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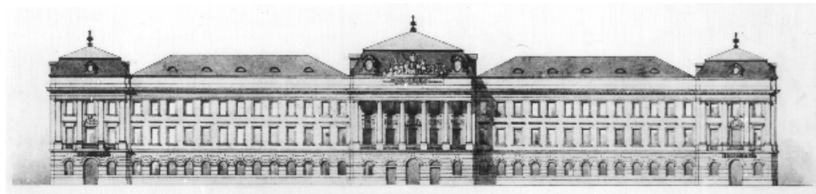

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Abstract Argumentation via Monadic Second Order Logic

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Wolfgang Dvořák **Stefan Szeider** **Stefan Woltran**

Institut für Informationssysteme
Abteilung Datenbanken und
Artificial Intelligence
Technische Universität Wien
Favoritenstr. 9
A-1040 Vienna, Austria
Tel: +43-1-58801-18403
Fax: +43-1-58801-18493
sekret@dbai.tuwien.ac.at
www.dbai.tuwien.ac.at

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Abstract Argumentation via Monadic Second Order Logic

Wolfgang Dvořák [1] **Stefan Szeider**[1] **Stefan Woltran** [1]

Abstract. We propose the formalism of "Monadic Second-Order Logic" as a unifying framework for representing and reasoning with various semantics of abstract argumentation. We express a wide range of semantics within the proposed framework, including the semantics proposed by Dung, semi-stable, stage, cf2, and resolution-based semantics. We provide building blocks which make it easy and straight-forward to express further semantics. Expressing reasoning problems in abstract argumentation within Monadic Second-Order Logic not only shows that this logic can serve as a *lingua franca* for further investigations, but also directly yields important complexity results. In particular, we obtain that for argumentation frameworks with certain structural properties the main computational problems can all be solved in linear time.

[1]Institute of Information Systems, Technische Universität Wien, Favoritenstrasse 9-11, 1040 Vienna, Austria. E-mail: {dvorak, woltran }@dbai.tuwien.ac.at, stefan@szeider.net

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1 Introduction

Starting with the seminal work by Dung Dung (1995) the area of argumentation has evolved to one of the most active research branches within Artificial Intelligence (see, e.g., Bench-Capon and Dunne (2007)). Dung's abstract argumentation frameworks, where arguments are seen as abstract entities which are just investigated with respect to how they relate to each other, in terms of "attacks", are nowadays well understood and different semantics (i.e. the selection of sets of arguments which are jointly acceptable) have been proposed. In fact, there seems to be no single "one suits all" semantics, but it turned out that studying a particular setting within various semantics and to compare the results is a central research issue within the field. Different semantics give rise to different computational problems, such as deciding whether an argument is acceptable with respect to the semantics under consideration, that require different approaches for solving these problems.

This broad range of semantics for abstract argumentation demands for a *unifying framework* for representing and reasoning with the various semantics. Such a unifying framework would allow to see what the various semantics have in common, in what they differ, and ideally, it would offer generic methods for solving the computational problems that arise within the various semantics. Such a unifying framework should be general enough to accommodate most of the significant semantics, but simple enough to be decidable and computationally feasible.

In this paper we propose such a unifying framework. We express several semantics within the framework, and we study its properties. The proposed unifying framework is based on the local formalism of "Monadic Second-Order Logic (MSO)", which is a fragment of Second-Order logic, with relational variables restricted to unary. MSO provides higher expressiveness than First-Order Logic while it has more appealing algorithmic properties than full Second-Order Logic. Furthermore, MSO plays an important role in various parts of Computer Science, to give two examples: (i) by Büchi's Theorem, a formal language is regular if and only if it can be expressed by MSO (this also provides a link between MSO and finite automata); (ii) by Courcelle's Theorem, MSO expressible properties can be checked in linear time on structures of bounded treewidth.

Our main contributions can be summarized as follows:

- We express a wide range of semantics within our proposed framework, including the semantics proposed by Dung Dung (1995), semi-stable, stage, cf2, and resolution-based semantics.
- We provide MSO-building blocks which make it easy and straight-forward to express other semantics or to create new ones or variants. We also illustrate that any labeling-based semantics can be canonically expressed within our framework.
- We show that the main computational problems can be solved in linear time for all semantics expressible in our framework when restricted to argumentation frameworks of certain structures. This includes decision problems such as skeptical and brave acceptance, but also counting problems, for instance, determining how many extensions contain a given argument.

Our results show that MSO is indeed a suitable unifying framework for abstract argumentation and can serve as a *lingua franca* for further investigations. Furthermore, recent systems Kneis, Langer, and Rossmanith (2011); Langer et al. (2012) showed quite impressive performance for evaluating MSO formulas over graphs, thus the proposed framework can be exploited as a rapid-prototyping approach to experiment with established and novel argumentation semantics.

Related Work. Using MSO as a tool to express AI formalisms has been advocated in Gottlob and Szeider (2006); Gottlob, Pichler, and Wei (2010). In terms of abstract argumentation first MSO encodings were given by Dunne Dunne (2007) and also discussed in Dvořák and Woltran (2010). Implications in terms of parameterized complexity also appeared in Dvořák, Pichler, and Woltran (2010); Dvořák, Szeider, and Woltran (2010).

