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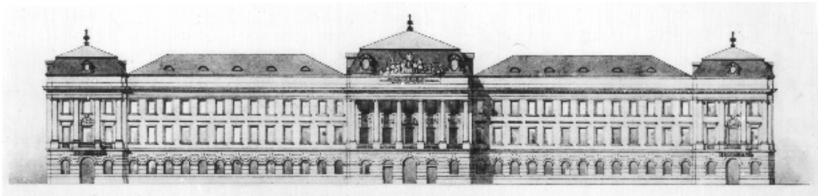

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Shaping Abstract Argumentation for the Argumentation Pipeline – How to avoid Multiple Arguments with the same Claim

DBAI-TR-2019-115

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DBAI TECHNICAL REPORT
2019

TECHNISCHE UNIVERSITÄT WIEN
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DBAI TECHNICAL REPORT
DBAI TECHNICAL REPORT DBAI-TR-2019-115, 2019

Shaping Abstract Argumentation for the Argumentation Pipeline – How to avoid Multiple Arguments with the same Claim

Wolfgang Dvořák [1] **Anna Rapberger** [2] **Stefan Woltran** [3]

Abstract. Abstract argumentation frameworks are a constitutive formalism within a more general argumentation process, where arguments and conflicts are instantiated from a given knowledge base and acceptable sets of arguments are obtained solely from the resulting attack relation between arguments. Inspecting the claims of the accepted arguments is often considered as final step in this process. However, a certain subtlety occurs when different arguments with the same claim are constructed during the instantiation process. This not only gives rise to different views on acceptance but can also have an impact on the complexity of acceptance problems, as recently shown. A natural goal is thus to investigate under which circumstances such situations can be avoided, i.e. when one can express the result of the instantiation with standard formalisms where each argument represents a unique claim. We study this problem on the general level of claim-augmented argumentation frameworks (CAFs). As a main result we show that, for all standard semantics, the class of well-formed CAFs, where arguments with the same claim have the same outgoing attacks, can be equivalently represented as argumentation frameworks with collective attacks.

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1 Introduction

The formal analysis of human reasoning facing uncertain information and conflicting beliefs is an important research area within AI. Abstract argumentation, as introduced by Dung [13], has been established as an important tool to analyze and evaluate the raw structure of argumentation systems by treating arguments as abstract entities. Depending on the particular task, various instantiation processes are used to model discourses, medical and legal cases [4], but also logic programs and non-monotonic reasoning formalisms [13, 11].

The general schema is often referred to as the argumentation pipeline and involves: (1) instantiation of a problem into an abstract argumentation framework (AF); (2) application of semantics yielding sets of collectively acceptable arguments (the extensions of the AF); (3) re-interpretation of the extensions in terms of the original problem. Different instantiation processes have been established, see e.g. [17, 22, 11]. They all have in common that each generated argument possesses a statement (*claim*). Step (3) hence usually consists of inspecting the claims of the arguments occurring in the extensions. The entire process is implemented, for instance, in the TOAST system [23].

Example 1. *We consider an instantiation procedure using ASPIC$^+$ [22]. Let $\mathcal{K}_p = \{b, \overline{b}, c, \overline{c}\}$ be the set of premises, $\mathcal{K}_s = \{\overline{b} \to a, \overline{c} \to a\}$ be the set of strict rules and let the pairs $(b, \overline{b})$, $(c, \overline{c})$, $(\overline{b}, \overline{c})$ be contradictory. The arguments and the resulting AF are given as follows.*

Arg.	*Structure*	*claim*
A_1	b	b
A_2	$\overline{b}$	$\overline{b}$
A_3	c	c
A_4	$\overline{c}$	$\overline{c}$
A_5	$A_2 \to a$	a
A_6	$A_4 \to a$	a

Figure 1: Arguments in Example 1.

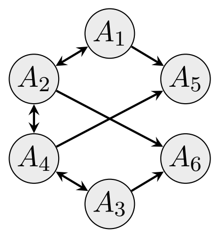

Figure 2: Resulting AF.

The evaluation of the AF in Example 1 under stable semantics[1] yields the sets $\{A_1, A_3\}$, $\{A_2, A_3, A_5\}$ and $\{A_1, A_4, A_6\}$. The re-interpretation in terms of claims gives us the sets $\{b, c\}$, $\{a, \overline{b}, c\}$ and $\{a, b, \overline{c}\}$.

Observing that arguments A_5 and A_6 refer to the same claim it is a natural question whether or not we can avoid such duplicates of claims. To put it in other words, to which extent can we simplify an AF such that the extensions (interpreted in terms of their claims) remain unchanged? It is evident that such questions are crucial for practical reasons, since smaller AFs can be processed more efficiently. In the best case, each claim then is represented by exactly one argument. This would avoid certain subtleties observed by several authors (see e.g. [18], Definition 2.18), namely that although a particular *claim* is covered by all extensions, there might be no *argument* which is contained in all extensions, making the skeptical acceptance problem on the AF level insufficient for a claim-centric view.

[1] A set S of arguments is stable if it attacks exactly those arguments which do not belong to S.

Coming back to our example, the affirmative answer to the question whether each claim can be represented by one argument is negative. In this case, we would need an AF over a, b, c, $\overline{b}$ and $\overline{c}$, such that the stable extensions are given by the sets $\{b, c\}$, $\{a, \overline{b}, c\}$ and $\{a, b, \overline{c}\}$. Results by Dunne et al. [14] on signatures show that such an AF cannot exist. However, recent results [15] on signatures for SETAFs (AFs with collective attacks [20]) show that a slightly more powerful abstract formalism is capable to deliver those sets as stable extensions.

Generally speaking, we are interested under which conditions one can express the result of an instantiation process in terms of an abstract formalism $\mathcal{F}$, such that the outcome of the original problem is directly given by the extensions of $\mathcal{F}$.

In the present paper, we utilize the concept of *claim-augmented argumentation frameworks*, CAFs for short, as introduced by Dvořák and Woltran [16] in order to base our studies independently from concrete instantiations. CAFs are AFs where arguments are associated with claims and semantics deliver those sets of claims attached to the arguments in the extensions under standard AF semantics. We will focus on the most common semantics, namely preferred, stable, complete, admissible and grounded semantics [13]. Compared to [16] who analyse the complexity of acceptance problems in terms of CAFs, we are interested in the *expressibility* of CAFs in order to understand in which situations we can avoid the necessity to have different arguments with the same claim.

To this end, we consider two independent restrictions on the attack relation in CAFs and investigate their effect on the expressibility of the CAF semantics. A CAF is *well-formed* if arguments with the same claim attack the same arguments (the AF from Example 1 is indeed well-formed). This constraint naturally occurs in instantiation processes: attacks are usually generated under consideration of the claim of the attacker.[2] The second restriction we study can be seen as dual to well-formed CAFs and requires arguments with the same claim to be attacked by the same arguments. We call such frameworks *attack-unitary CAFs*. Our main results are:

- We first provide a rewriting technique for CAFs in order to reduce the number of arguments with the same claim.

- We show that well-formed CAFs are equally powerful (w.r.t. the semantics under consideration) to SETAFs by providing translations in both directions. As a consequence, each well-formed CAF can be equivalently represented as SETAF where each argument refers to a unique claim.

- We strengthen this result for attack-unitary CAFs under admissibility-based semantics and we provide a transformation to standard AFs for this particular class.

- Finally, we show for unrestricted CAFs that the expressibility of the considered semantics increases drastically which indicates that compilation of arbitrary CAFs into purely abstract formalisms cannot be expected.

[2] Exceptions are instantiation procedures which allow rule and claim preferences (cf. ASPIC$^+$).

2 Preliminaries

In this section, we introduce argumentation frameworks [13] and recall the semantics we study (for a comprehensive introduction, see [5]). We fix U as countable infinite domain of arguments.

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where $A \subseteq U$ is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. Given an argument a, we use $a^+_R = \{b \mid (a, b) \in R\}$ and $a^-_R = \{b \mid (b, a) \in R\}$; we extend both notions to sets S as expected: $S^+_R = \bigcup_{a \in S} a^+_R$, $S^-_R = \bigcup_{a \in S} a^-_R$. Argument $a \in A$ is* defended *(in F) by $S \subseteq A$ if $a^-_R \subseteq S^+_R$.*

Semantics for AFs are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions cf, adm, com, grd, stb, and prf, which stand for conflict-free, admissible, complete, grounded, stable, and preferred extensions, respectively.

Definition 2. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F)*, if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of sets being conflict-free in F. For a conflict-free set $S \in cf(F)$, it holds that*

- *$S \in adm(F)$, if each $a \in S$ is defended by S in F;*
- *$S \in com(F)$, if $S \in adm(F)$ and each $a \in A$ defended by S in F is contained in S;*
- *$S \in grd(F)$, if $S = \bigcap_{T \in com(F)} T$;*
- *$S \in stb(F)$, if each $a \in A \setminus S$ is attacked by S in F;*
- *$S \in prf(F)$, if $S \in adm(F)$ and there is no $T \supset S$ such that $T \in adm(F)$.*

We recall that for each AF F, $stb(F) \subseteq prf(F) \subseteq com(F) \subseteq adm(F)$, and $grd(F)$ yields a unique extension. Moreover, semantics $\sigma \in \{stb, prf\}$ deliver *incomparable* sets, i.e. for all $S, T \in \sigma(F)$, $S \subseteq T$ implies $S = T$.

3 Argumentation Frameworks with Claims

Dvořák and Woltran [16] have recently introduced *claim-augmented argumentation frameworks*, CAFs for short, in order to analyse the complexity of abstract argumentation under a claim-centric view. The idea is to assign each argument in an AF a claim, i.e. an element from a countable infinite domain of claims C. Hence, different arguments can have the same claim, but no further information about claims is available. Figure 3 shows the CAF for Example 1.

