On the Intertranslatability of Argumentation Semantics◇

Argumentation Christmas Meeting (Vienna)

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Motivation (ctd.)

“Plethora” of Argumentation Semantics

- Properties of different semantics are well understood, but relations (and translations) between them not “well“ investigated yet
- Current Situation: Similar as NonMon in the late 80ies

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- To reuse sophisticated solver for other Semantics.
- Categorise Semantics w.r.t. Expressibility.

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Why consider translations between Argumentation Semantics ?

- To reuse sophisticated solver for other Semantics.
- Categorise Semantics w.r.t. Expressibility.
- Merge AFs modeled with different Semantics.
- Interchange AFs between agents (using different semantics).
- Other Multi-agent, Meta-Argumentation applications ...

Reuse Solvers via Translations

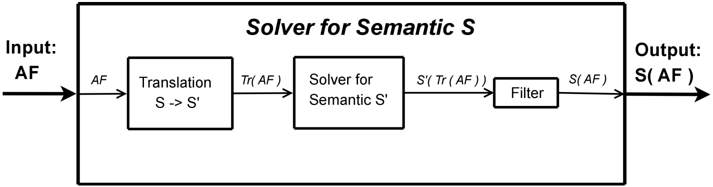

Figure: A Solver for a semantic S, using a translation for $S \Rightarrow S'$

Expressibility

Expressibility vs. Computational Complexity

σ	Cred_σ
ground	P-c
stable	NP-c
adm	NP-c
comp	NP-c
pref	NP-c
semi	Σ_2^p-c
stage	Σ_2^p-c

Expressibility

Expressibility vs. Computational Complexity

σ	Cred_σ	Skept_σ
ground	P-c	P-c
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c

Expressibility

Expressibility vs. Computational Complexity

σ	Cred_σ	Skept_σ	Ver_σ	Exists_σ	$\text{Exists}_\sigma^{\neg\emptyset}$
ground	P-c	P-c	P-c	trivial	in L
stable	NP-c	co-NP-c	in L	NP-c	NP-c
adm	NP-c	trivial	in L	trivial	NP-c
comp	NP-c	P-c	in L	trivial	NP-c
pref	NP-c	Π_2^p-c	co-NP-c	trivial	NP-c
semi	Σ_2^p-c	Π_2^p-c	co-NP-c	trivial	NP-c
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The complexity of a decision problem is not the appropriate measure for the expressibility of a semantic.

Outline

Argumentation Frameworks

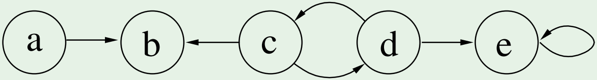
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing "attacks" ("defeats")

Example

F=({a,b,c,d,e} , {(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)})

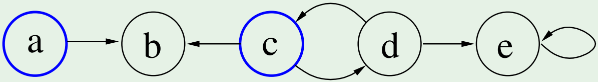


Argumentation Frameworks (ctd.)

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

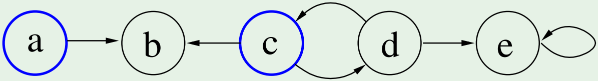
Argumentation Frameworks (ctd.)

Admissible Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$adm(F) = \{\{a, c\}, \{a, d\},$ ~~$\{b, d\}$~~$, \{a\},$ ~~$\{b\}$~~$, \{c\}, \{d\}, \emptyset\}$

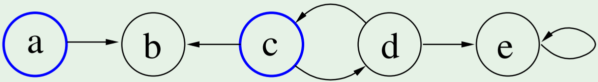
Argumentation Frameworks (ctd.)

Preferred Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



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Argumentation Frameworks (ctd.)

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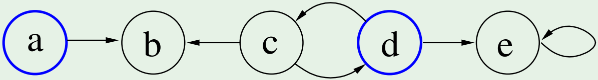

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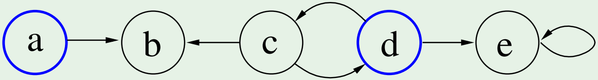
Semantics (ctd.)

Stable Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Example



$stable(F) = \{\sout{\{a, c\}}, \{a, d\}\}$

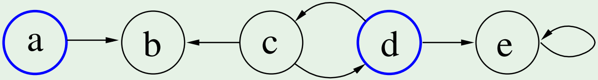
Semantics (ctd.)