Finding a uniform logical representation for abstract argumentation has been subject of several papers. While Besnard and Doutre Besnard and Doutre (2004) used propositional logic for this purpose, Egly and Woltran Egly and Woltran (2006) showed that quantified propositional logic allows for complexity-adequate representations. Another branch of research focuses on logic programming as common grounds for different argumentation semantics, see Toni and Sergot (2011) for a survey. Finally also the use of CSP was suggested Amgoud and Devred (2011); Bistarelli, Campli, and Santini (2011). All these works were mainly motivated by implementation issues and lead to systems as ASPARTIX Egly, Gaggl, and Woltran (2010). As mentioned above also MSO can serve this purpose, but in addition yields further results "for free", in particular in terms of computational complexity.

2 Background

2.1 Argumentation

In this section we introduce (abstract) argumentation frameworks Dung (1995) and recall the semantics we study in this paper (see also Baroni and Giacomin (2009)).

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where A is a set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. We say that an argument $a \in A$ is* defended *(in F) by a set $S \subseteq A$ if, for each $b \in A$ such that $(b, a) \in R$, there exists a $c \in S$ such that $(c, b) \in R$.*

Semantics for argumentation frameworks are given via a function σ which assigns to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions.

We first consider for σ the functions $naive$, stb, adm, com, prf, grd, stg, and sem which stand for naive, stable, admissible, complete, preferred, grounded, stage, and semi-stable semantics, respectively. Towards the definition of these semantics we introduce two more formal concepts.

Definition 2. *Given an AF $F = (A, R)$, the characteristic function $\mathcal{F}_F : 2^A \Rightarrow 2^A$ of F is defined as $\mathcal{F}_F(S) = \{x \in A \mid x \text{ is defended by } S\}$. For a set $S \subseteq A$ and an argument $a \in A$, we write $S \rightarrowtail^R a$ (resp. $a \rightarrowtail^R S$) in case there is an argument $b \in S$, such that $(b, a) \in R$ (resp. $(a, b) \in R$). Moreover, for a set $S \subseteq A$, we denote the set of arguments attacked by S as $S^\oplus_R = \{x \mid S \rightarrowtail^R x\}$, and resp. $S^\ominus_R = \{x \mid x \rightarrowtail^R S\}$, and define the* range of S *as $S^+_R = S \cup S^\oplus_R$.*

Definition 3. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F)*, if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of conflict-free sets of F. For a conflict-free set $S \in cf(F)$, it holds that*

- *$S \in naive(F)$, if there is no $T \in cf(F)$ with $T \supset S$;*
- *$S \in stb(F)$, if $S_R^+ = A$;*
- *$S \in adm(F)$, if $S \subseteq \mathcal{F}_F(S)$;*
- *$S \in com(F)$, if $S = \mathcal{F}_F(S)$;*
- *$S \in grd(F)$, if $S \in com(F)$ and there is no $T \in com(F)$ with $T \subset S$;*
- *$S \in prf(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S \subset T$;*
- *$S \in sem(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S_R^+ \subset T_R^+$;*
- *$S \in stg(F)$, if there is no $T \in cf(F)$, with $S_R^+ \subset T_R^+$.*

We recall that for each AF F, the grounded semantics yields a unique extension, the grounded extension, which is the least fix-point of the characteristic function $\mathcal{F}_F$.

On the base of these semantics one can define the family of resolution-based semantics Baroni, Dunne, and Giacomin (2011), with the resolution-based grounded semantics being its most popular instance.

Definition 4. *Given AF $F = (A, R)$, a resolution $\beta \subset R$ of F is a $\subseteq$-minimal set of attacks such that for each pair $\{(a, b), (b, a)\} \subseteq R$ ($a \neq b$) either $(a, b) \in \beta$ or $(b, a) \in \beta$. We denote the set of all resolutions of an AF F by $\gamma(F)$. Given a semantics σ, the corresponding resolution-based semantics σ^* is given by $\sigma^*(F) = \min_{\subseteq} \bigcup_{\beta \in \gamma(F)} \{\sigma((A, R \setminus \beta))\}$.*

Finally, let us consider the semantics cf2, which was introduced by Baroni, Giacomin, and Guida Baroni, Giacomin, and Guida (2005) as part of a general schema for argumentation semantics. cf2 semantics gained some interest as it handles even and odd length cycles of attacks in a similar way. Towards a definition of cf2 semantics we need the following concepts.

Definition 5. *Given AF $F = (A, R)$ and a set $S \subseteq A$. By $SCC(F)$ we denote the set of all strongly connected components of F. $D_F(S)$ denotes the set of arguments $a \in A$ attacked by an argument $b \in S$ occurring in a different component. Finally, for $F = (A, R)$ and a set S of arguments, $F|_S := (A \cap S, R \cap (S \times S))$ and $F - S := F|_{A \setminus S}$.*

Definition 6. *Given AF $F = (A, R)$. for $S \subseteq A$ we have that $S \in cf2(F)$ if one of the following conditions holds:*

- *$|SCC(F)| = 1$ and $S \in naive(F)$*
- *$\forall C \in SCC(F) : C \cap S \in cf2(F|_C - D_F(S))$*

Labeling-based semantics. So far we have considered so-called extension-based semantics. However, there are several approaches defining argumentation semantics via certain kind of labelings instead of extensions. As an example we consider the approach by Caminada and Gabbay Caminada and Gabbay (2009) and their complete labelings.

