Logic Argument and Dialectic

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Outline of Talk

- Brief review of Dung's argumentation theory and its **dialectical** characterisation of non-monotonic Inference
- Limitations of **structured** argumentation for a resource-bounded practical account of dialectical reasoning
- A new account of **classical logic** argumentation that accommodates real-world modes of dialectical reasoning and is rational under resource bounds

Argumentation and Non-monotonic Reasoning

Dung's Argumentation Theory [1]

- Given a set Δ of wff in some logic L :

  1) Construct arguments (*Args*) from Δ

  2) Define conflict based defeat relation (*Def*) amongst *Args*

  3) Evaluate justified (winning) arguments in directed graph (*Args*,*Def*)

1. P. M. Dung. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77(2):321–358, 1995

Argumentation-based characterisations of non-monotonic inference relations

$(Args,Def)_{\Delta}$ $\alpha \mid\!\sim$ (the claim of a justified argument)

$$\Updownarrow$$

$$\Delta \mid\!\sim_{nml} \alpha$$

E.g. *nml* = Logic Programming, Default Logic, *Prioritised* Default Logic, Defeasible Logic, Preferred Subtheories ….

Why do we bother ?

- Basis for individual agent epistemic and practical reasoning that uses intuitive principle of reinstatement familiar in everyday reasoning

- Basis for formalising dialogue amongst *computational and/or human* agents engaging dialectically in distributed epistemic/practical reasoning

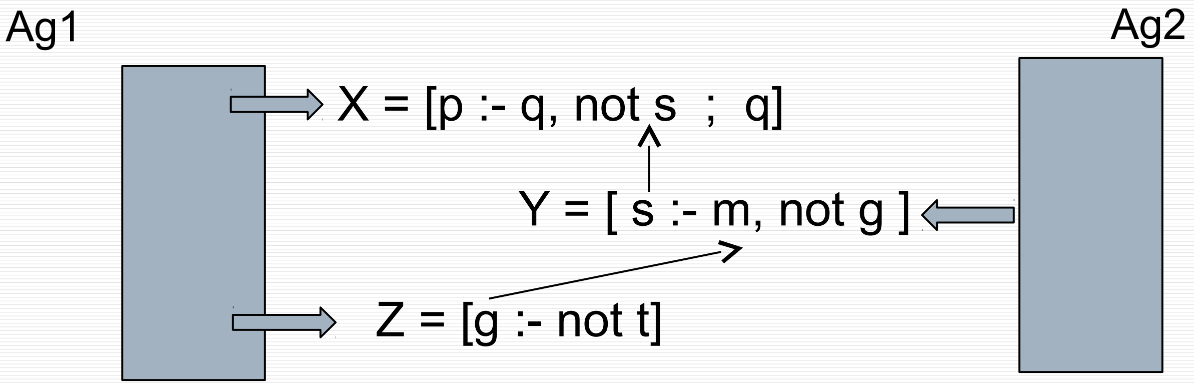
- Dialectic:

  Merriam Webster = *discussion and reasoning by dialogue as*

  a method of intellectual investigation

Distributed non-monotonic reasoning through argumentation-based dialogue

- Argumentation-based dialogue establishing *p* in which Δ *incrementally defined* by contents of locutions



Desirable Properties of Argumentation

What are desirable properties for argumentation formalisms that underpin dialectical reasoning by **real world** *computational/human agents* ?

- *Logic-based rationality criteria*: e.g claims of arguments in an extension are mutually consistent

- *Practical desiderata*: formalise dialectical reasoning as used **in practice** by **resource bounded** agents

But these rational and practical desiderata are incompatible for argumentation based characterisations of non-monotonic inference

Limitations of Structured Argumentation

Structured Argumentation

- ASPIC+ [1] captures various structured approaches to argumentation and provides dialectical characterisations of non-monotonic inference relations (*Logic Programming*,*Default Logic*, *Prioritised Default Logic*, *Preferred Subtheories*)

- Arguments consist of premises and defeasible and/or strict inference rules where strict inference rules encode inference in some deductive logic, e.g.

$$\alpha_1 , \ldots , \alpha_n \rightarrow \alpha \text{ iff } \alpha_1 , \ldots , \alpha_n \vdash_{CL} \alpha$$

- Preferences over arguments determine which attacks succeed as defeats and ASPIC+ identifies sufficient conditions for satisfaction of rationality postulates

