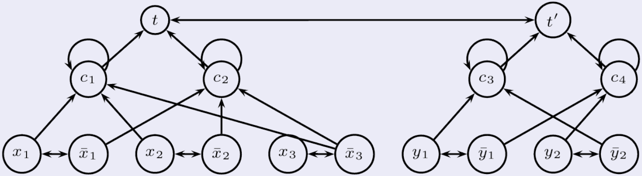
Computational Complexity - Hardness

Theorem

The problems JS_{comp}, GJS_{comp}, JS_{adm}, GJS_{adm} are DP*-hard, i.e.* NP $\wedge$ co-NP*-hard.*

Proof Idea.

We prove hardness by reducing the (DP-hard) SAT-UNSAT problem to JS_{comp} (resp. JS_{adm}).



The reduction builds on slightly modified standard translations of both formulas and adds a mutual attack between them. □

Computational Complexity

σ	*ground*	*adm*	*comp*	*stable*	*pref*	*semi*	*stage*
Cred_σ	P-c	NP-c	NP-c	NP-c	NP-c	Σ_2^p-c	Σ_2^p-c
Skept_σ	P-c	trivial	P-c	co-NP/DP-c	Π_2^p-c	Π_2^p-c	Π_2^p-c
JS_σ	P-c	DP-c	DP-c	DP-c	$\mathrm{P}^{\Sigma_2^p[1]}$-c	DP_2-c	DP_2-c
GJS_σ	P-c	DP-c	DP-c	DP-c	$\mathrm{P}^{\Sigma_2^p[1]}$-c	DP_2-c	DP_2-c

Table: Complexity Results ($\mathcal{C}$-c denotes completeness for class $\mathcal{C}$)

Relations between the above complexity classes:

$$\mathrm{P} \subseteq \begin{matrix}\mathrm{NP} \\ \text{co-NP}\end{matrix} \subseteq \mathrm{DP} \subseteq \begin{matrix}\Sigma_2^p \\ \Pi_2^p\end{matrix} \subseteq \mathrm{P}^{\Sigma_2^p[1]} \subseteq \mathrm{DP}_2$$

Conclusion

- We generalised the concept of the justification status of an argument to arbitrary semantics.
- Using the Justification Status in general increases the complexity.
- Two sources of complexity:

  We have to determine that

  - some labels are in the justification status
  - some labels are not in the justification status
- There are several problem classes where these decision problems are easier, e.g. Credulous and Skeptical Acceptance.