Definition 7. *Given an AF $F = (A, R)$, a function $\mathcal{L} : A \to \{in, out, undec\}$ is a* complete labeling *iff the following conditions hold:*

- $\mathcal{L}(a) = in$ *iff for each* b *with* $(b, a) \in R$, $\mathcal{L}(b) = out$
- $\mathcal{L}(b) = out$ *iff there exists* b *with* $(b, a) \in R$, $\mathcal{L}(b) = in$

There is a one-to-one mapping between complete extensions and complete labelings, such that the set of arguments labeled with in corresponds to a complete extension.

2.2 Monadic Second Order Logic

Informally Monadic Second Order Logic (MSO) is an extension of first order logic (FO) that allows for quantification over sets.

FO is build from variables $x, y, z, \ldots$ referring to elements of the universe, atomic formulas $R(t_1, \ldots, t_k)$, $t_1 = t_2$, with t_i being variables or constants, the usual Boolean connectives, and quantification $\exists x$, $\forall x$. MSO_1 extends the language of FO by set variables $X, Y, Z, \ldots$, atomic formulas $t \in X$ with t a variable or constant, and quantification over set variables. We further consider MSO_2 an extension of MSO_1 which is only defined on graphs (which is perfectly fine for our purposes). MSO_2 adds variables $X^E, Y^E, Z^E, \ldots$ ranging over sets of edges of the graph and quantification over such variables. In the following when talking about MSO we refer to MSO_2.

For an MSO formula ϕ We usually write $\phi(x_1, \ldots, x_i, X_1, \ldots X_j)$ to denote that the free variables of ϕ are $x_1, \ldots, x_i, X_1, \ldots X_j$. For a graph $G = (V, E)$ and $v_k \in V$ and $A_k \subseteq V$ we write $G \models \phi(v_1, \ldots, v_i, A_1, \ldots A_j)$ to denote that G models ϕ if x_i is assigned with v_i and X_i is assigned with A_i.

3 Encoding Argumentation Semantics in MSO

Building Blocks. We first introduce some shorthands simplifying notation when dealing with subset relations and the range of extensions.

$$
\begin{aligned}
x \notin X &= \neg(x \in X)\\
X \subseteq Y &= \forall x\,(x \in X \rightarrow x \in Y)\\
X \subset Y &= X \subseteq Y \wedge \neg(Y \subseteq X)\\
X \not\subseteq Y &= \neg(X \subseteq Y)\\
X \not\subset Y &= \neg(X \subset Y)\\
x \in X^+_R &= x \in X \vee \exists y(y \in X \wedge (y,x) \in R)\\
X \subseteq^+_R Y &= \forall x\,(x \in X^+_R \rightarrow x \in Y^+_R)\\
X \subset^+_R Y &= X \subseteq^+_R Y \wedge \neg(Y \subseteq^+_R X)
\end{aligned}
$$

Another important notion that underlies argumentation semantics is the notion of a set being conflict-free. The following MSO formula encodes that a set X is conflict-free w.r.t. the attack relation R:

$$
\mathit{cf}_R(X) = \forall x, y\,((x,y) \in R \rightarrow (\neg x \in X \vee \neg y \in X))
$$

Next we give a building block for maximizing extension using an (MSO expressible) order $\sqsubseteq$:

$$
\max{}_{A,P(.),\sqsubseteq}(X) = P(X) \wedge \neg\exists Y\big(Y \subseteq A \wedge P(Y) \wedge X \sqsubset Y\big)
$$

Clearly we can also implement minimization by inverting the order, i.e. $\min{}_{A,P(.),\sqsubseteq}(X) = \max{}_{A,P(.),\sqsupseteq}(X)$.

Standard Encodings. In the following we provide MSO-characterizations for the different argumentation semantics. The characterizations for $\mathit{adm}, \mathit{stb}, \mathit{prf}$ are borrowed from Dunne (2007) while those for $\mathit{sem}, \mathit{stg}$ are borrowed from Dvořák and Woltran (2010).

$$
\begin{aligned}
\mathit{naive}_{A,R}(X) &= \max{}_{A,\mathit{cf}_R(.),\subseteq}(X)\\
\mathit{adm}_R(X) &= \mathit{cf}_R(X) \wedge \forall x, y\,\big(((x,y) \in R \wedge y \in X) \rightarrow\\
&\qquad \exists z(z \in X \wedge (z,x) \in R)\big)\\
\mathit{com}_{A,R}(X) &= \mathit{adm}_R(X) \wedge \forall x((x \in A \wedge x \notin X) \rightarrow\\
&\qquad \exists y((y,x) \in R \wedge \neg\exists z(z \in X \wedge (z,y) \in R)))\\
\mathit{grd}_{A,R}(X) &= \min{}_{A,\mathit{com}_{A,R}(.),\subseteq}(X)\\
\mathit{stb}_{A,R}(X) &= \mathit{cf}_R(X) \wedge \forall x(x \in A \rightarrow x \in X^+_R)\\
\mathit{prf}_{A,R}(X) &= \max{}_{A,\mathit{adm}_R(.),\subseteq}(X)\\
\mathit{sem}_{A,R}(X) &= \max{}_{A,\mathit{adm}_R(.),\subseteq^+_R}(X)\\
\mathit{stg}_{A,R}(X) &= \max{}_{A,\mathit{cf}_R(.),\subseteq^+_R}(X)
\end{aligned}
$$

These characterisations are straight-forward translations of the definitions and thus can be easily checked to be correct.