- However these conditions are incompatible with satisfaction of practical desiderata because of deductive (in particular *classical logic*) reasoning used in arguments

1. S.Modgil and H. Prakken. A General Account of Argumentation and Preferences. *Artificial Intelligence,*195(0):361–397, 2013

Classical Logic Argumentation with Preferences

Let Δ be a possibly inconsistent set of classical formulae

- X$\in$ *Args* is a pair (Γ,α) such that *premises* $\Gamma \subseteq \Delta$, and :

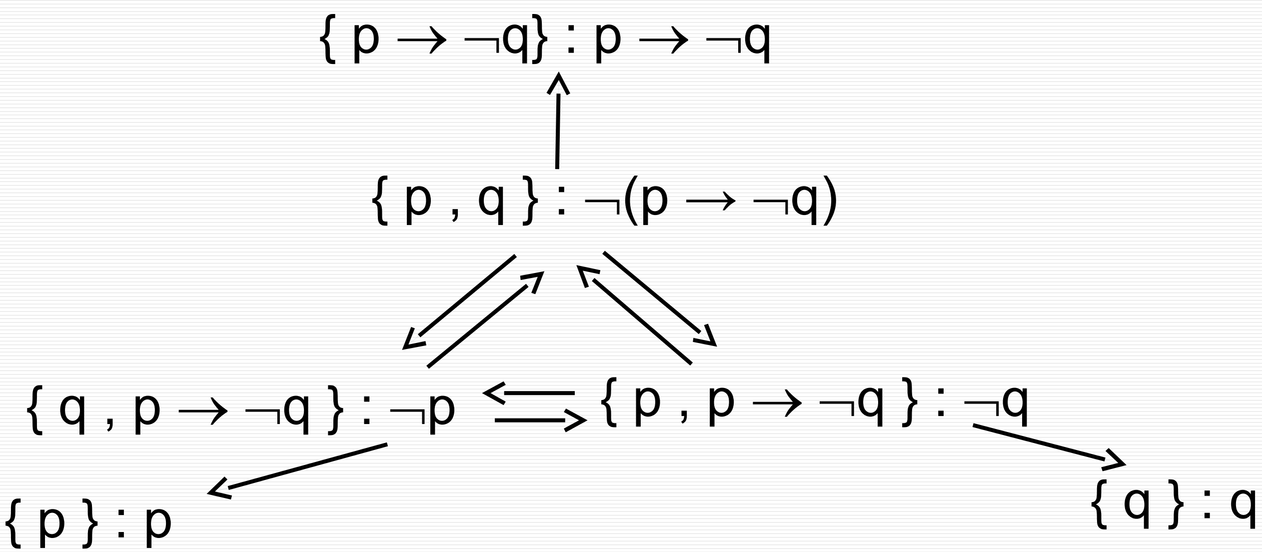
1) $\Gamma \quad \alpha \quad \vdash_{CL}$
2) Γ is consistent
3) No proper subset of Γ entails α

- (Γ,α) *attacks* (Σ,β) if *premise* $\neg\alpha \in \Sigma$

- (Γ,α) *defeats* (Σ,β) if (Γ,α) *attacks* (Σ,β) and $(\Gamma,\alpha) \nprec (\{\neg\alpha\}, \neg\alpha)$

