On the Intertranslatability of Argumentation Semantics◇

London Argumentation Forum

Wolfgang Dvořák

Database and Artificial Intelligence Group
Institut für Informationssysteme
Technische Universität Wien

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What I'm doing in my PhD

Computational Aspects of Abstract Argumentation

1. Complexity Analysis: Studying worst case complexity of the common reasoning problems w.r.t. different argumentation semantics.
2. Identifying Tractable Fragments of in general hard problems, i.e. classes of instances on which a reasoning tasks is tractable.
   $\hookrightarrow$ We also consider Fixed-Parameter Tractability.
3. Intertranslatability of argumentation semantics. A translation for semantics σ, σ' modifies each AF such that the σ-extensions of the AF correspond to the σ'-extensions of the modified AF.

Motivation

- “Plethora” of Argumentation Semantics
- Properties of different semantics are well understood, but relations (and translations) between them not “well“ investigated yet

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Why consider translations between Argumentation Semantics ?

- To reuse sophisticated solver for other semantics.
- Categorise semantics w.r.t. Expressibility.

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Why consider translations between Argumentation Semantics ?

- To reuse sophisticated solver for other semantics.
- Categorise semantics w.r.t. Expressibility.
- Merge AFs modeled with different semantics.
- Interchange AFs between agents (using different semantics).
- Further Meta-Argumentation applications . . .

Reuse Solvers via Translations

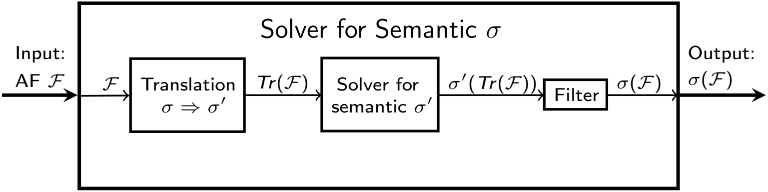

Figure: A Solver for a semantic σ, using a translation for $\sigma \Rightarrow \sigma'$

Expressibility

Expressibility vs. Computational Complexity

σ	Cred_σ	Skept_σ
ground	P-c	P-c
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c

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The complexity of a decision problem is **not** a fully satisfiying measure for the expressibility of a semantic.

Argumentation Frameworks

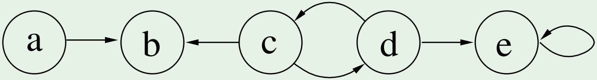
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing "attacks" ("defeats")

Example

F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})

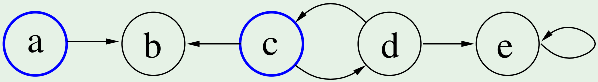


Argumentation Frameworks (ctd.)

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

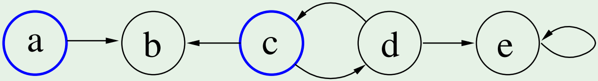
Semantics

Admissible Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \emptyset\}$

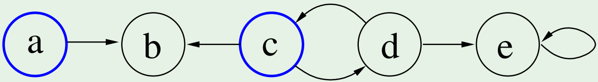
Semantics (ctd.)

Preferred Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



$pref(F) = \{\{a, c\}, \{a, d\}, \sout{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Semantics (ctd.)

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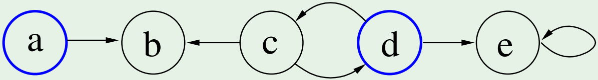


$\mathit{pref}(F) = \{\{a, c\}, \{a, d\},$ ~~$\{a\}, \{c\}, \{d\}, \emptyset$~~ $\}$

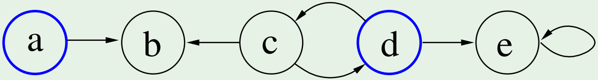
Semantics (ctd.)

Stable Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Example



$stable(F) = \{\sout{\{a, c\}}, \{a, d\}\}$

Translations

Definition

A *Translation* *Tr* is a function mapping (finite) AFs to (finite) AFs.

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We want translations to satisfy certain properties:

Basic Properties of a Translation Tr

- efficient: for every AF F, $Tr(F)$ can be computed using logarithmic space wrt. to $|F|$
- embedding: for any AF $F = (A, R)$: $A \subseteq A_{Tr(F)}$, $R = R_{Tr(F)} \cap (A \times A)$
- monotone: for any AFs F, F': $F \subseteq F'$ implies $Tr(F) \subseteq Tr(F')$

Translations

Next we connect translations with semantics.

“Levels of Faithfulness” (for semantics σ, σ')

- exact: for every AF F, $\sigma(F) = \sigma'(Tr(F))$
- faithful: for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.
- weakly exact: there is a fixed collection $\mathcal{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathcal{S}$;

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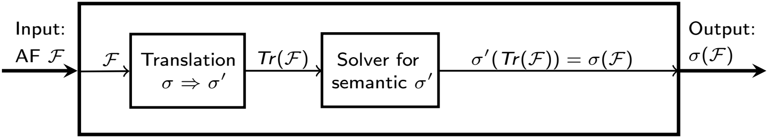

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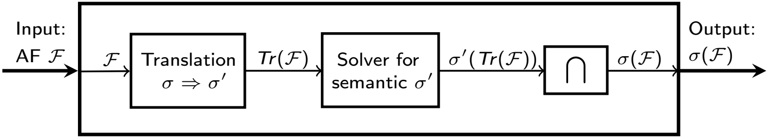