Based on the above encodings we build up encodings for the resolution-based semantics as follows. Via $res_R(X^E)$, given as

$$\forall x, y\big(X^E \subseteq R \wedge (x,x) \in R \rightarrow (x,x) \in X^E \wedge \\ (x \neq y \wedge (x,y) \in R) \rightarrow ((x,y) \in X^E \leftrightarrow (y,x) \notin X^E)\big),$$

we express modified frameworks $(A, R \setminus \beta)$ where β is a resolution according to Definition 4. Now resolution-based semantics are characterised by

$$\sigma^*_{A,R}(X) = \exists X^E(res_R(X^E) \wedge \sigma_{A,X^E}(X) \wedge \\ \forall Y \forall Y^E(res_R(Y^E) \wedge \sigma_{A,Y^E}(Y) \rightarrow Y \not\subset X)). \tag{1}$$

Labeling-based semantics. There are several approaches to define argument semantics via different kind of argumentation labelings and almost all argumentation semantics allow for a characterization via argument labelings. The general concept behind labelings is to use a fixed set of labels and assign to each argument a subset of them, or just a single label. Such labelings are valid if for each argument the assigned labels satisfy certain (qualitative) conditions concerning the labels of attacking arguments and the labels of the attacked arguments. Additionally one might demand that the set of arguments labeled by a specific label is maximal or minimal. All these things can be easily expressed in MSO, which we illustrate for complete labelings. We encode a $in, out, undec$ labeling $\mathcal{L}$ as a triple $(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec})$ where $\mathcal{L}_l := \{a \in A \mid \mathcal{L}(a) = l\}$. To have these three sets disjoint, one uses the formula $\varphi = \forall x \in A((x \notin \mathcal{L}_{in} \vee x \notin \mathcal{L}_{out}) \wedge (x \notin \mathcal{L}_{in} \vee x \notin \mathcal{L}_{undec}) \wedge (x \notin \mathcal{L}_{undec} \vee x \notin \mathcal{L}_{out}))$. Now we can give an MSO formula $com_{A,R}(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec})$ expressing whether such a triple $(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec})$ is a complete labeling:

$$\varphi \wedge \forall x \in X(x \in \mathcal{L}_{in} \leftrightarrow (\forall y \in X((y,x) \in R \rightarrow \mathcal{L}_{out}))) \\ \wedge \forall x \in X(x \in \mathcal{L}_{out} \leftrightarrow (\exists y \in X((y,x) \in R \wedge \mathcal{L}_{in})))$$

Further, one can directly encode preferred labelings, which are defined as complete labelings with maximal $\mathcal{L}_{in}$.

$prf_{A,R}(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec}) = com_{A,R}(\mathcal{L}_{in}, \mathcal{L}_{out}, \mathcal{L}_{undec}) \wedge \neg \exists \mathcal{L}'_{in}, \mathcal{L}'_{out}, \mathcal{L}'_{und}(\mathcal{L}_{in} \subset \mathcal{L}'_{in} \wedge com_{A,R}(\mathcal{L}'_{in}, \mathcal{L}'_{out}, \mathcal{L}'_{und}))$

MSO-characterization for $cf2$. The original definition of $cf2$ semantics is of recursive nature and thus not well suitable for a direct MSO characterisation. Hence we use an alternative characterisation of $cf2$ Gaggl and Woltran (2010). For this characterisation we need the following definitions.

Definition 8. *Given an AF $F = (A, R)$, $B \subseteq A$, and $a, b \in A$, $a \Rightarrow^B_F b$ if there exists a sequence $(c_i)_{1 \leq i \leq n}$ with $c_i \in B$, $c_1 = a$, $c_n = b$ and $(c_i, c_{i+1}) \in R$.*

The relation $\Rightarrow_F^B$ can be encoded in MSO by first defining a relation $\hat{R}_{R,B}(u,v) = (u,v) \in R \wedge u \in B \wedge v \in B$ capturing the allowed attacks and borrowing the following MSO encoding for reachability Courcelle and Engelfriet (2011): $\mathit{reach}_R(x,y) = \forall X(x \in X \wedge [\forall u,v(u \in X \wedge R(u,v) \to v \in X)] \to y \in X)$. Finally we obtain $\Rightarrow_R^B (x,y) = \mathit{reach}_{\hat{R}_{R,B}}(x,y)$.

Definition 9. *For AF $F = (A,R)$ and sets $D,S \subseteq A$ we define: $\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b,a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}$. $\Delta_{F,S}$ denotes the least fixed-point of $\Delta_{F,S}(.)$.*

One can directly encode whether an argument x is in the operator $\Delta_{F,S}(D)$ by $\Delta_{A,R,S,D}(x) = x \in A \wedge \exists b \in S(b \neq x \wedge (b,x) \in R \wedge \neg \Rightarrow_F^{A \setminus D} (x,b))\}$ and thus also whether x is in the least fixed-point $\Delta_{F,S}$, by $\Delta_{A,R,S}(x) = \exists X \subseteq A(x \in X \wedge \forall a(a \in X \leftrightarrow \Delta_{A,R,S,X}(a)) \wedge \neg\exists Y \subset X(\forall b(b \in Y \leftrightarrow \Delta_{A,R,S,Y}(b))))$.

Definition 10. *For AF F we define the separation of F as $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$.*

The attack relation of the separation of an AF (A,R) is given by $R_{[[(A,R)]]}(x,y) = x \in A \wedge y \in A \wedge (x,y) \in R \wedge \Rightarrow_R^A (y,x)$.