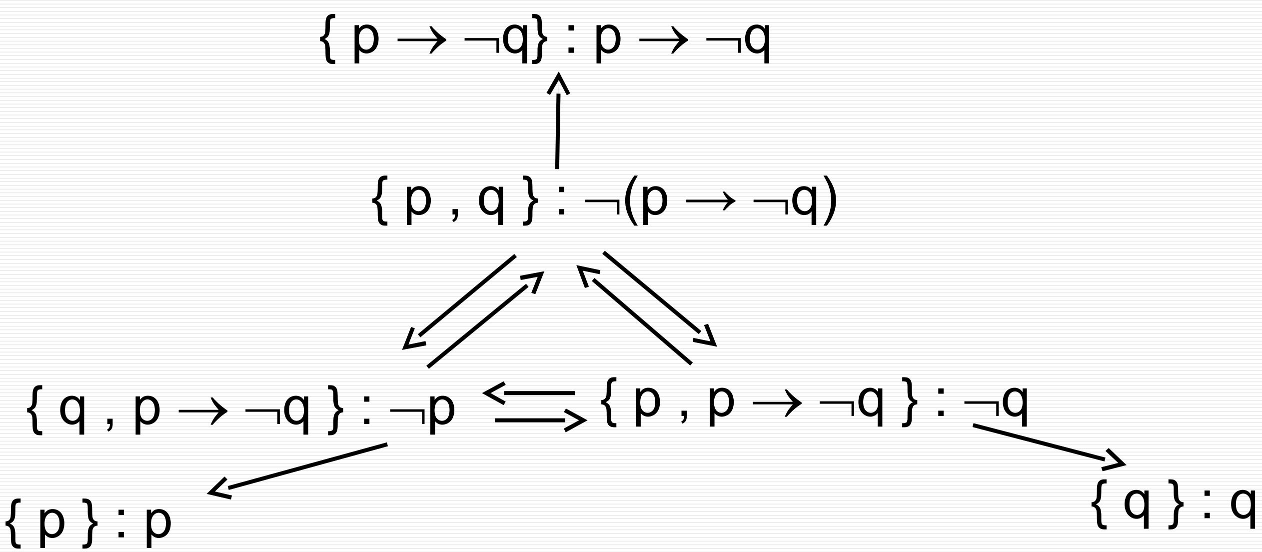
Classical Logic Argumentation : An Example

- (*Args*,*Attacks*) defined by $\Delta = (p\ ,\ q\ ,\ p \rightarrow \neg q)$



Classical Logic Argumentation : An Example

- (*Args*,*Attacks*) defined by $\Delta = (p\ ,\ q\ ,\ p \rightarrow \neg q)$ and $p \rightarrow \neg q < p \approx q$



Classical Logic Argumentation : An Example

- (*Args*,*Defeats*) defined by $\Delta = (p\ ,\ q\ ,\ p \rightarrow \neg q)$ and $p \rightarrow \neg q < p \approx q$

$\{\ p \rightarrow \neg q\} : p \rightarrow \neg q$

$\{\ p\ ,\ q\ \} : \neg(p \rightarrow \neg q)$

$\{\ q\ ,\ p \rightarrow \neg q\ \} : \neg p$ $\quad \{\ p\ ,\ p \rightarrow \neg q\ \} : \neg q$

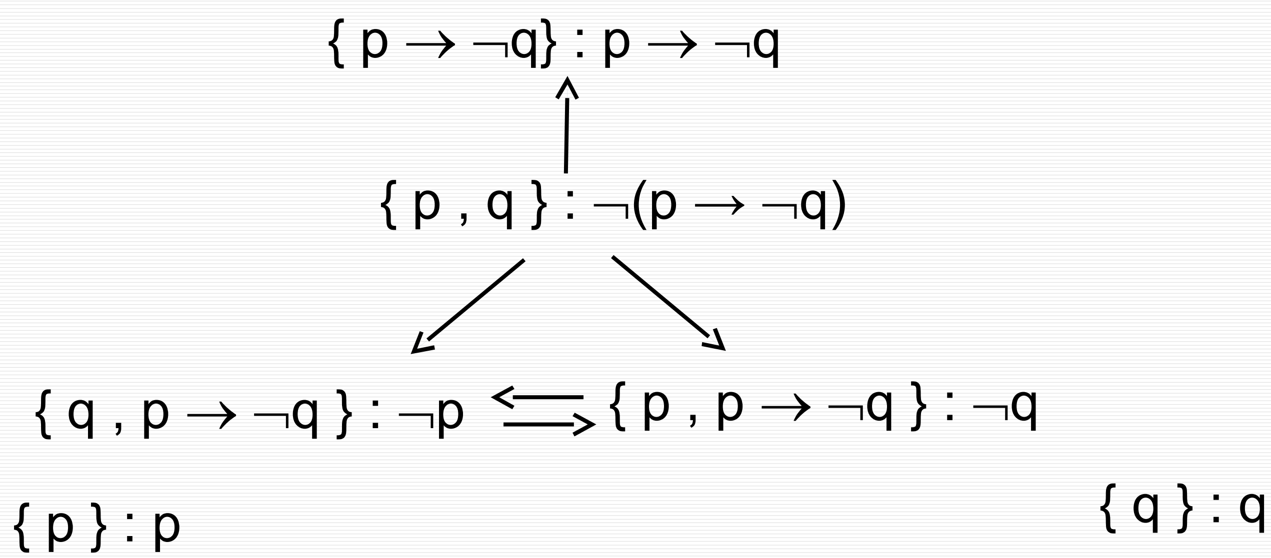
$\{\ p\ \} : p$ $\quad \{\ q\ \} : q$

Single preferred extension containing $\{\ \{p\} : p\ ,\ \{q\} : q\ ,\ \{p,q\} : \neg(p \rightarrow \neg q)\ \}$