Semi-Stable Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of F, if

- S is admissible in F
- the set $S^+ = S \cup \{a \in A \mid \exists b \in S : (b, a) \in R\}$ is $\subseteq$-maximal

Example



$semi(F) = \{\{a, d\}\}$

Translations

Definition

A *Translation* Tr is a function mapping (finite) AFs to (finite) AFs.

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We want translations to satisfy certain properties:

Basic Properties of a Translation Tr

- efficient: for every AF F, $Tr(F)$ can be computed using logarithmic space wrt. to $|F|$
- embedding: for any AF $F = (A, R)$: $A \subseteq A_{Tr(F)}$, $R = R_{Tr(F)} \cap (A \times A)$
- monotone: for any AFs F, F': $F \subseteq F'$ implies $Tr(F) \subseteq Tr(F')$

Translations

Next we connect translations with semantics.

"Levels of Faithfulness" (for semantics σ, σ')

- exact: for every AF F, $\sigma(F) = \sigma'(Tr(F))$
- faithful: for every AF F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.
- weakly exact: there is a fixed $\mathcal{S}$ of sets of arguments, such that for any AF F, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathcal{S}$;

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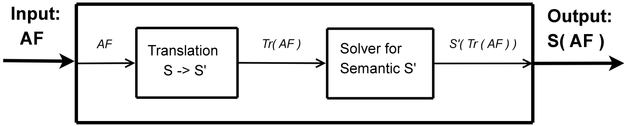

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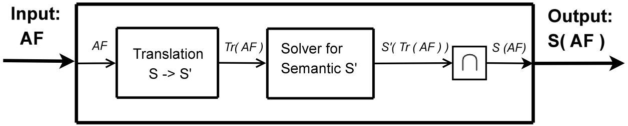

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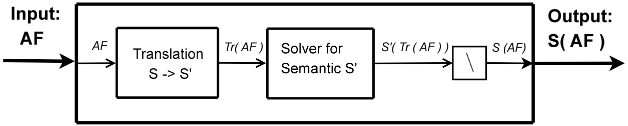


Contribution

Main Contributions:

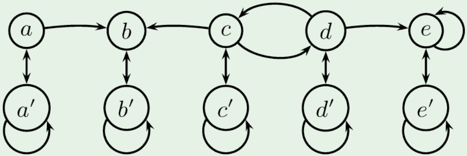
- Consider 7 of the most important semantics (Dung's original + two alternative)
- Provide (efficient) translations, whenever possible
- Impossibility results, in particular wrt. efficient translations.

Example Translation 1

Definition

For AF F, let $Tr_1(F) = (A^*, R^*)$ where $A^* = A_F \cup A'_F$ and $R^* = R_F \cup \{(a, a'), (a', a), (a', a') \mid a \in A_F\}$, with $A'_F = \{a' \mid a \in A_F\}$.

Example



Result:

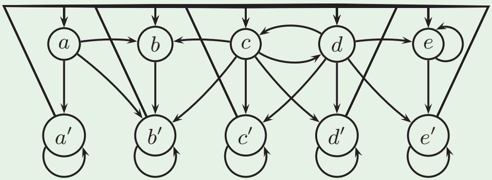
Tr_1 is an exact translation for *pref* $\Rightarrow$ *semi*.

Example Translation 2

Definition

For AF F, let $Tr_2(F) = (A^*, R^*)$ where $A^* = A_F \cup A'_F$ and $R^* = R_F \cup \{(b', a) \mid a, b \in A_F\} \cup \{(a', a'), (a, a') \mid a \in A_F\} \cup \{(a, b') \mid (a, b) \in R_F\}$.

Example



Result:

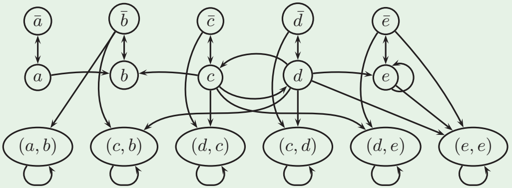
Tr_2 is a weakly exact translation for *stable* $\Rightarrow \sigma$ with $\sigma \in \{$*adm*, *pref*, *semi*$\}$.

Example Translation 3

Definition

For AF F, $Tr_3(F) = (A^*, R^*)$ where $A^* = A_F \cup \bar{A}_F \cup R_F$ and
$R^* = R_F \cup \{(a, \bar{a}), (\bar{a}, a) \mid a \in A_F\} \cup \{(r, r) \mid r \in R_F\} \cup$
$\{(\bar{a}, r) \mid r = (y, a) \in R_F\} \cup \{(a, r) \mid r = (z, y) \in R_F, (a, z) \in R_F\}$.