The following result provides an alternative characterization for *cf2* semantics that can be encoded in MSO_1.

Proposition 1. *Gaggl and Woltran (2010) For any AF F, $\mathit{cf2}(F) = \mathit{cf}(F) \cap \mathit{naive}([[F - \Delta_{F,S}]])$.*

We obtain an MSO_1 characterisation of $\mathit{cf2}$.

$$\hat{A}(x) = x \in A \wedge \neg\Delta_{A,R,S}(x)$$
$$\mathit{cf2}(X) = \mathit{cf}_R(X) \wedge \mathit{naive}_{\hat{A},R_{[[(\hat{A},R)]]}}(X)$$

4 Algorithmic Implications

Most computational problems studied for AFs are computationally intractable (see, e.g. Dunne (2007)), while the importance of efficient algorithms is evident. An approach to deal with intractable problems comes from parameterized complexity theory and is based on the fact, that many hard problems become polynomial-time tractable if some problem parameter is bounded by a fixed constant. In case the order of the polynomial bound is independent of the parameter one speaks of *fixed-parameter tractability* (FPT).

One popular parameter for graph based problems is the parameter of *tree-width* Bodlaender (1993) which intuitively measures how tree-like a graph is. One weakness of tree-width is that it only captures sparse graphs. The parameter *clique-width* Courcelle, Engelfriet, and Rozenberg (1991) generalizes tree-width, in the sense that each graph class of bounded tree-width has also bounded clique-width, and also captures a wide range of dense graphs.[1]

[1] As we do not make direct use of them, we omit the formal definitions of tree-width and clique-width here; the interested reader is referred to Dunne (2007); Dvořák, Szeider, and Woltran (2010). We just note that these parameters are originally defined for undirected graphs, but can directly be used for directed graphs, and thus for AFs, as well.

Both parameters have already been considered for abstract argumentation Dunne (2007); Dvořák, Pichler, and Woltran (2010); Dvořák, Szeider, and Woltran (2010) and are closely related to MSO by means of meta-theorems. One such meta-theorem is due to Courcelle, Makowsky, and Rotics Courcelle, Makowsky, and Rotics (2000) and shows that one can solve any graph problem that can be expressed in MSO_1 in linear time for graphs of clique-width bounded by some fixed constant k, when given together with a certain algebraic representation of the graph, a so called k-expression. A similar result is Courcelle's seminal meta-theorem Courcelle (1987, 1990) for MSO_2 and tree-width (which is also based on a certain structural decomposition of the graph, a so called tree-decomposition). Together with results from Bodlaender (1996); Oum and Seymour (2006) stating that also k-expressions and tree-decompositions can be computed in linear time if k is bounded by a constant we get the following meta-theorem.

Theorem 1. *Given an integer k and a MSO formula $\phi(x_1, \ldots, x_i, X_1, \ldots X_j, X_1^E, \ldots X_l^E)$, there is a linear time algorithm, given a graph (V, E) of tree-width $\leq c$, $v_k \in V$, $A_k \subseteq V, B_k \subseteq E$ deciding whether $(V, E) \models \phi(v_1, \ldots, v_i, A_1, \ldots A_j, B_1, \ldots B_l)$. If ϕ is in MSO_1 this also holds for graphs of clique-width $\leq c$.*

The theorem can be extended to capture counting and enumeration problems Arnborg, Lagergren, and Seese (1991); Courcelle, Makowsky, and Rotics (2001).

In the next theorem we give fixed-parameter tractability results w.r.t. the parameters tree-width and clique-width for the main reasoning problems in abstract argumentation.

Theorem 2. *Given an argumentation semantics σ that is expressible in MSO, the following tasks are fixed-parameter tractable w.r.t. the tree-width of the given AF:*

- *Deciding whether an argument $a \in A$ is in at least one σ-extension (Credulous acceptance).*
- *Deciding whether an argument $a \in A$ is in each σ-extension (Skeptical acceptance).*
- *Verifying that a set $E \subseteq A$ is a σ-extension (Verification).*
- *Deciding whether there exists a σ-extension (Existence).*
- *Deciding whether there exists a non-empty σ-extension (Nonempty).*
- *Deciding whether there is a unique σ-extension (Unique).*

If σ is expressible in MSO_1 the above tasks are also fixed-parameter tractable w.r.t. the clique-width of the AF.

Proof. By Theorem 1 and the MSO - encodings below:
Credulous acceptance: $\phi^{\sigma}_{\mathsf{Cred}}(x) = \exists X\ (x \in X \wedge \sigma_R(X))$
Skeptical acceptance: $\phi^{\sigma}_{\mathsf{Skept}}(x) = \forall X\ (\sigma_R(X) \rightarrow x \in X)$
Verification: $\phi^{\sigma}_{\mathsf{Ver}}(X) = \sigma_R(X)$
Existence: $\phi^{\sigma}_{\mathsf{Exists}} = \exists X \sigma_R(X)$
Nonempty: $\phi^{\sigma}_{\mathsf{Exists}^{\neg\emptyset}} = \exists X \exists x (\sigma_R(X) \wedge x \in X)$

Unique: $\phi_{\mathsf{U}}^{\sigma} = \exists X \sigma_R(X)) \wedge \neg\exists Y(Y \neq X \wedge \sigma_R(Y))$
Note that these encodings do not use quantification over edge sets whenever σ is free of such a quantification. □

Moreover MSO is also a gentle tool to study the relation between different semantics, as illustrated by Theorem 3.