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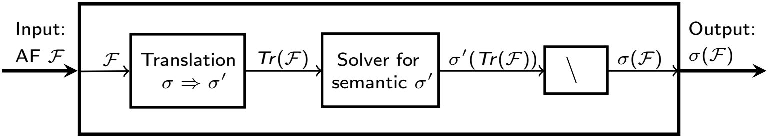


Contribution

Main Contributions:

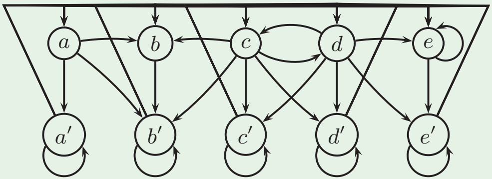
- Consider 7 of the most important semantics (Dung's original + two alternative)
- Provide (efficient) translations, whenever possible
- Impossibility results, in particular wrt. efficient translations.

Example Translation 1

Definition

For AF F, let $Tr_1(F) = (A^*, R^*)$ where $A^* = A_F \cup A'_F$ and $R^* = R_F \cup \{(b', a) \mid a, b \in A_F\} \cup \{(a', a'), (a, a') \mid a \in A_F\} \cup \{(a, b') \mid (a, b) \in R_F\}$.

Example



Result:

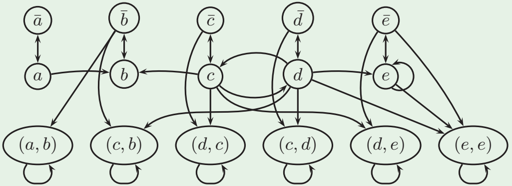
Tr_1 is a weakly exact translation for *stable* $\Rightarrow \sigma$ with $\sigma \in \{adm, pref\}$.

Example Translation 2

Definition

For AF F, $Tr_2(F) = (A^*, R^*)$ where $A^* = A_F \cup \bar{A}_F \cup R_F$ and
$R^* = R_F \cup \{(a, \bar{a}), (\bar{a}, a) \mid a \in A_F\} \cup \{(r, r) \mid r \in R_F\} \cup$
$\{(\bar{a}, r) \mid r = (y, a) \in R_F\} \cup \{(a, r) \mid r = (z, y) \in R_F, (a, z) \in R_F\}$.

Example



Result:

Tr_2 is a faithful translation for *adm* ⇒ *stable*.

Impossibility Results

Proposition

There is no (weakly) exact translation for $adm \Rightarrow \sigma$, $\sigma \in \{stable, pref\}$.

Admissible sets may be in a $\subset$ relation, while preferred (resp. stable) extensions are incomparable.

Impossibility Results

Proposition

There is no (weakly) exact translation for *adm* $\Rightarrow \sigma$, $\sigma \in \{\mathit{stable}, \mathit{pref}\}$.

Admissible sets may be in a $\subset$ relation, while preferred (resp. stable) extensions are incomparable.

Proposition

There is no efficient (weakly) faithful translation for *pref* $\Rightarrow \sigma$, $\sigma \in \{\mathit{adm}, \mathit{stable}\}$, unless $\Sigma_2^p = \mathrm{NP}$.

Follows from known complexity results.

Impossibility Results

proof sketch.

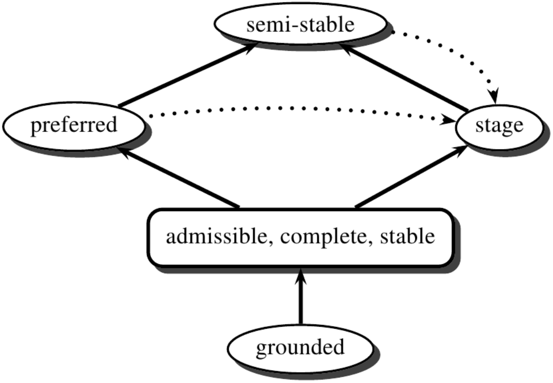
pref $\not\Rightarrow \sigma$, $\sigma \in \{adm, stable\}$ unless $\Sigma_2^p = \text{NP}$:

Given an efficient weakly faithful translation *Tr* with remainder collection S for *pref* $\Rightarrow \sigma$.
The problem Skept_{pref} is translated to the problem Skept_σ^S, deciding whether an argument is in each σ-extension which is not in the set S. As $\text{Ver}_\sigma \in \text{P}$, one can show that the problem Skept_σ^S is in co-NP (by standard guess and check).
But Skept_{pref} is Π_2^p-hard, while Skept_σ^S is co-NP-easy, thus $\Sigma_2^p = \text{NP}$. □

Results (Snapshot)

	admissible	stable	preferred
admissible	id	Tr_2 / -	$Tr_1 \circ Tr_2$ / -
stable	Tr_1	id	Tr_1
preferred	–	–	id

Results (big picture)



Intertranslatability w.r.t. (weakly) faithful translations

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Investigation of intertranslations between different semantics for abstract argumentation:

- complements results about comparing semantics
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W. Dvořák and S. Woltran.
On the Intertranslatability of Argumentation Semantics.
In *Proceedings of NonMon@30, 2010*