Definition 3. *A* Claim-augmented Argumentation Framework (CAF) *is a triple $(A, R, claim)$ where (A, R) is an AF and $claim\colon A \to C$ assigns a claim to each argument of A.*

Semantics of CAFs are defined from the standard semantics of the underlying AF, but interpret the extensions in terms of the claims of their arguments. To this end, we extend the claim function to sets, i.e. $claim(S) = \{claim(s) \mid s \in S\}$.

Definition 4. *For a semantics σ, we define its claim-based variant σ_c as follows. For any CAF $CF = (A, R, claim)$, $\sigma_c(CF) = \{claim(S) \mid S \in \sigma((A, R))\}$. Given $S \in \sigma_c(CF)$, we say that $E \subseteq A$ is a σ-*realization *of S in CF if $claim(E) = S$ and $E \in \sigma((A, R))$.*

Example 2. *Let $CF = (A, R, claim)$ be given with (A, R) as depicted in Figure 4, and $claim(x_i) = x$, $claim(y_i) = y$ for $i = 1, 2$. We have $com_c(CF) = adm_c(CF) = \{\emptyset, \{x\}, \{x, y\}\}$, $grd_c(CF) = \{\emptyset\}$, and $stb_c(CF) = prf_c(CF) = \{\{x\}, \{x, y\}\}$. Note that $\{x\} \in adm_c(CF)$ has two adm-realizations, namely $E_1 = \{x_1\}$ and $E_2 = \{x_2\}$.*

Some basic relations between different semantics carry over from standard AFs. In fact, we have for any CAF CF

$$stb_c(CF) \subseteq prf_c(CF) \subseteq com_c(CF) \subseteq adm_c(CF) \tag{1}$$

and $grd_c(CF)$ is unique and contained in $com_c(CF)$. Moreover, the claim-based grounded extension $S \in grd_c(CF)$ is still the unique minimal claim-based complete extension.

As Example 2 shows, claim-based semantics loose some basic properties, for instance incomparability of stable and preferred extensions. However, this can be circumvented with a particular subclass called *well-formed CAFs* which has been defined in [16].

Definition 5. *A CAF $(A, R, claim)$ is called* well-formed *if $a^+_R = b^+_R$ for any $a, b \in A$ with $claim(a) = claim(b)$, i.e. arguments with the same claim attack the same arguments.*

Proposition 1. *For any well-formed CAF CF, $S \in prf_c(CF)$ iff $S \in com_c(CF)$ and there is no $T \in com_c(CF)$ with $T \supset S$.*

Proof. Consider a well-formed CAF $CF = (A, R, claim)$. We show that for $D, E \in com((A, R))$, $D \subseteq E$ iff $claim(D) \subseteq claim(E)$. The assertion then follows immediately. As $claim(.)$ is monotone we have that $D \subseteq E$ implies $claim(D) \subseteq claim(E)$. We next show the converse, i.e. that $claim(D) \subseteq claim(E)$ implies $D \subseteq E$. As CF is well-formed, $claim(D) \subseteq claim(E)$ implies $D^+_R \subseteq E^+_R$. That is all arguments defended by D in (A, R) are also defended by E in (A, R). Finally as D defends all its arguments and E contains all arguments it defends we have $D \subseteq E$. □

Notice that the above results generalise similar observations for instantiations from logic programming [11] and ABA [10].

Proposition 2. *For $\sigma \in \{stb, prf\}$ and every well-formed CAF $CF = (A, R, claim)$, we have (a) $\sigma_c(CF)$ is incomparable, and (b) $|\sigma((A, R))| = |\sigma_c(CF)|$.*

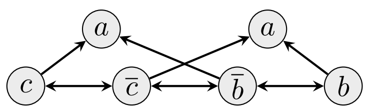

Figure 3: The AF F from Example 1 as CAF; here, claims are depicted instead of argument names.

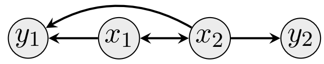

Figure 4: The AF from Example 2.

4 Translations

In this section, we provide our main results: we show that well-formed CAFs can be equivalently expressed without multiple claims when we allow for collective attacks (Section 4.2); moreover, we give conditions under which CAFs can be equivalently expressed as AFs, i.e. without multiple claims (Section 4.3). We start however, with some simplification steps for CAFs and define a canonical form.

4.1 A Normalform for CAFs

We first show that under certain conditions we can safely remove one of the arguments which have the same claim.

Definition 6. *Let* $CF = (A, R, claim)$ *be a CAF. Argument* $a \in A$ *is called* redundant *(in* CF*) w.r.t. argument* $b \in A$ *if* $a \neq b$*,* $claim(a) = claim(b)$*,* $a_R^+ = b_R^+$*, and* $a_R^- \supseteq b_R^-$*.*

In the CAF CF from Example 2, argument y_1 is redundant w.r.t. y_2: Indeed, we have that $y_1^+ = y_2^+ = \emptyset$ and $y_1^- \supseteq y_2^-$.

Proposition 3. *Let* $CF = (A, R, claim)$ *be a CAF,* $a \in A$ *be redundant in* CF *w.r.t. some* $b \in A$*, and let* $CF' = (A', R', claim)$ *with* $A' = A \setminus \{a\}$*, and* $R' = R \cap (A' \times A')$*. Then, for* $\sigma \in \{cf, adm, com, grd, stb, prf\}$*,* $\sigma_c(CF) = \sigma_c(CF')$*.*

Proof (for $\sigma = stb$*).* Let $S \in stb_c(CF)$ and let E be a stb-realization of S in CF, i.e. $E \in cf((A, R))$ and $E_R^+ = A \setminus E$. In case $a \notin E$, it is easy to see that $E \in cf((A', R'))$ and $E_{R'}^+ = A' \setminus E$; thus E is a stb-realization of S in CF'. In case $a \in E$, we observe that $b \in E$ holds, too: otherwise, $b \in E_R^+$ and therefore, by $a_R^- \supseteq b_R^-$, also $a \in E_R^+$. A contradiction to $E \in cf((A, R))$. We show that $E' = E \setminus \{a\}$ is a stb-realization of S in CF'. Clearly, $claim(E') = claim(E) = S$. Moreover, $E' \in cf((A', R'))$, and $E'^+_{R'} = A' \setminus E' = A \setminus E$. Hence, $E' \in stb((A', R'))$.

For the other direction, let $S \in stb_c(CF')$ and let E be a stb-realization of S in CF'. Clearly $E \in cf((A, R))$. If $a \in E_R^+$, we immediately get $E \in stb((A, R))$. So suppose $a \notin E_R^+$. Since $a_R^- \supseteq b_R^-$, $b \notin E_R^+$, and it follows that $b \in E$, since E is stable in (A', R'). We conclude that $(b, a) \notin R$ and thus, since $a_R^+ = b_R^+$, $(a, a) \notin R$. Moreover, as $b \notin E_R^-$ we have $a \notin E_R^-$ as well. That is, we have $(a, a) \notin R$, $a \notin E_R^+$, $a \notin E_R^-$, and thus $(E \cup \{a\}) \in cf((A, R))$. Moreover, in (A, R) the set $E \cup \{a\}$ attacks each argument $c \in A \setminus E$ and, since $b \in E$, $claim(E \cup \{a\}) = claim(E) = S$. Hence, $E \cup \{a\}$ is a stb-realization of S in CF. □

Definition 7. *A CAF* $CF = (A, R, claim)$ *is called* normalized *if there are no redundant arguments in* CF*.*

The following result is by repetitive application of Prop. 3.

Theorem 1. *Any CAF CF can be transformed into an normalized CAF CF', such that $\sigma_c(CF) = \sigma_c(CF')$, for $\sigma \in \{cf, adm, com, grd, stb, prf\}$.*

4.2 Expressing Well-formed CAFs as SETAFs

In well-formed CAFs, arguments with the same claim are indistinguishable in terms of their outgoing attacks. Hence, one can speak about claims attacking arguments, which one cannot in general CAFs. We will use this advantage to connect well-formed CAFs to a well-studied extension of AFs.

SETAFs and Collective Attacks. SETAFs, as introduced by Nielsen and Parsons [19], generalize the binary attack-relation in AFs to collective attacks of arguments. The formalism captures situations in which a single argument might be too weak to attack more powerful statements.

Definition 8. *A SETAF is a pair $SF = (A, R)$ where $A \subseteq U$ is finite, and $R \subseteq (2^A \setminus \{\emptyset\}) \times A$ is the attack relation.*

Given a SETAF $SF = (A, R)$, then $S \subseteq A$ *attacks* a if there is a set $S' \subseteq S$ with $(S', a) \in R$. S is *conflicting* in SF if S attacks some $a \in S$; S is *conflict-free* in SF, if S is not conflicting in SF, i.e. $S' \cup \{a\} \not\subseteq S$ for each $(S', a) \in R$. Finally, $a \in A$ is *defended* by S if for each set $B \subseteq A$ with $(B, a) \in R$, there is some $b \in B$ such that S attacks b. With these extended notions of conflict and defense, the semantics of AFs generalize to SETAFs as follows.

Definition 9. *Given a SETAF $SF = (A, R)$, we denote the set of all conflict-free sets in SF as $cf_s(SF)$. For $S \in cf_s(SF)$,*

- *$S \in adm_s(SF)$ if each $a \in S$ is defended by S in SF;*
- *$S \in com_s(SF)$, if $S \in adm_s(SF)$ and $a \in S$ for all $a \in A$ defended by S in SF;*
- *$S \in grd_s(SF)$, if $S = \bigcap_{T \in com_s(SF)} T$;*
- *$S \in stb_s(SF)$, if each $a \in A \setminus S$ is attacked by S in SF;*
- *$S \in prf_s(SF)$, if $S \in adm_s(SF)$ and there is no $T \in adm_s(SF)$ s.t. $T \supset S$.*

Towards our translation we introduce attack formulas as an alternative formalisation of the attack-structure in SETAFs.