$\Longrightarrow\ \Delta\ |\!\sim\ Cn(p,q)$

Classical Logic Argumentation : An Example

- (*Args*,*Defeats*) defined by $\Delta = (p\ ,\ q\ ,\ p \rightarrow \neg q)$ and $p \rightarrow \neg q < p \approx q$



$(\Delta, \leq)\ |\!\sim_{nml}\ Cn(p,q)$ *nml = Brewka's Preferred Subtheories*

1. S.Modgil and H. Prakken. A General Account of Argumentation and Preferences. *Artificial Intelligence,*195(0):361–397, 2013

Is Structured (Classical Logic) Argumentation suitable for real-world applications ?

- X$\in$ $\mathcal{A}rgs$ is a pair (Γ,α) such that $\Gamma \subseteq \Delta$, and :

1) $\Gamma \vdash_{CL} \alpha$
2) Γ is consistent
3) No proper subset of Γ entails α

Practicality

- We want an account of argumentation for dialectical reasoning by resource bounded computational/human agents

1) Consistency checks are **computationally expensive**

2) In practice we typically don’t interrogate the consistency of our arguments’ premises prior to use in discussion and debate

Rather, we witness the Socratic **dialectical move** whereby the premises of an interlocutor’s arguments are shown to be inconsistent (generalising the case where the premises of an interlocutor’s individual argument is shown to be inconsistent)

Rationality versus Practicality

- But consistency check on premises prevents violation of ***non-contamination*** postulates [1]

For example

$$\Delta = \{p, \neg p, s\}$$

If we allow $(\{p, \neg p\}, \neg s)$ then $(\{s\},s)$ is not justified under grounded semantics !

1. M. Caminada, W. Carnielli, and P. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 2011

Practicality

- We want an account of argumentation for dialectical reasoning by resource bounded computational/human agents

- Subset minimality checks are intended to ensure premises are relevant with respect to deriving the claim

- However checking for subset minimality is **computationally unfeasible**

- Moreover, checking for subset minimality fails to account for the fact that the **relevance** of a set of premises w.r.t deriving a claim is **addressed proof-theoretically**

Rationality versus Practicality

- But subset minimality check on premises prevents ***contamination***

For example

$$\Delta = \{p, \neg p, s\}$$

- $(\{\neg p\}, \neg p) < (\{p\}, p)$ and so $(\{\neg p\}, \neg p)$ **does not *defeat*** $(\{p\}, p)$ which is therefore justified under grounded semantics

- But suppose we allow $(\{\neg p,s\}, \neg p)$ and $(\{\neg p,s\}, \neg p) \nless (\{p\}, p)$ and so $(\{\neg p,s\}, \neg p)$ **does defeat** $(\{p\}, p)$

Then $(\{p\},p)$ is not justified under grounded semantics !

Practicality

- We want an account of argumentation for dialectical reasoning by resource bounded computational/human agents

- Tacit assumption that ***all*** arguments defined by a set Δ are included in ($Args$,Def)

i.e., every (Γ,α) s.t. Γ is a minimal consistent subset of Δ that entails α !

- Computationally unfeasible for resource-bounded agents

Suppose we drop this assumption:

Rationality versus Practicality

- (*Args*,*Attack*) defined by $\Delta = (p\ ,\ q\ ,\ q \rightarrow \neg p)$

$$A = \{\ q\ ,\ q \rightarrow \neg p\ \} : \neg p$$

$$\downarrow$$

$$B = \{\ p\ \} : p$$

Rationality versus Practicality

- (*Args*,*Defeat*) defined by $\Delta = (p\ ,\ q\ ,\ q \rightarrow \neg p)$ and $A < B$

$$A = \{\, q\ ,\ q \rightarrow \neg p \,\} : \neg p$$

$$\not\downarrow$$

$$B = \{\, p \,\} : p$$

Rationality versus Practicality

- (*Args*,*Defeat*) defined by $\Delta = (p \, , \, q \, , \, q \rightarrow \neg p)$ and $A < B$

$$A = \{ q , q \rightarrow \neg p \} : \neg p$$

$$B = \{ p \} : p$$

- Single inconsistent preferred extension {A,B}
- To guarantee consistency need to construct following arguments attacking A:

