Parametric Properties of Ideal Semantics

IJCAI 2011, Barcelona

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July 22, 2011

\textsuperscript{\dagger}Supported by the Vienna Science and Technology Fund (WWTF) under grant ICT08-028 and by the Austrian Science Fund (FWF) under grant P20704-N18.
Motivation

“Ideal semantics” as an alternative basis for skeptical reasoning in abstract argumentation [Dung, Mancarella and Toni, 2007].
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Ideal acceptance

Informally, ideal acceptance requires an argument to be in an admissible set all of whose arguments are also skeptically accepted.
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“Ideal semantics” as an alternative basis for skeptical reasoning in abstract argumentation [Dung, Mancarella and Toni, 2007].

Ideal acceptance

Informally, ideal acceptance requires an argument to be in an admissible set all of whose arguments are also skeptically accepted.

- Similar to the concept of prudent reasoning in nonmonotonic reasoning.
- The original proposal was couched in terms of preferred semantics.
- Has been applied to semi-stable semantics (⇒ eager semantics) [Caminada 2007].
2. Background

Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing “attacks”

Example

\[ F = (\{a, b, c, d\}, \{(a, b), (b, a), (a, c), (b, c), (c, d)\}) \]
Conflict-Free Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

\[ \mathcal{E}_{cf}(F) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}\} \]
2. Background

Argumentation Semantics (ctd.)

Admissible Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **admissible** in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is **defended** by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$E_{adm}(F) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, d\}\}$
Preferred Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of $F$, if
- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S \not\subseteq T$

Example

$\mathcal{E}_{pr}(F) = \{\emptyset, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
Argumentation Semantics (ctd.)

Ideal Semantics

Given an AF $F = (A, R)$. The ideal extension is the $\subseteq$-maximal admissible set, that is contained in all preferred extensions.

Example

- $\mathcal{E}_{pr}(F) = \{\{a, d\}, \{b, d\}\}$
- Credulous accepted arguments: $\{a, b, d\}$
- Skeptical accepted arguments: $\{d\}$
- Ideal extension: $\emptyset$
Argumentation Semantics (landscape)

**Definition**

For an AF $F = \langle \mathcal{X}, \mathcal{A} \rangle$ we define the following semantics:

- $E_{cf}(F) = \{ S \subseteq \mathcal{X} \mid \forall x, y \in S, \langle x, y \rangle \not\in \mathcal{A} \}$
- $E_{adm}(F) = \{ S \in E_{cf}(F) \mid S \subseteq \mathcal{F}(S) \}$
- $E_{comp}(F) = \{ S \in E_{adm}(F) \mid \mathcal{F}(S) \subseteq S \}$
- $E_{gr}(F) = \mathcal{F}^k(\emptyset)$, for $k$ such that $\mathcal{F}^k(\emptyset) = \mathcal{F}^{k+1}(\emptyset)$
- $E_{naive}(F) = \{ S \in E_{cf}(F) \mid S \subseteq T \Rightarrow T \not\in E_{cf}(F) \}$
- $E_{pr}(F) = \{ S \in E_{adm}(F) \mid S \subseteq T \Rightarrow T \not\in E_{adm}(F) \}$
- $E_{sst}(F) = \{ S \in E_{adm}(F) \mid S \cup S^+ \subseteq T \cup T^+ \Rightarrow T \not\in E_{adm}(F) \}$
- $E_{stage}(F) = \{ S \in E_{cf}(F) \mid S \cup S^+ \subseteq T \cup T^+ \Rightarrow T \not\in E_{cf}(F) \}$
- $E_{gr^*}(F) = \min \bigcup_{\beta \in \gamma(<\mathcal{X}, \mathcal{A}>) \setminus \beta} \{ E_{gr}(\langle \mathcal{X}, \mathcal{A} \setminus \beta \rangle) \}$

Parametric Properties of Ideal Semantics
Parameterised Ideal Semantics

Let \( \langle X, \mathcal{A} \rangle \) be an AF and \( \sigma \) a semantics that for every AF promises at least one extension.

**Definition**

\( S \subseteq X \) is an **ideal set** w.r.t. base semantics \( \sigma \) of \( \langle X, \mathcal{A} \rangle \) iff:

1. \( S \in \mathcal{E}_{adm}(\langle X, \mathcal{A} \rangle) \)
2. \( S \subseteq \bigcap_{T \in \mathcal{E}_\sigma(\langle X, \mathcal{A} \rangle)} T \)

\( S \) is an **ideal extension** wrt \( \sigma \), if \( S \) is a \( \subseteq \)-maximal ideal set wrt \( \sigma \).
Parameterised Ideal Semantics

Let $\langle \mathcal{X}, \mathcal{A} \rangle$ be an AF and $\sigma$ a semantics that for every AF promises at least one extension.