Definition 10. *For any SETAF $SF = (A, R)$ and $a \in A$, let*

$$\mathcal{D}_a^{SF} = \bigvee_{B,(B,a) \in R} \bigwedge_{b \in B} b$$

denote the attack-formula *of a in SF. By $\mathcal{CD}_a^{SF}$ we denote any equivalent formula in CNF over the same set of variables. $\mathcal{CD}_a^{SF}$ is called CNF-attack-formula of a in SF.*

For each $s \in A$, the models of the attack-formula $\mathcal{D}_s^{SF}$ coincide with the sets $S \subseteq A$ such that S attacks s in SF. Using this identity, the semantics for SETAFs can be rephrased in terms of attack-formulas. For example, a set S is stable in SF, if for each $s \in A$, we have that $s \in S$ if and only if $\delta \not\subseteq S$ for all $\delta \in \mathcal{D}_s^{SF}$ (following standard conventions, we will occasionally identify formulas in CNF or DNF as a collection of sets of literals; in our case, atoms).

Translations. We show that each well-formed CAF can be reduced to an equivalent SETAF by identifying claims in well-formed CAFs with arguments in SETAFs. To this end, we will introduce attack formulas for each claim c which intuitively capture all possible sets of claims which jointly contradict each occurrence of claim c.

Definition 11. *Given a well-formed CAF $CF = (A, R, claim)$, then for each claim $c \in claim(A)$, the* CNF-attack-formula of c in CF *is defined as*

$$\mathcal{CD}_c^{CF} = \bigwedge_{a \in A, claim(a) = c} \quad \bigvee_{(x,a) \in R} claim(x).$$

$\mathcal{D}_c^{CF}$ denotes any equivalent formula in DNF over the same set of variables and is called DNF-attack-formula of c in CF.

Note that formula $\mathcal{CD}_c^{CF}$ is unsatisfiable iff there exists an argument x in CF with $claim(x) = c$ such that $x_R^- = \emptyset$.

Similarly to SETAFs, attack formulas allow for an exact characterization of well-formed CAFs, i.e. each well-formed CAF $CF = (A, R, claim)$ is uniquely determined (modulo argument names) via its attack formulas $\mathcal{CD}_c^{CF}$.

Example 3. *Consider the following CNF-attack formulas:*

$$\begin{array}{lcllcl} \mathcal{CD}_a^{CF} & = & (c \vee \overline{b}) \wedge (\overline{c} \vee b) & & & \\ \mathcal{CD}_b^{CF} & = & \overline{b} & \mathcal{CD}_c^{CF} & = & \overline{c} \\ \mathcal{CD}_{\overline{b}}^{CF} & = & b \vee \overline{c} & \mathcal{CD}_{\overline{c}}^{CF} & = & \overline{b} \vee c \end{array}$$

The two conjuncts of $\mathcal{CD}_a^{CF}$ determine that we have two arguments with claim a, while the remaining claims appear only once. The disjunctions refer to the attackers of the arguments. In fact, we obtain the CAF as depicted in Figure 3.

We are now ready to formally state our translation $\mathtt{T}_{cts}$, which maps each well-formed CAF CF to a corresponding SETAF $\mathtt{T}_{cts}(CF)$. Each claim c in the original framework CF corresponds to an argument in $\mathtt{T}_{cts}(CF)$, furthermore we identify each disjunct of the DNF-attack-formula $\mathcal{D}_c^{CF}$ with a collective attack against c. Consequently, the formula $\mathcal{D}_c^{CF}$ coincides with the attack-formula $\mathcal{D}_c^{\mathtt{T}_{cts}(CF)}$ of the resulting SETAF $\mathtt{T}_{cts}(CF)$. Therefore, the SETAF $\mathtt{T}_{cts}(CF)$ solely depends on the choice of the formula $\mathcal{D}_c^{CF}$.

Translation 1. *For a well-formed CAF $CF = (A, R, claim)$ we define $\mathtt{T}_{cts}(CF) = (A', R')$ with $A' = claim(A)$ and*

$$R' = \{(\delta, c) \mid c \in A', \delta \in \mathcal{D}_c^{CF}\}.$$

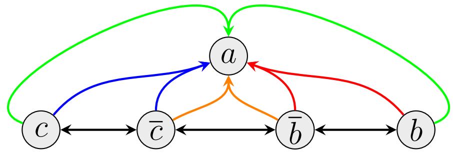

Figure 5: SETAF $\mathtt{T}_{cts}(CF)$ from Example 4.

Example 4. *In Example 3 we have already provided the attack formulas for the CAF CF depicted in Figure 3. The attack formula $\mathcal{CD}_a^{CF}$ for claim a in DNF representation yields $\mathcal{D}_a^{CF} = (c \wedge \overline{c}) \vee (c \wedge b) \vee (\overline{b} \wedge \overline{c}) \vee (\overline{b} \wedge b)$. The attack formulas for the remaining claims are readily given in DNF. Applying Translation 1 yields the SETAF given in Figure 5: for a, we need a collective attack for each disjunct in $\mathcal{D}_a^{CF}$; for the remaining arguments, each disjunct contains one atom, thus the incoming attacks remain binary as in CF.*

Notice that the translation links multiple occurrences of a claim with collective attacks on a corresponding single argument. Therefore, it interlinks claim-based extensions of CAFs with extensions (on the argument level) of SETAFs. As we show next, this is performed in a faithful way, that is, the reviewed semantics of well-formed CAFs can be reduced to their counterparts in SETAFs.

Proposition 4. *For each well-formed CAF CF and $\sigma \in \{cf, adm, com, grd, prf, stb\}$, $\sigma_c(CF) = \sigma_s(\mathcal{T}_{cts}(CF))$.*

Proof (for $\sigma \in \{cf, stb\}$). First, let $\sigma = cf$. We rephrase conflict-freeness in terms of CNF- resp. DNF-attack-formulas. In well-formed CAFs, $S \in cf_c(CF)$ iff for each $s \in S$ there is an $a \in A$ with $claim(a) = s$ such that a is not attacked by any argument b with $claim(b) \in S$. Note that, for any $s \in claim(A)$, $\mathcal{CD}_s^{CF}$ identifies each clause with the set of attacking claims for a particular occurrence of s in well-formed CAFs. That is, $S \in cf_c(CF)$ iff for each $s \in S$,

$$\textit{there is some } \gamma \in \mathcal{CD}_s^{CF} \textit{ such that } \gamma \cap S = \emptyset. \tag{C1}$$

In a SETAF (A', R'), a set $S \subseteq A'$ is conflict-free iff for all $S' \subseteq S$ and all $s \in S$, $(S', s) \notin R'$. In terms of attack formulas, $S \in cf_s(SF)$ iff for each $s \in S$, it holds that

$$\textit{for all } \delta \in \mathcal{D}_s^{SF} \textit{ we have } \delta \not\subseteq S. \tag{C2}$$

We have (i) $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for each $s \in A'$ by construction, and (ii) that no $\delta \in \mathcal{D}_s^{CF}$ is a subset of S iff there exists $\gamma \in \mathcal{CD}_c^{CF}$ such that $\gamma \cap S = \emptyset$, and thus we obtain that (C1) is equivalent to (C2), hence the statement follows.

Let $\sigma = stb$. A set S is stable on claim-level in CF if for each $s \in claim(A)$, it holds that $s \in S$ *if and only if* (C1). Similarly, S is stable in $\mathtt{T}_{cts}(CF)$ if for each $s \in A' = claim(A)$, it holds that $s \in S$ if and only if (C2). Again, the statement follows by the equivalence of (C2) and (C1). □

The results show that multiple occurrences of claims in AFs can be equivalently treated as collective attacks, if the framework satisfies well-formedness. Indeed, in our running example (cf. Figure 5), the stable extensions $stb_s(\mathtt{T}_{cts}(CF))$ are given by the sets $\{a, \overline{b}, c\}$, $\{a, b, \overline{c}\}$ and $\{b, c\}$.

Our next results show that it is equally possible to map each SETAF to a well-formed CAF while preserving the reviewed semantics. We will provide a translation $\mathtt{T}_{stc}$ which maps each SETAF SF to an equivalent well-formed CAF using attack formulas. Each argument a will correspond to a claim in the resulting CAF; furthermore, we introduce for each clause γ in the attack formula $\mathcal{CD}_a^{SF}$ an argument a_γ labelled with claim a. The clause γ also determines the set of attackers of the argument a_γ. Again, we see that the resulting CAF depends on the choice of the particular CNF attack-formulas.

Translation 2. *For each SETAF $SF = (A', R')$, we define $\mathtt{T}_{stc}(SF) = (A, R, claim)$ as follows:*

$$\begin{aligned}
&A = \{a_\gamma \mid a \in A', \gamma \in \mathcal{CD}_a^{SF}\} \cup \{a_\emptyset \mid a \in A', \mathcal{CD}_a^{SF} = \emptyset\},\\
&R = \{(x, a_\gamma) \mid a \in A', \gamma \in \mathcal{CD}_a^{SF}, claim(x) \in \gamma\},\\
&claim(a_\gamma) = a.
\end{aligned}$$

By restricting both Translations $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ to the class of all normalized well-formed CAFs, and respectively, SETAFs in minimal form[3], it can be shown that $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ are each others inverse w.r.t. a fixed conversion from CNF- to DNF-formulas and vice versa. With this observation the following result holds.