Example



Result:

Tr_3 is a faithful translation for *adm* $\Rightarrow \sigma$ with $\sigma \in \{$*stable*, *semi*$\}$.

Impossibility Results

Proposition

There is no (weakly) exact translation for *adm* $\Rightarrow \sigma$, $\sigma \in \{stable, pref, semi\}$.

Admissible sets may be in a $\subset$ relation, while preferred, stable and semi-stable extensions are incomparable.

Impossibility Results

Proposition

There is no (weakly) exact translation for $adm \Rightarrow \sigma$, $\sigma \in \{stable, pref, semi\}$.

Admissible sets may be in a $\subset$ relation, while preferred, stable and semi-stable extensions are incomparable.

Proposition

There is no efficient (weakly) faithful translation for

1. $pref \Rightarrow \sigma$, $\sigma \in \{adm, stable\}$,
2. $semi \Rightarrow \sigma$, $\sigma \in \{adm, stable, pref\}$,

unless $\Sigma_2^p = \mathrm{NP}$.

Follows from known complexity results (details on the next slide).

Impossibility Results

proof sketch.

1. *pref* $\not\Rightarrow \sigma$, $\sigma \in \{adm, stable\}$ unless $\Sigma_2^p = \mathrm{NP}$:

   Given an efficient weakly faithful translation Tr with remainder set S for *pref* $\Rightarrow \sigma$. Skept_{pref} is translated to the problem Skept_σ^S, deciding whether an argument is in each σ-extension which is not in the set S. One can show that the problem Skept_σ^S is in co-NP (by standard guess and check). But Skept_{pref} is Π_2^p-hard, while Skept_σ^S is co-NP-easy $\lightning$.

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2. *semi* $\not\Rightarrow \sigma$, $\sigma \in \{adm, stable, pref\}$ unless $\Sigma_2^p = \text{NP}$:

   Let Tr be an efficient (weakly) faithful translation for $semi \Rightarrow \sigma$. By definition Tr is L-computable and reduces Cred_{semi} to Cred_σ. But Cred_{semi} is Σ_2^p-hard, while Cred_σ is NP-easy ↯.

□

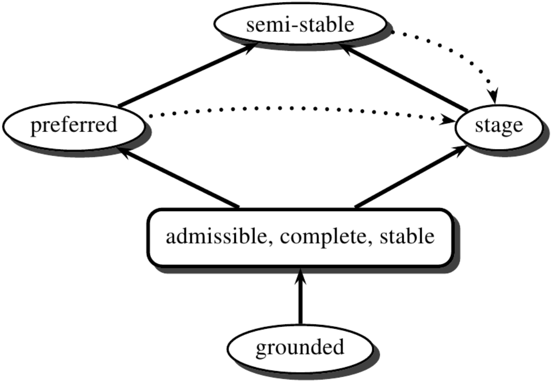
Results (Snapshot)

	admissible	stable	preferred	semi-stable
admissible	id	Tr_3 / -	$Tr_2 \circ Tr_3$ / -	Tr_3 / -
stable	Tr_2	id	Tr_2	Tr_2
preferred	–	–	id	Tr_1
semi-stable	–	–	–	id

Results (in the paper)

	ground	**adm**	**stable**	*comp*	**pref**	**semi**	*stage*
ground	id	$\surd$ / -	$\surd$ / -	$\surd$ / -	$\surd$ / ?	$\surd$ / ?	$\surd$ / ?
adm	–	id	Tr_3 / -	Tr_1	$Tr_2 \circ Tr_3$ / -	Tr_3 / -	Tr_3 / -
stable	–	Tr_2	id	Tr_2	Tr_2	Tr_2	$\surd$
comp	–	$\surd$ / -	$\surd$ / -	id	$\surd$ / -	$\surd$ / -	$\surd$ / -
pref	–	–	–	–	id	Tr_1	?
semi	–	–	–	–	–	id	?
stage	–	–	–	–	–	$\surd$	id

Results (in the paper) ctd.



Intertranslatability w.r.t. (weak) faithful translations

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Investigation of intertranslations between different semantics for abstract argumentation:

- complements results about comparing semantics
- provides new insight into "meta-argumentation" (express semantical concepts within argumentation frameworks)

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- extend to other important semantics

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