**Definition**

$S \subseteq \mathcal{X}$ is an ideal set w.r.t. base semantics $\sigma$ of $\langle \mathcal{X}, \mathcal{A} \rangle$ iff:

- $S \in \mathcal{E}_{adm}(\langle \mathcal{X}, \mathcal{A} \rangle)$
- $S \subseteq \bigcap_{T \in \mathcal{E}_\sigma(\langle \mathcal{X}, \mathcal{A} \rangle)} T$

$S$ is an ideal extension wrt $\sigma$, if $S$ is a $\subseteq$-maximal ideal set wrt $\sigma$.

Some Notation:

- $E_{\sigma}^{ie}$ denotes an ideal extension wrt $\sigma$.
- $\sigma^{ie}$ denotes the corresponding semantics.
Parameterised Ideal Semantics - Basic Properties

We show that standard properties of classical ideal semantics continue to hold for any “reasonable” extension-based base-semantics $\sigma$. 

- **Theorem**
  - If every $\sigma$-extension is conflict-free then $\sigma$ is a unique status semantics.

- **Theorem**
  - If $\sigma$ satisfies the reinstatement property $a$ then the ideal extension $E_{\sigma}$ is a complete extension.

A semantics $\sigma$ satisfies reinstatement if for every AF $\langle X, A \rangle$ and $E \in E(\sigma)(X, A)$, we have that if $E$ defends $x \in X$ then $x \in E$. 

3. Parameterised Ideal Semantics
Parameterised Ideal Semantics - Basic Properties

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*If every $\sigma$-extension is conflict-free then $\sigma^{ie}$ is a unique status semantics.*
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**Theorem**

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$^a$A semantics $\sigma$ satisfies reinstatement iff for every AF $\langle X, A \rangle$ and $E \in E_\sigma(X, A)$, we have that if $E$ defends $x \in X$ then $x \in E$. 
Parameterised Ideal Semantics - Algorithmic Aspects

Algorithms

We provide two algorithms for computing ideal extensions:

- A generalisation of the algorithm presented by Dunne (2009) that uses a proof procedure for $CA_\sigma$.
- A new algorithm using proof procedures for $SA_\sigma$.

Computational Complexity

- We give generic upper bounds for the complexity of several decision problems associated with ideal semantics.
- Moreover we provide generic hardness results for some of the decision problems.
We study several instantiations of parametric ideal semantics and the relations between those.

Theorem

For any AF $F = \langle X, A \rangle$ the following $\subseteq$-relations hold:

$$E_{comp}^{IE}(F) \subseteq E_{gr}(F) \subseteq E_{pr}^{IE}(F) \subseteq E_{sst}^{IE}(F)$$

We have that:

- $E_{comp}^{IE}(F)$ is the grounded semantics.
- $E_{pr}^{IE}(F)$ is the standard ideal semantics.
- $E_{sst}^{IE}(F)$ is the eager semantics.
## Complexity Landscape

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\text{VER}_{\sigma}^{idl}$</th>
<th>$\text{CA}_{\sigma}^{idl}$</th>
<th>$\text{NE}_{\sigma}^{idl}$</th>
<th>$\text{VER}_{\sigma}^{ie}$</th>
<th>$\text{CONS}_{\sigma}^{ie}$</th>
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<td>in L</td>
<td>P-c</td>
<td>in FP</td>
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<tr>
<td>$\text{pr}$</td>
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<td>in $\Theta_2^P$</td>
<td>in $\Theta_2^P$</td>
<td>in $\Theta_2^P$</td>
<td>FP$_{\parallel}^{NP}$-c</td>
</tr>
<tr>
<td>$\text{sst}$</td>
<td>$\Pi_2^P$-c</td>
<td>$\Pi_2^P$-c</td>
<td>$\Pi_2^P$-c</td>
<td>DP$_2$-c</td>
<td>FP$_{\parallel}^{\Sigma_2^P}$-c</td>
</tr>
<tr>
<td>$\text{stage}$</td>
<td>$\Pi_2^P$-c</td>
<td>$\Pi_2^P$-c</td>
<td>$\Pi_2^P$-c</td>
<td>DP$_2$-c</td>
<td>FP$_{\parallel}^{\Sigma_2^P}$-c</td>
</tr>
<tr>
<td>$\text{gr*}$</td>
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<td>co-NP-c</td>
<td>co-NP-c</td>
<td>DP-c</td>
<td>FP$_{\parallel}^{NP}$-c</td>
</tr>
<tr>
<td>$\text{naive}$</td>
<td>in L</td>
<td>P-c</td>
<td>P-c</td>
<td>P-c</td>
<td>in FP</td>
</tr>
</tbody>
</table>
Conclusion

In this work we:

- Argue that the notion of "ideal acceptability" is applicable to arbitrary semantics.
- Justify this claim by showing that standard properties of classical ideal semantics continue to hold.
- Categorise the relationship between the diverse concepts of "ideal extension wrt semantics $\sigma$".
- Give a comprehensive analysis of algorithmic and complexity issues.
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- Give a comprehensive analysis of algorithmic and complexity issues.

Future research directions:

- Ideal Reasoning in generalizations of AFs (VAF, EAF, AFRA)
- in particular: Uncontested Semantics for Value-based Argumentation