Proposition 5. *Let $\sigma \in \{cf, adm, com, grd, prf, stb\}$. For each SETAF SF in minimal form, $\sigma_s(SF) = \sigma_c(\mathtt{T}_{stc}(SF))$.*

Since each SETAF can be reduced to a minimal form [21][Thm. 4.2.5] and, similarly, each well-formed CAF has a normalized representative, the following theorem is an immediate consequence of the Propositions 4 and 5:

Theorem 2. *Let $\sigma \in \{cf, adm, com, grd, prf, stb\}$. For any well-formed CAF CF, there is a SETAF SF such that $\sigma_c(CF) = \sigma_s(SF)$, and vice versa.*

4.3 Classes of CAFs Expressible as AFs

In this section we investigate classes of CAFs that can be directly expressed by AFs. An example are CAFs such that their normal form (cf. Definition 7) contains each claim at most once (then only replacing each argument a by $claim(a)$ does the job). Well-formed CAFs $(A, R, claim)$ where R is symmetric fall into this category (we have $a_R^+ = b_R^+$, $a_R^- = b_R^-$ for any arguments a, b with the same claim; thus Proposition 3 can be applied exhaustively). However, as we show next, a weaker condition on the attack structure is sufficient.

Definition 12. *A CAF $(A, R, claim)$ is called* attacker-unitary (att-unitary) *if, for any $a, b \in A$ with $claim(a) = claim(b)$, it holds that $a_R^- = b_R^-$, i.e. arguments with the same claim are attacked by the same arguments.*

[3]A SETAF $SF = (A, R)$ is in minimal form if it has no attacks $(A, a), (B, a) \in R$ such that $A \subset B$.

In att-unitary CAFs, a set of arguments E defends either all or no occurrences of a claim c. The next lemma states that, for admissible-based semantics, each claim-extension S can be realized by a maximal representative.

Lemma 1. *Let $CF = (A, R, claim)$ be att-unitary and let $S \in \sigma_c(CF)$ for $\sigma \in \{adm, com, grd, prf, stb\}$. Then $E_S^{\max} = \{x \in A \mid claim(x) \in S\} \in \sigma((A, R))$.*

Next, we state translation $\mathtt{T}_{cta}$ from CAFs to AFs. Given CAF $CF = (A, R, claim)$, each claim $c \in claim(A)$ is mapped to a single argument c in the resulting AF $\mathtt{T}_{cta}(CF)$, wherein c attacks d if at least one argument with claim c attacks the arguments with claim d in (A, R).

Translation 3. *For an att-unitary CAF $CF = (A, R, claim)$, we define $\mathtt{T}_{cta}(CF) = (claim(A), R')$ with*

$$R' = \{(claim(x), claim(y)) \mid (x, y) \in R\}.$$

Proposition 6. *Let $CF = (A, R, claim)$ be att-unitary. Then $\sigma_c(CF) = \sigma(\mathtt{T}_{cta}(CF))$ for $\sigma \in \{adm, com, grd, prf, stb\}$.*

Proof (for $\sigma = stb$). Let $F = \mathtt{T}_{cta}(CF) = (A', R')$ and consider $S \in stb_c(CF)$. By Lemma 1, $E_S^{max} \in stb((A, R))$. Note that $S \in cf(F)$ by definition of R'. For each $c \in A' \setminus S$ there is an $x \in A \setminus E_S^{max}$ with $claim(x) = c$ and there is $y \in E_S^{max}$ such that $(y, x) \in R$. Consequently $(claim(y), claim(x)) \in R'$, and S attacks in F all arguments $a \in A' \setminus S$.

Now, let $S \in stb(F)$ and let $c \in A' \setminus S$. Then there is some argument $a \in S$ such that $(a, c) \in R'$. By definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = c$, such that $(x, y) \in R$. By att-unitaryness, $(x, z) \in R$ for each $z \in A$ such that $claim(z) = c$. Hence each argument $y \in A \setminus E_S^{max}$ is attacked by E_S^{max}. □

For an AF $F = (A, R)$ the CAF $CF = (A, R, id)$ with $id(a) = a$ is an equivalent att-unitary CAF. Together with above proposition this yields the following result.

Theorem 3. *Let $\sigma \in \{adm, com, grd, prf, stb\}$. For any att-unitary CAF CF there is an equivalent AF F, i.e. $\sigma_c(CF) = \sigma(F)$, and vice versa.*

Notice that Theorem 3 does not extend to cf semantics. For cf the orientation of attacks is immaterial and thus well-formed and att-unitary CAFs are intertranslateable.

5 Expressiveness of CAFs

Dunne et al. [14] introduced the concept of signatures in order to compare the expressiveness of semantics for AFs. For a semantics σ, its signature is defined as $\Sigma_\sigma^{AF} = \{\sigma(F) \mid F \text{ is an AF}\}$, thus capturing all possible outcomes which can be obtained by AFs when evaluated under σ. We consider Σ_σ^{CAF}, Σ_σ^{wf} and Σ_σ^{unit} as the claim-based counter-parts for unrestricted, well-formed and att-unitary CAFs, e.g. $\Sigma_\sigma^{wf} = \{\sigma_c(CF) \mid CF \text{ is a well-formed CAF}\}$. Note that Σ_σ^{AF} yields a collection of sets of arguments while Σ_σ^{CAF}, Σ_σ^{wf} and Σ_σ^{unit} yield a collection of sets of claims. In order to compare Σ^{AF} with Σ^{CAF}, Σ^{wf} and Σ^{unit} we assume that the domains of identifiers for arguments and claims coincide.

Note that for any semantics σ, we have $\Sigma_\sigma^{AF} \subseteq \Sigma_\sigma^{\alpha} \subseteq \Sigma_\sigma^{CAF}$ with $\alpha \in \{wf, unit\}$. $\Sigma_\sigma^{\alpha} \subseteq \Sigma_\sigma^{CAF}$ is immediate by definition; $\Sigma_\sigma^{AF} \subseteq \Sigma_\sigma^{\alpha}$ holds since using the claim function $id(a) = a$, we have $\sigma((A, R)) = \sigma_c((A, R, \mathit{claim}))$.

We first will discuss signatures for well-formed resp. att-unitary CAFs. As an immediate consequence of Theorem 2, we have that signatures Σ_σ^{wf} of well-formed CAFs and signatures Σ_σ^{SET} of SETAFs [15] coincide. By Theorem 3, the signatures of att-unitary CAFs for admissible, complete, grounded, preferred and stable semantics correspond to the signatures of AFs. Furthermore, the signature of well-formed and att-unitary CAFs for conflict-free semantics coincide (the orientation of attacks can be ignored here).

Theorem 4. *$\Sigma_\sigma^{wf} = \Sigma^{SET}$ and $\Sigma_\sigma^{unit} = \Sigma_\sigma^{AF}$ for $\sigma \in \{adm, com, grd, prf, stb\}$; and $\Sigma_{cf}^{wf} = \Sigma_{cf}^{unit} = \Sigma_{cf}^{SET}$.*

It follows that $\Sigma_\sigma^{AF} \subseteq \Sigma_\sigma^{unit} \subseteq \Sigma_\sigma^{wf}$. Dvořák et al. [15] have shown that Σ_{stb}^{SET} contains all incomparable sets of extensions and this carries over to the signatures of well-formed CAFs, while Σ_{stb}^{AF} is weaker. Coming back to our introductory example, this reflects the fact that we cannot express the claim-based stable extensions of the CAF of Example 1 in terms of AFs and stable semantics, but can do it with SETAFs.

Expressiveness of General CAFs. We next show that almost all sets of claim-based extensions can be realised in arbitrary CAFs with stable and preferred semantics.

Theorem 5. *The following characterisations hold:*

$$\begin{aligned}
\Sigma_{stb}^{CAF} &= \{\mathbb{S} \subseteq 2^C \mid \mathbb{S} = \{\emptyset\} \text{ or } \emptyset \notin \mathbb{S}\} \\
\Sigma_{prf}^{CAF} &= \Sigma_{stb}^{CAF} \setminus \{\emptyset\}
\end{aligned}$$

Proof. The conditions are necessary, since for any $CF = (A, R, \mathit{claim})$, $\sigma_{prf}(CF) \neq \emptyset$ and $\emptyset \in \sigma_c(CF)$ implies $\sigma(A, R) = \{\emptyset\}$ and thus $\sigma_c((A, R, \mathit{claim})) = \{\emptyset\}$.

Now we show that the above conditions are also sufficient by giving an actual construction of a realising CAF. If $\mathbb{S} = \emptyset$ (this only applies to stable semantics) simply use any AF which has no stable extension. If $\mathbb{S} \neq \emptyset$ construct a CAF $CF = (A, R, \mathit{claim})$ as follows (we use $[\mathbb{S}] = \bigcup_{S \in \mathbb{S}} S$).

$$\begin{aligned}
A =& \{a_S \mid S \in \mathbb{S}\} \cup \{a_c \mid c \in [\mathbb{S}]\} \\
R =& \{(a_S, a_{S'}) \mid S, S' \in \mathbb{S}, S \neq S'\} \cup \\
& \{(a_S, a_c) \mid S \in \mathbb{S}, c \in [\mathbb{S}] \setminus S\}
\end{aligned}$$

and $\mathit{claim}(a_c) = c$ and $\mathit{claim}(a_S) \in S$, i.e. for a_S one can pick an arbitrary claim from the set S. It can be verified that $stb_c(CF) = prf_c(CF) = \mathbb{S}$. □

Finally, we give the characterisations for the remaining semantics. We call a set $\mathbb{S}$ downwards-closed if for any $S \in \mathbb{S}$, all subsets of S are also contained in $\mathbb{S}$.