  $C = \{p, q \rightarrow \neg p \} : \neg q$ and $D = \{p, q\} : \neg(q \rightarrow \neg p)$

Rationality versus Practicality

- (*Args*,*Defeat*) defined by $\Delta = (p\ ,\ q\ ,\ q \rightarrow \neg p)$ and $A < B$

$$A = \{ q\ ,\ q \rightarrow \neg p \} : \neg p$$

$$B = \{ p \} : p$$

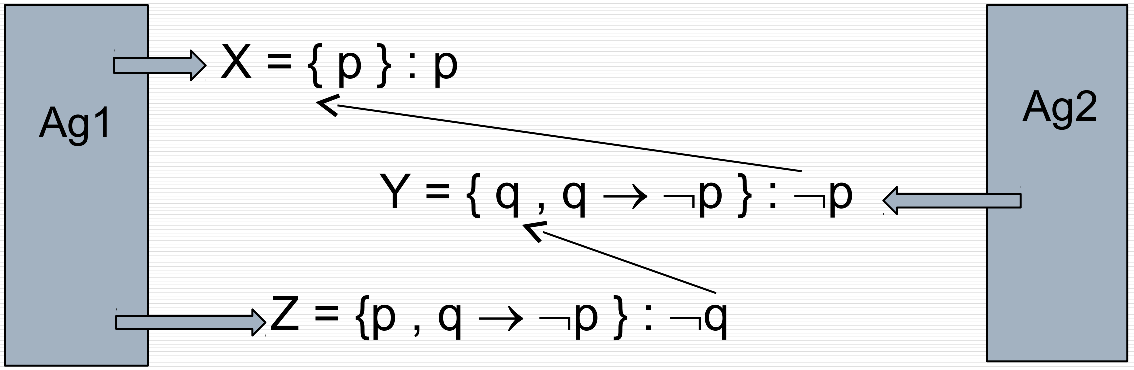
- Single inconsistent preferred extension {A,B}
- To guarantee consistency need to construct following arguments attacking A:

  $C = \{p,\ q \rightarrow \neg p \} : \neg q$ and $D = \{p,\ q\} : \neg(q \rightarrow \neg p\ )$

and assume a *reasonable* preference relation such that either C $\nprec$ A or D $\nprec$ A

The Foreign Commitment Problem

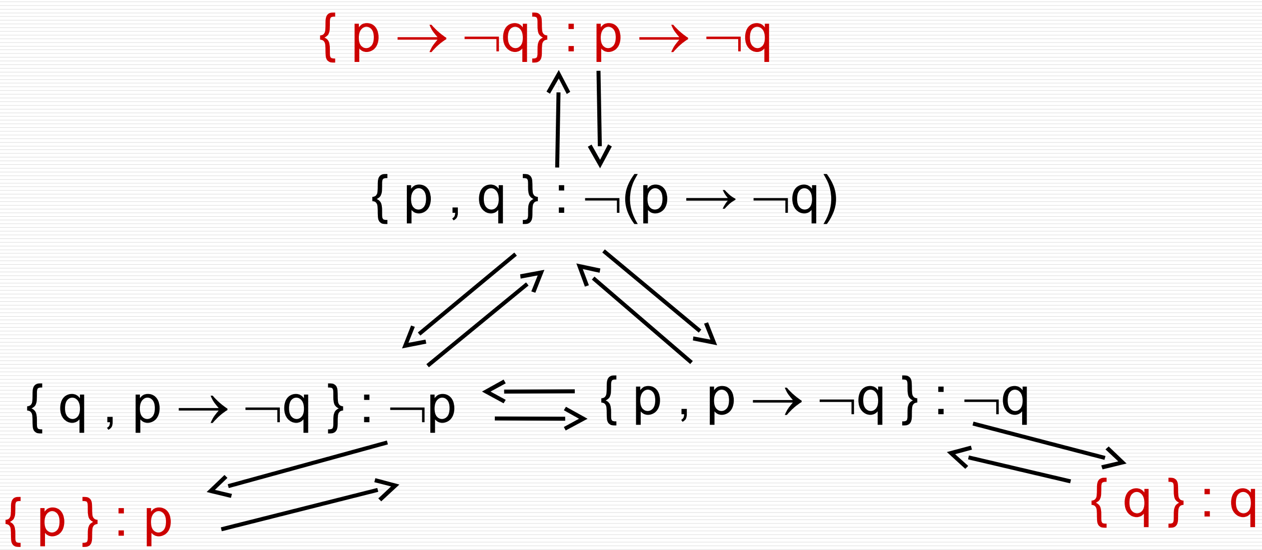
- Ag1 defends X by constructing an admissible extension containing X, but Ag1 cannot counter Y with X as attacks can only target premises