Theorem 3. *Given argumentation semantics σ, σ' expressible in MSO, the following tasks are fixed-parameter tractable w.r.t. the tree-width of the AF.*

- *Deciding whether $\sigma(F) = \sigma'(F)$ (Coincidence).*
- *Deciding whether arguments skeptically accepted w.r.t. σ are also skeptically accepted w.r.t. σ' (Skepticism 1).*
- *Deciding whether arguments credulously accepted w.r.t. σ are also credulously accepted w.r.t. σ' (Skepticism 2).*
- *Deciding whether $\sigma(F) \subseteq \sigma'(F)$ (Skepticism 3).*

If σ is expressible in MSO$_1$ the above tasks are also fixed-parameter tractable w.r.t. the clique-width of the AF.

Proof. By Theorem 1 and the MSO - encodings below:
Coincidence: $\phi_{\mathsf{Coin}}^{\sigma}(x) = \forall X\ (\sigma_R(X) \leftrightarrow \sigma'_R(X))$
Skepticism 1: $\phi_{\mathsf{sk1}}^{\sigma}(x) = \forall x(\phi_{\mathsf{Skept}}^{\sigma}(x) \rightarrow \phi_{\mathsf{Skept}}^{\sigma'}(x))$
Skepticism 2: $\phi_{\mathsf{sk1}}^{\sigma}(x) = \forall x(\phi_{\mathsf{Cred}}^{\sigma}(x) \rightarrow \phi_{\mathsf{Cred}}^{\sigma'}(x))$
Skepticism 3: $\phi_{\mathsf{sk1}}^{\sigma}(x) = \forall X(\sigma_{A,R}(X) \rightarrow \sigma'_{A,R}(X))$ □

One prominent instantiation of the first problem mentioned in Theorem 3 is deciding whether an AF is coherent, i.e. whether stable and preferred extensions coincide.

Most of the characterizations we have provided so far are actually in MSO$_1$ and by the above results we obtain fixed-parameter tractability for tree-width and clique-width. The notable exception is the schema (1) we provided for the resolution-based semantics. Obviously, it is not straight forward to reduce this MSO$_2$ formula into MSO$_1$ (and thus providing complexity results in terms of clique-width). Next we address this question for the resolution-based grounded semantics and present a corresponding MSO$_1$ encoding.

5 MSO$_1$-characterization for grd^*

We provide a novel characterisation of resolution-based grounded semantics which eliminates the quantification over sets of attacks in schema (1), exploiting results from Baroni, Dunne, and Giacomin (2011).

To this end we first restrict the class of resolutions we have to consider when showing that a set of arguments is a complete extension of some resolved AF.

Lemma 1. *Given AF $F = (A, R)$ and $E \in grd^*(F)$, then there exists a resolution β with $\{(b, a) \mid a \in E, b \notin E, \{(a, b), (b, a)\} \subseteq R\} \subseteq \beta$ such that $E \in com(A, R \setminus \beta)$.*

Proof. As $E \in grd^*(F)$ we have that there exists a resolution β' such that $E \in grd(A, R \setminus \beta')$. Now let us define β as $\{(b, a) \mid a \in E, \{(a, b), (b, a)\} \subseteq R\} \cup (\beta' \cap (A \setminus E \times A \setminus E))$. Clearly E is conflict-free in $(A, R \setminus \beta)$. Next we show that (i) $E^{\oplus}_{R\setminus\beta'} = E^{\oplus}_{R\setminus\beta}$ and (ii) $E^{\ominus}_{R\setminus\beta'} \supseteq E^{\ominus}_{R\setminus\beta}$.

For (i), let us first consider $b \in E^{\oplus}_{R\setminus\beta'}$. Then there exists $(a, b) \in R \setminus \beta'$ with $a \in E$ and by construction also $(a, b) \in R \setminus \beta$ and thus $b \in E^{\oplus}_{R\setminus\beta}$. Now let us consider $b \in E^{\oplus}_{R\setminus\beta}$. Then there exists $(a, b) \in R \setminus \beta$ with $a \in E$ and by construction either $(a, b) \in R \setminus \beta'$ or $(b, a) \in R \setminus \beta'$. In the first case clearly $b \in E^{\oplus}_{R\setminus\beta'}$. In the latter case b attacks E and as E is admissible in $(A, R \setminus \beta')$ there exists $c \in E$ such that $(c, b) \in R \setminus \beta'$, hence $b \in E^{\oplus}_{R\setminus\beta'}$. For (ii) consider $b \in E^{\ominus}_{R\setminus\beta}$, i.e. exists $a \in E$ such that $(b, a) \in R \setminus \beta$. By the construction of β we have that $(a, b) \notin R$ and therefore $(b, a) \in R \setminus \beta'$. Hence also $b \in E^{\ominus}_{R\setminus\beta'}$.