Theorem 6. *The following characterisations hold:*

$$\Sigma_{cf}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \mathbb{S} \neq \emptyset, \mathbb{S} \textit{ is downwards-closed}\}$$
$$\Sigma_{adm}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \emptyset \in \mathbb{S}\}$$
$$\Sigma_{com}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \mathbb{S} \neq \emptyset, \bigcap_{S \in \mathbb{S}} S \in \mathbb{S}\}$$

6 Discussion

Related Work. The work by Amgoud et al. [3] is probably closest to ours. They investigate equivalence in logic-based argumentation systems and study conditions under which arguments can be removed from a system without affecting its semantics. Their setting is more limiting as they require arguments to have both equivalent support and equivalent claims in order to remove one of them. Also related are semantics-preserving translations to standard AFs that have been investigated for several generalizations of AFs, e.g. [12, 7, 6]. In contrast to our main results where claims are mapped to arguments, all of these translations concern the argument level only. An exception is the work by Strass [24] on expressiveness of AFs compared to the expressiveness of logic programs and propositional logic, where arguments are mapped to propositional atoms. Finally, we mention studies [8, 17, 2, 1] that analyse whether particular properties on the level of claims (rationality and consistency postulates) are fulfilled by AF extensions in certain instantiation scenarios.

Summary and Outlook. In this work we addressed the question under which conditions argumentation frameworks can be simplified such that multiple arguments that share the same claim can be represented by a single argument. Apart from identifying classes of CAFs which can be expressed as standard AFs, we showed that well-formed CAFs are equally powerful to SETAFs with respect to the reviewed semantics. We complemented these findings by settling the expressiveness of unrestricted CAFs.

Directions for future research include: (a) extending our studies to further argumentation semantics, in particular naïve, stage and semi-stable semantics [25] in well-formed CAFs; (b) a systematic analysis of instantiation processes in order to identify other structural restrictions on CAFs; (c) an adaption of our formalism towards 3-valued labellings [9] instead of extensions.

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A Appendix

In this appendix we provide full proofs for the main results of the paper. We first show an alternative characterization of grd_c-semantics that we will exploit later on.

Proposition 7. *For any CAF CF, $S \in grd_c(CF)$ iff $S \in com_c(CF)$ and $S \subseteq T$ for all $T \in com_c(CF)$.*

Proof. Let $CF = (A, R, claim)$ and consider the grounded extension $G \in grd((A, R))$. We have that $G \subseteq E$ for all $E \in com((A, R))$ and, by the monotonicity of $claim(.)$, $claim(G) \subseteq S$ for all $S \in com_c(CF)$. □

Proofs of Section 3

Proposition 2 (restated). For $\sigma \in \{stb, prf\}$ and every well-formed CAF $CF = (A, R, claim)$, we have (a) $\sigma_c(CF)$ is incomparable, and (b) $|\sigma((A, R))| = |\sigma_c(CF)|$.

Proof. Let $F = (A, R)$. Then $\sigma(F)$ is an incomparable set, $\sigma(F) \subseteq com(F)$, and, as we have shown in the proof of Proposition 1, for $E, E' \in com(F)$: $E \subseteq E'$ iff $claim(E) \subseteq claim(E')$. Hence, for $S, T \in \sigma(F)$, $S \neq T$ implies $claim(S) \not\subseteq claim(T)$. The result follows. □

Proofs of Section 4.1

Proposition 3 (restated). Let $CF = (A, R, claim)$ be a CAF, with $a \in A$ redundant in CF w.r.t. some $b \in A$, and let $CF' = (A', R', claim)$ with $A' = A \setminus \{a\}$, and $R' = R \cap (A' \times A')$. Then, for $\sigma \in \{cf, adm, com, grd, stb, prf\}$, $\sigma_c(CF) = \sigma_c(CF')$.

Proof. We show the result step-by-step for the different semantics.

(1) Let $S \in cf_c(CF)$ and let E be a cf-realization of S in CF. If $a \notin E$, then E is a cf-realization of S in CF' as well and thus $S \in cf_c(CF')$. If $a \in E$, consider $E' = (E \setminus \{a\}) \cup \{b\}$; by definition $claim(E') = claim(E)$, thus it remains to show that $E' \in cf((A', R'))$. First, we have $(b, b) \notin R$ as otherwise $(b, a) \in R$ (since $a_R^- \supseteq b_R^-$) and also $(a, a) \in R$ (since $a_R^+ = b_R^+$). By $a \in E \in cf((A, R)$ we have $a_R^- \cap E = emptyset$ and $a_R^+ \cap E = emptyset$ and, since $a_R^- \supseteq b_R^-$ and $a_R^+ = b_R^+$, we obtain $E' \in cf((A, R)$ and thus $E' \in cf((A', R')$. For the other direction, let $S \in cf_c(CF')$ and E' a cf-realization of S in CF'. Clearly, E' remains conflict-free in (A, R) and thus $S \in cf_c(CF)$.

(2) Let $S \in adm_c(CF)$ and let E be an adm-realization of S in CF. If $a \notin E$, then E is conflict-free in CF'. Furthermore, since $E_R^- \subseteq E_R^+$, we have that $E_{R'}^- = E_R^- \setminus \{a\} \subseteq E_R^+ \setminus \{a\} = E_{R'}^+$, hence E defends itself in (A', R') and thus adm-realizes S in CF'. If $a \in E$, define $E' = (E \setminus \{a\}) \cup \{b\}$. We already know that E' is a cf-realization of S in CF', thus it remains to show that E' defends itself in (A', R'). First observe that $E_R'^- \subseteq E_R^-$, since $b_R^- \subseteq a_R^-$, and $E_R^+ = E_R'^+$, since $b_R^+ = a_R^+$. Second, $E_{R'}'^- = E_R'^-$: otherwise $(a, c) \in R$ for some $c \in E'$, but since $E \in cf((A, R))$,

this implies $c = b$ which would yield $(a,a) \in R$ via $a_R^+ = b_R^+$. Finally, $E_R'^+ = E_{R'}'^+$: otherwise $(c,a) \in R$ for some $c \in E'$; this implies $(b,a) \in R$ (since $E \in \mathit{cf}((A,R))$) and $(a,a) \in R$ via $a_R^- \supseteq b_R^-$. Together with $E_R^- \subseteq E_R^+$, we thus obtain $E_{R'}'^- = E_R'^- \subseteq E_R^- \subseteq E_R^+ = E_R'^+ = E_{R'}'^+$ and hence E' defends itself in (A',R'). To show the other direction, let $S \in adm_c(CF')$. Then there is a set E' which adm-realizes S in CF'. We show that E' is an adm-realization of S in CF: Clearly, E' is conflict-free in (A,R). It remains to show that E' defends itself in (A,R). Suppose this is not the case. Then $a \notin E_R'^+$ and $(a,c) \in R$ for some $c \in E'$. It follows that $b \notin E'$ since $b_R^+ = a_R^+$ and $E' \in \mathit{cf}(A',R')$. Since $E' \in adm((A',R'))$, it then also follows that $b \in E_{R'}'^+ = E_R'^+$. Because of $a_R^- \supseteq b_R^-$, we arrive at $a \in E_R'^+$, a contradiction.

(3) Let $S \in com_c(CF)$ and let E be a com-realization of S in CF. If $a \notin E$, we know from above that E is an adm-realization of S in CF'. It remains to show that E contains all arguments it defends in (A',R'). Let $d \in A'$ be defended by E in (A',R'); we have that $d_{R'}^- \subseteq E_{R'}^+ = E_R^+ \setminus \{a\}$. If $(a,d) \notin R$, then $d_R^- = d_{R'}^-$, consequently $d_R^- \subseteq E_R^+$, and therefore $d \in E$. If $(a,d) \in R$, then $(b,d) \in R$ (since $a_R^+ = b_R^+$), and thus $(b,d) \in R'$. Since $a_R^- \supseteq b_R^-$, we conclude that d is defended by E in (A,R) and therefore $d \in E$. Now, if $a \in E$, then $b \in E$, since $a_R^- \supseteq b_R^-$. Let $E' = E \setminus \{a\}$. We know from above that E' is an adm-realization of S in CF'. Moreover, for each $d \in A'$ which is defended by E' in (A',R'), it holds that $d_R^- = d_{R'}^- \subseteq E_{R'}'^+ = E_R'^+ = E_R^+$, therefore $d \in E'$. Thus $E' \in com((A',R'))$. To show the other direction, let $S \in com_c(CF')$ and let E' be a com-realization of S in CF'. We already have that E' is admissible in (A,R). If $b \notin E'$, then $E' \in com((A,R))$: Consider $d \in A$ such that $d_R^- \subseteq E_R'^+$. First note that $d \neq a$, otherwise $b \in E'$ since $a_R^- \supseteq b_R^-$. Furthermore, $d_{R'}^- = d_R^- \setminus \{a\} \subseteq E_R'^+ \setminus \{a\} = E_{R'}'^+$, thus $d \in E'$. If $b \in E'$, then either E' or $E = E' \cup \{a\}$ is complete in (A,R). For each argument $c \in A \setminus \{a\}$, if $c_R^- \subseteq E_R'^+$, then $c_{R'}^- \subseteq E_{R'}'^+$, i.e. E' contains all arguments from $A \setminus \{a\}$ it defends in CF. Hence it remains to check whether a is defended by E'. If this is the case, i.e. if $a_R^- \subseteq E_R'^+$, then E is conflict-free in CF: Towards a contradiction, assume that a is in conflict with some $c \in E$. If $(a,c) \in R$, then also $(b,c) \in R$ by $a_R^+ = b_R^+$. Since $E \in \mathit{cf}((A,R))$, we have that $c = a$, i.e. $(b,a) \in R$. But since E' defends a, we get that $(e,b) \in R$ for some $e \in E'$, contradiction. If $(c,a) \in R$, then there is an argument $d \in E'$ such that $(d,c) \in R$. Since $E \in \mathit{cf}((A,R))$ it follows that $c = a$, i.e. $(a,a) \in R$ and, therefore, $(b,a) \in R$, which leads to the same contradiction. It follows that E is admissible in (A,R). Furthermore, $E \in com((A,R))$, since E and E' defend the same arguments. If $a_R^- \not\subseteq E_R'^+$, then $E' \in com((A,R))$. In both cases, S has a com-realization in CF. We thus have shown that $com_c(CF) = com_c(CF')$.