Classical Logic Argumentation with Attacks Targeting Conclusions

- Inconsistent preferred extension containing $\{p \rightarrow \neg q\} : p \rightarrow \neg q$

  $\{ p \} : p$ and $\{ q \} : q$



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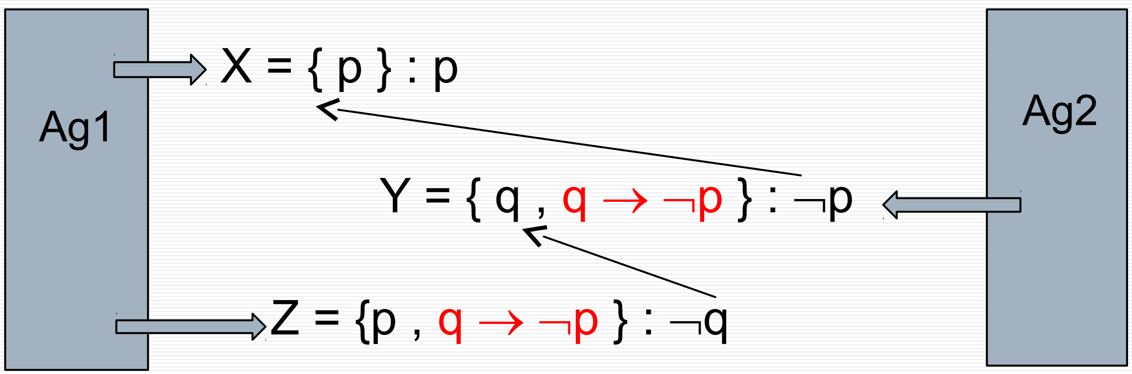


Ag1 is forced to counter with either {p , q } : ¬(q → ¬p) or Z.

In either case Ag1 makes a “foreign commitment” to the premise of his interlocutor (q → ¬p in Z)

Practicality and Rationality

- (*Args*,*Defeat*) defined by $\Delta = (p\ ,\ q\ ,\ q \rightarrow \neg p)$ and $A < B$

$$A = \{\, q\ ,\ q \rightarrow \neg p \,\} : \neg p$$

$$B = \{\, p \,\} : p$$

- Ideally one would want to preclude A and B being jointly acceptable by simply recognising that conflicting conclusions imply mutually inconsistent premises

Want to avoid assuming that resources available to construct $\{p,\ q \rightarrow \neg p\ \} : \neg q$

$\{p,\ q\} : \neg(q \rightarrow \neg p\ )$ (and the implied foreign commitment problem) and shouldn't have to assume *reasonable* preference relation

Rationality *and* Practicality

- We want an account of argumentation for dialectical reasoning by resource bounded agents:

  1) Drops computationally expensive consistency and subset minimality checks on arguments, while preserving rationality (non-contamination)

  2) Enables dialectical move of showing that interlocutor contradicts himself

  3) Accommodates resource bounded agents who do not construct all arguments, and can make use of *any* preference relation, while still satisfying consistency postulates

Dialectical Classical Logic Argumentation [1,2]

Joint work with Marcello D'Agostino,
Dept. of Philosophy, University of Milan

1. M. D'Agostino and S.Modgil *A Rational Account of Classical Logic Argumentation for Real-world Agents.*, In: **European Conference on Artificial Intelligence (ECAI 2016)**, 141 - 149, 2016.
2. M. D'Agostino and S.Modgil *Classical Logic, Argument and Dialectic.*, Submitted to: **Artificial Intelligence**

A dialectical ontology for arguments

- The solution is to define an ontology and evaluation of arguments that accounts for a ubiquitous feature of *dialectical* reasoning.
- In practice, arguments are of the following form :