As $E \in adm(A, R \setminus \beta')$ we have that $E^{\ominus}_{R\setminus\beta'} \subseteq E^{\oplus}_{R\setminus\beta'}$ and by the above observations then also $E^{\ominus}_{R\setminus\beta} \subseteq E^{\oplus}_{R\setminus\beta}$. Thus E is an admissible set. Finally let us consider an argument $a \in A \setminus E^{\oplus}_{R\setminus\beta}$. In the construction of β the incident attacks of a are not effected and hence $\{a\}^{\ominus}_{R\setminus\beta'} = \{a\}^{\ominus}_{R\setminus\beta}$. That is E defends a in $(A, R \setminus \beta)$ iff E defends a in $(A, R \setminus \beta')$. Now as $E \in com(A, R \setminus \beta')$ we have that a is not defended and hence $E \in com(A, R \setminus \beta)$. □

With this result at hand, we can give an alternative characterization for resolution-based grounded semantics.

Lemma 2. *Given AF $F = (A, R)$ and $E \subseteq A$, Then $E \in grd^*(F)$ iff the following conditions hold*

1. *there exists a resolution β with $\{(b, a) \mid a \in E, \{(a, b), (b, a)\} \subseteq R\} \subseteq \beta$ and $E \in com(A, R \setminus \beta)$*
2. *E is $\subseteq$-minimal w.r.t. (1).*

Proof. Let us first recall that by definition the grounded extension is the $\subseteq$-minimal complete extension and hence $grd^* = com^*$.

$\Rightarrow$: Let $E \in grd^*(F)$. Then by Lemma 1, E fulfills condition (1). Further we have that each set E satisfying (1) is a complete extension of a resolved AF. As by definition E is $\subseteq$-minimal in the set of all complete extensions of all resolved AFs it is also minimal for those satisfying (1).

$\Leftarrow$: As E satisfies (1) it is a complete extension of a resolved AF. Now towards a contradiction let us assume it is not a resolution-based grounded extension. Then there exists $G \in grd^*(F)$ with $G \subset E$. But by Lemma 1 G fulfills condition (1) and thus $G \subset E$ contradicts (2). □

In the next step we look for an easier characterization of condition (1) in the above lemma.

Lemma 3. *For an AF $F = (A, R)$ and $E \subseteq A$ the following statements are equivalent*

1. *There exists a resolution β with $\{(b, a) \mid a \in E, \{(a, b), (b, a)\} \subseteq R\} \subseteq \beta$ and $E \in com(A, R \setminus \beta)$*

2. $E \in com(A, R \setminus \{(b,a) \mid a \in E, \{(a,b),(b,a)\} \subseteq R\})$ *and* $grd^*(A \setminus E_R^+, R \cap ((A \setminus E_R^+) \times (A \setminus E_R^+))) = \{\emptyset\}$.

Proof. In the following we will use the following shorthands, $R^* = R \setminus \{(b,a) \mid a \in E, \{(a,b),(b,a)\} \subseteq R\}$ and $(A', R') = (A \setminus E_R^+, R \cap ((A \setminus E_R^+) \times (A \setminus E_R^+)))$.

(1) $\Rightarrow$ (2): Consider a resolution β such that $E \in com(A, R \setminus \beta)$. We first show that then also $E \in com(A, R^*)$. By construction we have that for arbitrary $b \in A$ that (a) $E \rightarrowtail^R b$ iff $E \rightarrowtail^{R\setminus\beta} b$ iff $E \rightarrowtail^{R^*} b$, and (b) $b \rightarrowtail^{R\setminus\beta} E$ iff $b \rightarrowtail^{R^*} E$. Hence we have that (i) $E \in adm(A, R \setminus \beta)$ iff $E \in adm(A, R^*)$ and (ii) $E_R^+ = E_{R\setminus\beta}^+ = E_{R^*}^+$. By definition of complete semantics, $E \in com(A, R\setminus\beta)$ is equivalent to for each argument $b \in A \setminus E$ there exists an argument $c \in A$ such that $c \rightarrowtail^{R\setminus\beta} b$ and $E \not\rightarrowtail^{R\setminus\beta} c$. As $R^* \supseteq R \setminus \beta$ we obtain that $(c,b) \in R \setminus \beta$ implies $(c,b) \in R^*$. Using (a) we obtain that $E \in com(A, R \setminus \beta)$ implies for each argument $b \in A \setminus E$ there exists an argument $c \in A$ such that $(c,b) \in R^*$ and $E \not\rightarrowtail^{R^*} c$, i.e. $E \in com(A, R^*)$.

Now addressing $grd^*(A', R') = \{\emptyset\}$ we again use the assumption $E \in com(A, R\setminus\beta)$, i.e. each argument which is defended by E is already contained in E, we have that $grd(A \setminus E_{R\setminus\beta}^+, R \setminus \beta \cap ((A \setminus E_R^+) \times (A \setminus E_R^+))) = grd(A', R' \setminus \beta) = \{\emptyset\}$. Note that $\beta' = \beta \cap R'$ is a resolution of (A', R') and that $grd(A', R' \setminus \beta) = grd(A', R' \setminus \beta') = \{\emptyset\}$. We can conclude that $grd^*(A', R') = \{\emptyset\}$.