(4) From $com_c(CF) = com_c(CF')$ and Proposition 7, it follows that $grd_c(CF) = grd_c(CF')$.

(5) Let $S \in prf_c(CF)$ and let E be a subset-maximal admissible set in (A,R) such that $claim(E) = S$. First consider the case $a \notin E$. From (2), we know that E is admissible in (A',R'). It remains to show there is no admissible set $F \subseteq A'$ in (A',R') such that $E \subset F$. Towards a contradiction, assume such F exists. Again, using the argument from (2), F is admissible in (A,R), a contradiction. If $a \in E$, then $b \in E$, since $a_R^- \supseteq b_R^-$ and each preferred set is also complete. Let $E' = E \setminus \{a\}$; in (3) we have shown that $E' \in com((A',R'))$ and likewise that for a complete set F of (A',R') with $b \in F$, $F \cup \{a\}$ is complete for (A,R) in case $a_R^- \subseteq F_R^+$. It follows that for each such F, $E \not\subset F$. Now, let $S \in prf_c(CF')$ and E' a prf-realization of S in CF'. Then E' is admissible in CF. We show that either E' or $E = E' \cup \{a\}$ is a maximal admissible set in

(A, R): Towards a contradiction, assume that there is an admissible set $F \in A$ in (A, R) such that $F \supset E'$. If $a \notin F$, then F is admissible in CF', contradiction to the maximality of E'. If $a \in F$, then $F' = (F \setminus \{a\}) \cup \{b\} \in adm((A', R'))$. Since E' is maximal in (A', R'), we conclude that $E' = F'$.

(6) Let $S \in stb_c(CF)$, let E be a stb-realization of S in CF, i.e. $E^+_R = A \setminus E$ and E is conflict-free. In case $a \notin E$, it is easy to see that $E \in cf((A', R'))$ and $E^+_{R'} = A' \setminus E$; thus E is a stb-realization of S in CF'. In case $a \in E$, we observe that $b \in E$ holds, too: otherwise, $b \in E^+_R$ and therefore, by $a^-_R \supseteq b^-_R$, also $a \in E^+_R$. A contradiction to $E \in cf((A, R))$. We show that $E' = E \setminus \{a\}$ is a stb-realization of S in CF'. Clearly, $claim(E') = claim(E) = S$. Moreover, $E' \in cf((A', R'))$, and $E'^+_{R'} = A' \setminus E' = A \setminus E$. Hence, $E' \in stb((A', R'))$. For the other direction, let $S \in stb_c(CF')$ and E a stb-realization of S in CF'. Clearly $E \in cf((A, R))$. If $a \in E^+_R$, we immediately get $E \in stb((A, R))$. So suppose $a \notin E^+_R$. Since $a^-_R \supseteq b^-_R$, $b \notin E^+_R$, and it follows that $b \in E$, since E is stable in (A', R'). We conclude that $(b, a) \notin R$ and thus, since $a^+_R = b^+_R$, $(a, a) \notin R$. Moreover, as $b \notin E^-_R$ we have $a \notin E^-_R$ as well. That is, we have $(a, a) \notin R$, $a \notin E^+_R$, $a \notin E^-_R$, and thus $(E \cup \{a\}) \in cf((A, R))$. Moreover, in (A, R) the set $E \cup \{a\}$ attacks each argument $c \in A \setminus E$ and, since $b \in E$, $claim(E \cup \{a\}) = claim(E) = S$. Hence, $E \cup \{a\}$ is a stb-realization of S in CF. □

Proofs of Section 4.2

Proposition 4 (restated). For each well-formed $CF = (A, R, claim)$, it holds that $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cts}(CF))$ for $\sigma \in \{cf, adm, com, grd, prf, stb\}$.

Proof. Let $SF = \mathtt{T}_{cts}(CF) = (A', R')$.

(1) Let $\sigma = cf$. We rephrase the property of being conflict-free in terms of CNF- resp. DNF-attack-formulas. In well-formed CAFs, $S \in cf_c(CF)$ iff for each $s \in S$ there is an $a \in A$ with $claim(a) = s$ such that a is not attacked by any argument b with $claim(b) \in S$. Note that, for any $s \in claim(A)$, the CNF-attack-formula $\mathcal{CD}^{CF}_s$ identifies each clause with the set of attacking claims for a particular occurrence of s in well-formed CAFs. That is, $S \in cf_c(CF)$ iff for each $s \in S$,

$$\textit{there is some } \gamma \in \mathcal{CD}^{CF}_s \textit{ such that } \gamma \cap S = \emptyset. \tag{C1}$$

In a SETAF (A', R'), a set $S \subseteq A'$ is conflict-free iff for all $S' \subseteq S$ and all $s \in S$, $(S', s) \notin R'$. In terms of attack formulas we have that S $\in cf_s(SF)$ iff for each $s \in S$, it holds that

$$\textit{for all } \delta \in \mathcal{D}^{SF}_s \textit{ it holds that } \delta \nsubseteq S. \tag{C2}$$

We have (i) $\mathcal{D}^{SF}_s = \mathcal{D}^{CF}_s$ for each $s \in A'$ by construction, and (ii) that no $\delta \in \mathcal{D}^{CF}_s$ is a subset of S iff there exists $\gamma \in \mathcal{CD}^{CF}_c$ such that $\gamma \cap S = \emptyset$, and thus we obtain that (C1) is equivalent to (C2), hence the statement follows.

(2) Let $\sigma = adm$. We will translate admissibility in well-formed CAFs resp. SETAFs to CNF- resp. DNF-attack-formulas. Let $S \subseteq claim(A)$. S is adm-realizable in CF if there exists a set

$E \subseteq A$, $claim(E) = S$, which is conflict-free and defends itself. Recall that in well-formed CAFs, arguments with the same claim attack the same arguments, which allows for speaking about claims attacking arguments. Using this advantage, we get that $S \in adm_c(CF)$ iff for each $s \in S$, there exists an $a \in A$, $claim(a) = s$, such that $(b, a) \notin R$ for any argument b with $claim(b) \in S$ and for all claims $d \in claim(A)$ which attack a, for each argument with claim d there is a claim $s' \in S$ which attacks the argument. Thus, in terms of CNF-attack-formulas: $S \in adm_c(CF)$ iff for each $s \in S$,

$$\begin{aligned}&\textit{there exists } \gamma \in \mathcal{CD}_s^{CF} \textit{ such that } \gamma \cap S = \emptyset,\\ &\textit{and for all } d \in \gamma, \textit{ for all } \gamma' \in \mathcal{CD}_d^{CF}\\ &\textit{it holds that } \gamma' \cap S \neq \emptyset.\end{aligned} \tag{A1}$$

A set $S \subseteq A'$ is admissible in SF iff it is conflict-free and defends itself. The latter is satisfied iff each attacking set B is attacked by some subset $S' \in S$, i.e. there is some $b \in B$ which gets attacked by S'. Thus, in terms of DNF-attack-formulas, a set S is admissible in SF iff for each $s \in S$, it holds that

$$\begin{aligned}&\textit{for all } \delta \in \mathcal{D}_s^{SF} \textit{ it holds that } \delta \nsubseteq S,\\ &\textit{and there exists } d \in \delta, \textit{ exists } \delta' \in \mathcal{D}_d^{SF},\\ &\textit{such that } \delta' \subseteq S.\end{aligned} \tag{A2}$$

By construction, we have that (i) $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for all $s \in A'$. Thus it remains to show that A1 and A2 are equivalent. Recall that (ii) for each $\gamma \in \mathcal{CD}_s^{CF}$ it holds that $\gamma \cap S \neq \emptyset$ if and only if there exists $\delta \in \mathcal{D}_s^{CF}$, such that $\delta \subseteq S$.

To show A1 $\Rightarrow$ A2, fix a witness $\gamma \in \mathcal{CD}_s^{CF}$ satisfying A1. We show that for all $\delta \in \mathcal{D}_s^{CF}$, $\delta \nsubseteq S$ and there exists $d \in \delta$ and $\delta' \in \mathcal{D}_d^{SF}$, such that $\delta' \subseteq S$. First note that $\delta \nsubseteq S$ follows immediately from (ii). Furthermore observe that each $\delta \in \mathcal{D}_s^{CF}$ contains some $d \in \gamma$, such that for every $\gamma' \in \mathcal{CD}_d^{CF}$, γ' has non-empty intersection with S. By (ii), the latter implies that there exists some $\delta' \in \mathcal{D}_d^{CF}$ such that $\delta' \subseteq S$.

To show A2 $\Rightarrow$ A1, let $\gamma = \{d \in \delta \mid \delta \in \mathcal{D}_s^{CF} \wedge \exists \delta' \in \mathcal{D}_d^{SF} \textit{ such that } \delta' \subseteq S\}$. We show that $\gamma \cap S = \emptyset$ and for all $d \in \gamma$, for all $\gamma' \in \mathcal{CD}_d^{CF}$ it holds that $\gamma' \cap S \neq \emptyset$. By (ii) and by definition of γ, the latter is satisfied, i.e. $\gamma' \cap S \neq \emptyset$ for each $\gamma' \in \mathcal{CD}_d^{CF}$, for all $d \in \gamma$. Now assume that $\gamma \cap S \neq \emptyset$. Let $c \in S \cap \gamma$. By definition of γ, $c \in \delta$ for some $\delta \in \mathcal{D}_s^{CF}$ and there exists $\delta' \in \mathcal{D}_c^{CF}$ such that $\delta' \subseteq S$. But since $c \in S$, it furthermore holds that $\delta' \nsubseteq S$ by A2, which is a contradiction.