  Given that I believe Π (premises) and supposing for the sake of argument your premises (suppositions) Γ, then it follows that α

- Given a set Δ of propositional classical wff, an argument is now a triple:

$$X = (\Pi, \Gamma, \alpha)$$

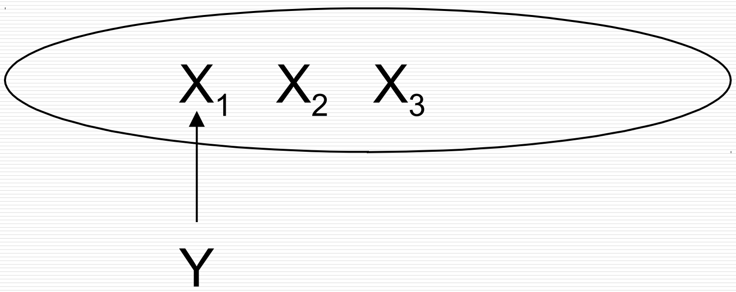
where $(\Pi \cup \Gamma) \subseteq \Delta$ and $\Pi \cup \Gamma \vdash_{CL} \alpha$

- Note that we drop the consistency (and subset minimality) check on *prem*(X) $\cup$ *supp*(X) = $\Pi \cup \Gamma$

Dialectical Defeat and Defense

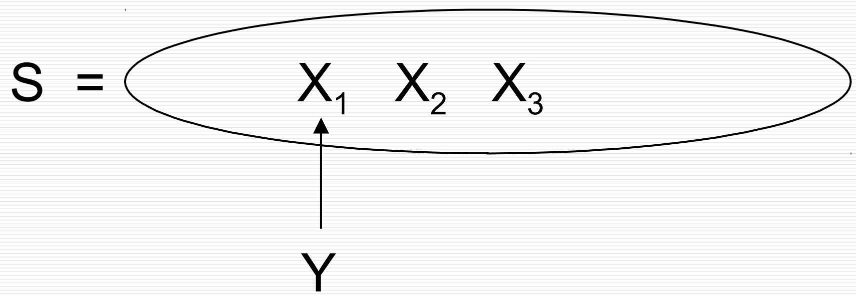
- Recall that an ‘extension’ S is a set of arguments that defend themselves against all defeats

$S =$



- $Y = (\Pi, \Gamma, \alpha)$ *dialectically defeats* $X_1 = (\Delta_1, \Sigma_1, \beta)$ if $\neg\alpha$ is in the **premises** $\boldsymbol{\Delta_1}$ of X_1

Dialectical Defeat and Defense

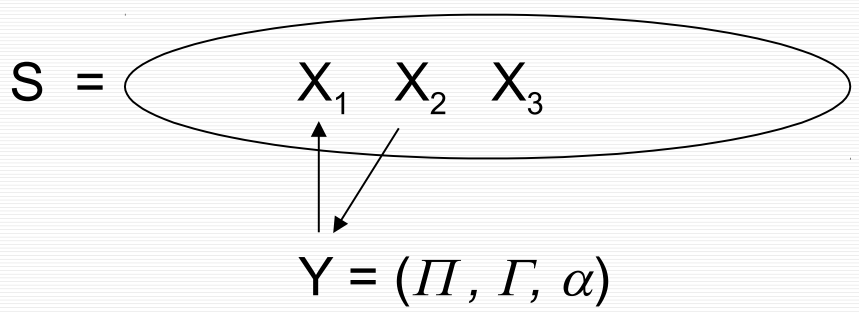


- Y = (Π , Γ, α) *dialectically defeats* X_1 = (Δ_1 , Σ_1 , β) if $\neg\, \alpha$ is in the **premises Δ_1** of X_1

  and $\Gamma \subseteq prem(X_1) \cup prem(X_2) \cup prem(X_3)$

- Intuitively, given my premises Π and supposing for the sake of argument the premises Γ you've committed to (in the arguments in S), then Y is a counter-argument to X_1

Dialectical Defeat and Defense



- X_2 counter-argues Y (and so defends X_1) if

  $X_2 = (\Delta_2 , \Sigma_2 , \gamma)$, $\neg \gamma \in \Pi$ *and* $\Sigma_2 \subseteq \Pi$

- Intuitively, given my premises Δ_2 and supposing for the sake of argument the premises Σ_2 you've committed to (in the premises Π of Y), then X_2 is a counter-argument to Y