(1) $\Leftarrow$ (2): Let β' be a resolution such that $grd(A', R' \setminus \beta') = \{\emptyset\}$; such a β' exists since $grd^*(A', R') = \{\emptyset\}$. Now consider the resolution $\beta = \{(b,a) \mid a \in E, \{(a,b),(b,a)\} \subseteq R\} \cup \beta'$. Again, by construction of β we have that for arbitrary $b \in A$: (a) $E \rightarrowtail^R b$ iff $E \rightarrowtail^{R\setminus\beta} b$ iff $E \rightarrowtail^{R^*} b$, and (b) $b \rightarrowtail^{R\setminus\beta} E$ iff $b \rightarrowtail^{R^*} E$. Hence we obtain that $E \in adm(A, R \setminus \beta)$. Using $R = E_{R\setminus\beta}^+ = E_{R^*}^+$ we have $grd(A \setminus E_{R\setminus\beta}^+, (R\setminus\beta) \cap ((A \setminus E_R^+) \times (A \setminus E_R^+))) = grd(A', R' \setminus \beta') = \{\emptyset\}$. Thus, $E \in com(A, R \setminus \beta)$. □

Proposition 2. *Baroni, Dunne, and Giacomin (2011) For an AF $F = (A, R)$, $grd^*(F) = \{\emptyset\}$ iff for each minimal SCC S of F one of the following conditions holds: S contains a self-attacking argument; S contains a non-symmetric attack; or S contains an undirected cycle*

Based on the above observations we obtain the following characterization of resolution-based grounded semantics

Theorem 4. *Given AF $F = (A, R)$, the grd^*-extensions are the $\subseteq$-minimal sets $E \subseteq A$ such that:*

- $E \in com(A, R')$ *with* $R' = R \setminus \{(b,a) \mid a \in E, \{(a,b),(b,a)\} \subseteq R\})$
- *Each minimal SCC S of $\hat{F} = (A \setminus E_R^+, R \cap A \setminus E_R^+ \times A \setminus E_R^+)$ satisfies one of the following conditions: S contains a self-attacking argument; S contains a non-symmetric attack; or S contains an undirected cycle*

Having Theorem 4 at hand we can build an MSO_1 encoding as follows. First we encode the attack relation R' as $R'_E(x,y) = (x,y) \in R \wedge \neg(x \in E \wedge y \notin E \wedge (x,y) \in R \wedge (y,x) \in R)$. Then the AF $\hat{F} = (\hat{A}, \hat{R})$ is given by:

$$\hat{A}_{A,R,E}(x) = x \in A \wedge x \notin E \wedge \neg\exists y \in E : R'_E(y,x)$$
$$\hat{R}_{E,R}(x,y) = (x,y) \in R \wedge A^*_{A,R,E}(x) \wedge A^*_{A,R,E}(y)$$

Based on reachability we can easily specify whether arguments are strongly connected $SC_R(x,y) = reach_R(x,y) \wedge reach_R(y,x)$, and a predicate that captures all arguments in minimal SCCs $minSCC_{A,R}(x) = A(x) \wedge \neg\exists y\ (A(y) \wedge reach_R(y,x) \wedge \neg reach_R(x,y))$. It remains to encode the check for each SCC.

$$
\begin{aligned}
C1_R(x) &= \exists y(SC_R(x,y) \wedge (y,y) \in R) \\
C2_R(x) &= \exists y,z(SC_R(x,y) \wedge SC_R(x,z) \\
&\qquad\qquad \wedge (y,z) \in R \wedge (z,y) \notin R) \\
C3_R(x) &= \exists X(\exists y \in X \wedge \forall y \in X[SC_R(x,y) \wedge \\
&\qquad\qquad \exists u,v \in X : u \neq v \wedge (u,y) \in R \wedge (y,v) \in R]) \\
C_R(x) &= C1_R(x) \vee C2_R(x) \vee C3_R(x)
\end{aligned}
$$

Finally using Theorem 4 we obtain an MSO_1 encoding for resolution-based grounded semantics:

$$grd^*{}_{A,R}(X) = cand_{A,R}(X) \wedge \neg\exists Y(cand_{A,R}(Y) \wedge Y \subset X)$$

where $cand_{A,R}(X)$ stands for $com_{A,R'_X}(X) \wedge \forall x(minSCC_{\hat{A}_{A,R,E},\hat{R}_{E,R}}(x) \rightarrow C_{\hat{R}_{E,R}}(x))$.

6 Conclusion

In this paper we have shown that the language of monadic second order logic (MSO) is a suitable unifying framework for abstract argumentation. We encoded the most popular semantics within MSO and gave building blocks illustrating that MSO can naturally capture several semantics concepts. This is vital for new semantics where MSO can be used as rapid prototyping tool.

Based on the work of Baroni, Dunne, and Giacomin Baroni, Dunne, and Giacomin (2011), we presented a new characterisation of resolution-based grounded semantics allowing for an MSO_1 encoding. This shows that reasoning in this semantics is tractable for frameworks of bounded clique-width. In fact, the collection of encodings we provided here shows that acceptance as well as other reasoning tasks are fixed parameter tractable for several semantics w.r.t. the clique-width (hence also for tree-width).

For future work we suggest to study whether also other instantiations of the resolution-based semantics can be expressed in MSO_1 (recall that we provided already a schema for MSO_2 encodings). Moreover, we will compare the performance of MSO tools with dedicated argumentation systems. Finally, we want to advocate the use of MSO for automated theorem discovery Tang and Lin (2011). In fact, our encodings allow to express meta-statements like "does it hold for AFs F that each σ-extension is also a σ'-extension". Although we have to face undecidability for such formulas, there is the possibility that MSO-theorem provers come up with a counter-model. Thus, MSO can be used to support the argumentation researcher in obtaining new insights concerning the wide range of different argumentation semantics.[2]

[2]In recent work related to this issue, Weydert Weydert (2011) used an first-order encoding of complete semantics to show certain properties for semi-stable semantics of infinite AFs.

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