(3) Let $\sigma = com$. We will express completeness of sets in well-formed CAFs and SETAFs in terms of CNF-, respectively, DNF-attack-formulas. Let $S \subseteq claim(A) = A'$. Observe that S is complete iff it is admissible and contains all arguments it defends. For well-formed CAFs, we already know from (2), that S is admissible in CF if for each $s \in S$, A1 is satisfied, i.e. there exists $\gamma \in \mathcal{CD}_s^{CF}$ such that $\gamma \cap S = \emptyset$, and for all $g \in \gamma$, for all $\gamma' \in \mathcal{CD}_g^{CF}$ it holds that $\gamma' \cap S \neq \emptyset$. Now, for complete sets, defense is not only necessary, but also a sufficient criteria for membership: If S defends an argument a, $claim(a) = s$, against any attacker $d \in claim(A)$, then $s \in S$. In terms of CNF-attack-formulas: If there is $\gamma \in \mathcal{CD}_s^{CF}$ such that for all $g \in \gamma$, for all $\gamma' \in \mathcal{CD}_g^{CF}$ it holds

that $\gamma' \cap S \neq \emptyset$, then $s \in S$. Combining both implications, we get that $s \in S$ *if and only if* A1 is satisfied. A similar reasoning also applies to complete sets in SETAFs: For any complete set S in SF it holds that $s \in S$ *if and only if* A2 for all $\delta \in \mathcal{D}_s^{CF}$, $\delta \not\subseteq S$ and there exists $d \in \delta$, $\delta' \in \mathcal{D}_d^{CF}$ such that $\delta' \subseteq S$.

Since $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for each $s \in A'$, and, furthermore, since A1 is equivalent to A2 as shown in (2), we obtain that indeed S $\in com_c(CF)$ iff $S \in com_s(SF)$ for any set $S \subseteq claim(A)$.

(4) Since $grd_c(CF)$ is the subset-minimal claim-based complete extension for any CAF CF by Proposition 7, it follows that $grd_c(CF) = grd_s(SF)$.

(5) We already know that $com_c(CF) = com_s(\mathtt{T}_{cts}(CF))$, by Proposition 1 the set $prf_c(CF)$ is given by the subset-maximal sets of $com_c(CF)$, and by definition $prf_s(\mathtt{T}_{cts}(CF))$ is given by the subset-maximal sets of $com_s(\mathtt{T}_{cts}(CF))$. Hence, we have $prf_c(CF) = prf_s(\mathtt{T}_{cts}(CF))$.

(6) Let $\sigma = stb$. A set S is stable on claim-level in CF if for each $s \in claim(A)$, it holds that $s \in S$ *if and only if* (C1) is satisfied. Similarly, S is stable in $\mathtt{T}_{cts}(CF)$ if for each $s \in A' = claim(A)$, it holds that $s \in S$ if and only if (C2) holds. The statement follows by the equivalence of (C2) and (C1). □

Proposition 5 (restated). Let $\sigma \in \{cf, adm, com, grd, prf, stb\}$. For each SETAF SF in minimal form, $\sigma_s(SF) = \sigma_c(\mathtt{T}_{stc}(SF))$.

Using an appropriate CNF-DNF-conversion, we will show that $\mathtt{T}_{stc}$ and $\mathtt{T}_{cts}$ are each others inverse when restricted to the class of all SETAFs in minimal form and to the class of all normalized CAFs, respectively. Recall that, by Proposition 4, $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cts}(CF))$ for each well-formed CAF and for $\sigma \in \{cf, adm, com, grd, prf, stb\}$, consequently we get that $\sigma_s(SF) = \sigma_s(\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF))) = \sigma_c(\mathtt{T}_{stc}(SF))$.

Let $\mathfrak{C}$ denote the class of all normalized well-formed CAFs and let $\mathfrak{S}$ denote the class of all SETAFs in minimal form. We show that there are CNF-DNF-formula conversions such that $\mathtt{T}_{stc}|_{\mathfrak{S}} = (\mathtt{T}_{cts}|_{\mathfrak{C}})^{-1}$.

To that end, we consider the following conversions.

Definition 13. *Let $X = \{\gamma_0, \dots, \gamma_n\}$ denote a CNF- respectively DNF-formula. We define the corresponding DNF- respectively CNF-formula $con(X)$ as the set of subset-minimal elements of $\{\delta \mid \forall i \leq n : |\delta \cap \gamma_i| \geq 1\}$.*

We call a formula X incomparable if all clauses $\gamma \in X$ are pairwise incomparable, i.e. for all $\gamma, \gamma' \in X$, $\gamma \not\subseteq \gamma'$. Observe that both conversions yield incomparable formulas. We will show that for each incomparable CNF- respectively DNF-formula X, the sequential application of both conversions yield the original formula X, i.e. $con(con(X)) = X$.

Lemma 2. *Let $X = \{\gamma_0, \dots, \gamma_n\}$ be incomparable then $con(con(X)) = X$.*

Proof. Let $Y = \{\delta_0, \dots, \delta_m\}$ denote the subset-minimal elements of $\{\delta \mid \forall i \leq n : |\delta \cap \gamma_i| \geq 1\}$, and let $L = \{\zeta \mid \forall j \leq m : |\zeta \cap \delta_j| \geq 1\}$. We show that the set of subset-minimal elements $\min_{\subseteq}(L)$ of L equals X.

$X \subseteq \min_{\subseteq}(L)$: First note that $\gamma \in L$ for all $\gamma \in X$, since $|\delta_j \cap \gamma| \geq 1$ for all $j \leq m$. For each $\gamma \in X$, for each $a \in \gamma$, there is some δ st $\delta \cap \gamma = \{a\}$. Take all γ_i such that $a \notin \gamma_i$, then there is some $\delta \subseteq \{b \mid b \in \gamma_i \setminus \gamma\} \cup \{a\}$ and $|\delta \cap \gamma| = 1$. Now, assume that there is some $\gamma' \subset \gamma$, $\gamma' \in \min_\subseteq(L)$. Let $a \in \gamma \setminus \gamma'$. Using the construction above, we get that there exists $\delta \in Y$ such that $\delta \cap \gamma = \{a\}$, consequently, $\gamma' \cap \delta = \emptyset$. It follows that $\gamma \in \min_\subseteq(L)$ for all $\gamma \in X$.

$\min_\subseteq(L) \subseteq X$: Towards a contradiction, let $\zeta \in \min_\subseteq(L) \setminus X$, and let $\delta \subseteq \bigcup_{i\leq n} \gamma_i \setminus \zeta$. Such a δ exists, otherwise $\zeta \supseteq \gamma$ for some $\gamma \in \min_\subseteq(L)$. But then $\delta \cap \zeta = \emptyset$. □

Proof of Proposition 5. Observe that (i) a well-formed CAF $CF = (A, R, claim)$ is normalized if and only if for each claim $c \in claim(A)$ it holds that $\mathcal{CD}_c^{CF}$ is incomparable. Similarly, (ii) a SETAF $SF = (A', R')$ is in minimal form if and only if for every argument $a \in A'$ it holds that $\mathcal{D}_a^{SF}$ is incomparable. Furthermore note that (iii) the output of both translations $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ solely depends on the choice of the CNF-DNF-conversion. Let $CF = (A, R, claim)$ and let $\mathtt{T}_{cts}$ and $\mathtt{T}'_{cts}$ denote translations using fixed CNF-DNF-conversions $\mathcal{D}_c^{CF}$ and respectively, $\mathcal{D}_c'^{CF}$ for each $c \in claim(A)$. Then $\mathtt{T}_{cts}(CF) = \mathtt{T}'_{cts}(CF)$ iff $\mathcal{D}_c^{CF} = \mathcal{D}_c'^{CF}$ for all $c \in claim(A)$. Similarly, for every SETAF $SF = (A', R')$, $\mathtt{T}_{stc}(SF) = \mathtt{T}'_{stc}(SF)$ iff $\mathcal{CD}_a^{SF} = \mathcal{CD}_a'^{SF}$ for all $a \in A'$. Since $con(con(X)) = X$ for each incomparable CNF- respectively DNF-formula X, we have that $\mathtt{T}_{stc}(\mathtt{T}_{cts}(CF)) = CF$ for each normalized well-formed CAF and, similar, $\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF)) = SF$ for each SETAF in minimal form. It follows that $\mathtt{T}_{stc}|_{\mathfrak{S}} = (\mathtt{T}_{cts}|_{\mathfrak{C}})^{-1}$.

By Proposition 4, we can conclude that $\sigma_s(SF) = \sigma_s(\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF))) = \sigma_c(\mathtt{T}_{stc}(SF))$ for each SETAF SF in minimal form. □

Proofs of Section 4.3

In order to prove Lemma 1 and Proposition 6, we show further properties of att-unitary CAFs.

Definition 14. *For any AF $F = (A, R)$, the* characteristic function $\mathcal{F}_F : 2^A \to 2^A$ *of F is defined as $\mathcal{F}_F(S) = \{x \in A \mid x$ is defended by $S\}$. For any CAF $CF = (A, R, claim)$, for $E \subseteq A$, we use $\mathcal{F}_{CF}(E)$ to abbreviate $\mathcal{F}_{(A,R)}(E)$.*

Lemma 3. *Let $CF = (A, R, claim)$ be att-unitary and let $E \subseteq A$. Then*

(1) $c \in claim(\mathcal{F}_{CF}(E))$ iff $x \in \mathcal{F}_{CF}(E)$ for all $x \in A$ such that $claim(x) = c$; and

(2) $|com_c(CF)| = |com((A, R))|$.

Proof. To prove (1), let $c \in claim(\mathcal{F}_{CF}(E))$, then there is an argument $x \in \mathcal{F}_{CF}(E)$, such that $claim(x) = c$. Let $y \in A$, $claim(y) = c$. Since $y^- = x^-$, we can conclude that E defends y, hence the statement follows. By definition of complete semantics, $E \in com((A, R))$ iff $\mathcal{F}_{CF}(E) = E$, consequently $c \in claim(E)$ iff $x \in E$ for all $x \in A$, $claim(x) = c$. Thus (2) follows. □

Lemma 1 (restated). Let $CF = (A, R, claim)$ be att-unitary and let $S \in \sigma_c(CF)$ for $\sigma \in \{adm, com, grd, prf, stb\}$. Then $E_S^{\max} = \{x \mid claim(x) \in S\} \in \sigma((A, R))$.

Proof. $S \in \sigma_c(CF)$ implies the existence of some set of arguments $E \subseteq A$, $claim(E) = S$, such that $E \in \sigma((A, R))$. Due to att-unitaryness, $E^- = (E_S^{\max})^-$. The statement follows for $\sigma = adm$ since $(E_S^{\max})^- = E^- \subseteq E^+ \subseteq (E_S^{\max})^+$. Let $\sigma = com$, then $\mathcal{F}_{CF}(E) = E$ by definition. By Lemma 3, $c \in S$ iff $x \in E$ for each argument x such that $claim(c) = x$. It follows that $E = E_S^{\max}$. Therefore, since $\sigma((A, R)) \subseteq com((A, R))$, it also holds that $E_S^{\max} \in \sigma((A, R))$ for $\sigma \in \{grd, prf, stb\}$. $\square$

Proposition 6 (restated). Let $CF = (A, R, claim)$ be att-unitary. Then $\sigma_c(CF) = \sigma(\mathtt{T}_{cta}(CF))$ for $\sigma \in \{adm, com, grd, prf, stb\}$.

Proof. Let $F = \mathtt{T}_{cta}(CF) = (A', R')$. For a set $S \subseteq claim(A) = A'$, we denote by $E_S^{\max} = \{x \mid claim(x) \in S\}$ the maximal representative of S in CF.

(1) Let $\sigma = adm$ and let $S \in adm_c(CF)$, then $E_S^{max} \in adm((A, R))$ by Lemma 1. We show that $S \in adm(F)$. First note that S is conflict-free since $(x, y) \notin R$ for all $x, y \in E_S^{max}$. To show that S defends itself, let $b \in S$ and let $(a, b) \in R'$ for some $a \in A'$. Then by definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = b$, such that $(x, y) \in R$. Since $y \in E_S^{\max}$ and $E_S^{\max} \in adm((A, R))$, y is defended by some $z \in E_S^{\max}$, i.e. $(z, x) \in R$ for some $z \in A$ such that $claim(z) \in S$. Consequently, $(claim(z), b) \in R'$ which shows that S defends itself.

To show the other direction, let $S \in adm(F)$. We show that E_S^{max} is admissible in (A, R): By definition of R', E_S^{max} does not contain any conflicts. Now, let $y \in E_S^{\max}$, $claim(y) = b$, and let $(x, y) \in R$ for some $x \in A$, $claim(x) = a$. By definition of R', we have that $(a, b) \in R'$. Since S defends itself, there is some $c \in S$ such that $(c, a) \in R'$, therefore there exist $z, x' \in A$, $claim(z) = c$, $claim(x') = a$, such that $(z, x') \in R$. By att-unitaryness, $x^- = x'^-$, thus $(z, x) \in R$. Note that $z \in E_S^{max}$ by definition, hence E_S^{max} defends itself.

(2) Let $\sigma = com$ and let $S \in com_c(CF)$. Then $E_S^{\max}$ is complete in (A, R) by Lemma 1. By (1), $S \in adm(F)$. We show that S contains all arguments it defends: Let $a \in A'$ be defended by S, i.e. for all $b \in A'$ such that $(b, a) \in R'$, there is some $c \in S$ such that $(c, b) \in R'$. By definition of Translation 3, it holds that for all $b \in A'$ such that $(b, a) \in R'$, (i) there is $y \in A$, $claim(y) = b$, such that $(y, x) \in R$, for some $x \in A$, $claim(x) = a$; and (ii) there are $y', z \in A$, $claim(y') = b$, $claim(z) = c$ for $c \in S$, such that $(z, y') \in R$. Since CF is att-unitary, $y^- = y'^-$, thus x is defended against y by $z \in E_S^{\max}$. Consequently, $x \in E_S^{\max}$ and therefore $a = claim(x) \in S$. in CF.

To show the other direction, let $S \in com(F)$. We show that $E_S^{\max} \in com((A, R))$. By (1), $E_S^{max} \in adm((A, R))$. To show that E_S^{max} contains all arguments it defends, let $x \in \mathcal{F}_{CF}(E_S^{\max})$, $claim(x) = a$. For each $y \in A$ such that $(y, x) \in R$, there is some $z \in E_S^{\max}$ such that $(z, y) \in R$. By construction of R', we have that $(claim(y), a), (claim(z), claim(y)) \in R'$ and $claim(z) \in S$, thus S defends a. Since $S \in com(F)$, we conclude that $a \in S$. By definition of $E_S^{\max}$ we have that $x \in E_S^{\max}$.

(3) Let $\sigma = grd$. By (2) and since $grd_c(CF)$ is the subset-minimal complete extension by Proposition 7, it follows that $grd_c(CF) = grd(F)$.

(4) Let $\sigma = prf$. Recall that $prf_c(CF) \subseteq com_c(CF)$. Since each $S \in com_c(CF)$ is realized by $E_S^{\max}$ and, by Lemma 3, $|com_c(CF)| = |com((A, R))|$, we have that for each $S, S' \in com_c(CF)$, $S \subseteq S'$ iff $E_S^{\max} \subseteq E_{S'}^{\max}$. By (2), $S \in com_c(CF)$ iff $S \in com(F)$, thus the statement follows.

(5) Let $\sigma = stb$ and $S \in stb_c(CF)$. By Lemma 1, $E_S^{max} \in stb((A, R))$. Furthermore S is conflict-free in $\mathtt{T}_{cta}(CF)$ by definition of R'. For each $x \in A \setminus E_S^{max}$, there is $y \in E_S^{max}$ such that $(y, x) \in R$, consequently $(claim(y), claim(x)) \in R'$, hence S attacks all arguments $a \in A' \setminus S$.

Now, let $S \in stb(F)$ and let $b \in A' \setminus S$. Then there is some argument $a \in S$ such that $(a, b) \in R'$. By definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = b$, such that $(x, y) \in R$. By att-unitaryness, $(x, y') \in R$ for each $y' \in A$ such that $claim(y') = b$. Hence each argument $y \in A \setminus E_S^{max}$ is attacked by E_S^{max}. □

Proofs of Section 5

Theorem 6 (restated). The following characterisations hold:

$$\Sigma_{cf}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \mathbb{S} \neq \emptyset, \mathbb{S} \text{ is downwards-closed}\}$$
$$\Sigma_{adm}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \emptyset \in \mathbb{S}\}$$
$$\Sigma_{com}^{CAF} = \{\mathbb{S} \subseteq 2^C \mid \mathbb{S} \neq \emptyset, \textstyle\bigcap_{S \in \mathbb{S}} S \in \mathbb{S}\}$$

Proof. In the following we use $[\mathbb{S}] = \bigcup_{S \in \mathbb{S}} S$.

(1) As the set of conflict-free sets is downwards-closed [14] for AFs, the same holds for the corresponding claim sets. To realise $\mathbb{S}$ construct a CAF $CF = (A, R, claim)$ with

$$A = \{a_{c,S} \mid S \in \mathbb{S} \setminus \{\emptyset\}, c \in S\}$$
$$R = \{(a_{c,S}, a_{c',S'}) \mid S, S' \in \mathbb{S}, c \in S, c' \in S', S \neq S'\}$$

and $claim(a_{c,S}) = c$. It holds that $CF_c(CF) = \mathbb{S}$.

(2) First, as the empty set is always admissible, the empty claim set has to be contained in each set of $\mathbb{S}$. Next we show that this condition is sufficient to realize a set $\mathbb{S}$ of claim sets. Construct a CAF $CF = (A, F, claim)$ as follows:

$$A = \{a_S \mid S \in \mathbb{S}\} \cup \{a_c, b_c \mid c \in [\mathbb{S}]\}$$
$$R = \{(a_S, a_{S'}) \mid S, S' \in \mathbb{S}, S \neq S'\} \cup \{(a_S, a_c) \mid S \in \mathbb{S}, c \in [\mathbb{S}] \setminus S\} \cup$$
$$\{(a_c, b_c) \mid c \in [\mathbb{S}]\} \cup \{(b_c, a_S) \mid S \in \mathbb{S}, c \in S\}$$

The function $claim$ is defined as $claim(a_c) = claim(b_c) = c$ and $claim(a_S) \in S$, i.e. for a_S one can pick an arbitrary claim from the set S. Now it is easy to verify that $adm_c(CF) = \mathbb{S}$.

(3) First, as the intersection of all complete extensions is a complete extension (the grounded extension) and there is a set of claims in $\mathbb{S}$ (the claims of the grounded extension) that is contained in all the other sets of $\mathbb{S}$. Next we show that this condition is sufficient to realise an extension-set $\mathbb{S}$. Let $G = \bigcap_{S \in \mathbb{S}} S$. Construct a CAF $CF = (A, F, claim)$ as follows:

$$A = \{a_S \mid S \in \mathbb{S} \setminus \{G\}\} \cup \{a_c \mid c \in [\mathbb{S}]\}$$
$$R = \{(a_S, a_{S'}) \mid S, S' \in \mathbb{S} \setminus \{G\}, S \neq S'\} \cup \{(a_S, a_c) \mid S \in \mathbb{S} \setminus \{G\}, c \in [\mathbb{S}] \setminus S\}$$

The function $claim$ is defined as $claim(a_c) = c$ and $claim(a_S) \in S$, i.e. for a_S one can pick an arbitrary claim from the set S. Now it is easy to verify that $com_c(CF) = \mathbb{S}$. □