Arguments showing inconsistency of opponent's premises

- Suppose Δ is an inconsistent set of premises committed to by an agent in a set S of arguments
- Then $(\emptyset, \Delta, \perp)$ attacks arguments in S that use premises Δ
- $(\emptyset, \Delta, \perp)$ is called a *falsum* argument
- Preferences over dialectical arguments are used in the usual way to define defeats, except that ***attacks from falsum arguments always succeed as defeats***

What we want is :

- An account of argumentation that :

  1) Drops computationally expensive consistency and subset minimality checks on arguments, while preserving rationality (non-contamination[1])

  2) Enables dialectical move of showing that an interlocutor has contradicted himself

  3) Accommodates resource bounded agents who cannot construct all arguments, while preserving rationality (consistency[2])

1. M. Caminada, W. Carnielli, and P. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 201
2. M. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artificial Intelligence,*171(5-6):286–310, 2007

Non-contamination postulates satisfied

$$S = \{ \quad A = (\{s\}, \emptyset, s) \quad , \quad B = (\emptyset, \{p, \neg p\}, \bot) \quad \}$$

$$C = (\{p, \neg p\}, \neg s)$$

- Contaminating argument C is countered by defending argument B
- Since B is a falsum argument, B's attack on C succeeds as a defeat independently of preferences over arguments
- Since B has empty premises it cannot be defeated by any argument and so is a member of **any** extension S

Non-contamination postulates satisfied

$$S = \{ \quad A = (\{p\}, \emptyset, p) \quad \}$$

$B < A$ $\quad$ $C \nless A$

$$B = (\{\neg p\}, \neg p) \qquad C = (\{\neg p, s\}, \neg p)$$

- We identify a notion of relevance that does not depend on subset minimality:

$(\Delta, \emptyset, \Box\Box)$ is *relevant* if no non-empty $\Gamma \subset \Delta$ syntactically disjoint from $\Delta - \Gamma \cup \{\Box\}$

one can define proof theories that only license construction of *relevant* proofs so that irrelevant proofs (arguments) such as C cannot be constructed using proof rules *

- M. D'Agostino, D.M. Gabbay, S. Modgil. Normality and non-contamination in depth-bounded natural deduction for classical propositional logic.',Technical Report, http://www.filosofia.unimi.it/dagostino/wp-content/uploads/2017/07/TR_final.pdf

Non-contamination postulates satisfied

$$S = \{ \quad A = (\{p\}, \varnothing, p) \quad \}$$

$B < A$ $\qquad$ $C \not< A$

$B = (\{\neg p\}, \neg p)$ $\qquad$ $C = (\{\neg p, s\}, \neg p)$

- If proof theory doesn't exclude *irrelevant* arguments (such as C) then non contamination satisfied only if preference relation does not make an argument stronger when including syntactically disjoint premises

- Some preference relations (e.g. well known *Elitist* preference) satisfy this property since arguments cannot be strengthened by adding premises

What we want is :

- An account of argumentation that :

  1) Drops computationally expensive consistency and subset minimality checks on arguments, while preserving rationality (non-contamination[1])

  2) Enables dialectical move of showing that an interlocutor has contradicted himself

  3) Accommodates resource bounded agents who cannot construct all arguments, while preserving rationality (consistency[2])

1. M. Caminada, W. Carnielli, and P. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 201
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Dialectical demonstration that premises of an argument (arguments) are mutually inconsistent

$$S = \{ \quad A = (\{ q , q \rightarrow \neg p\}, \emptyset, \neg p) \quad B = (\{p\} , \emptyset, p) \quad \} \qquad A < B$$

$$C = (\emptyset, \{p, q , q \rightarrow \neg p\} , \bot)$$

- C demonstrates that proponent of S = {A,B} has contradicted himself.
- C targets arguments (A and B) in S that use premises p, q , $q \rightarrow \neg p$, and since C is a falsum argument, these attacks always succeed as defeats
- Since C has empty premises, no argument in S can defend against defeats from C. Hence S cannot be an extension containing justified arguments
- Requires no assumptions as to the properties of the preference relation and does not assume construction of $\{p, q \rightarrow \neg p \} : \neg q$ and $\{p, q\} : \neg(q \rightarrow \neg p)$

Dialectical argumentation accommodates resource bounded agents

- What conditions on construction of arguments suffice to show rationality ?

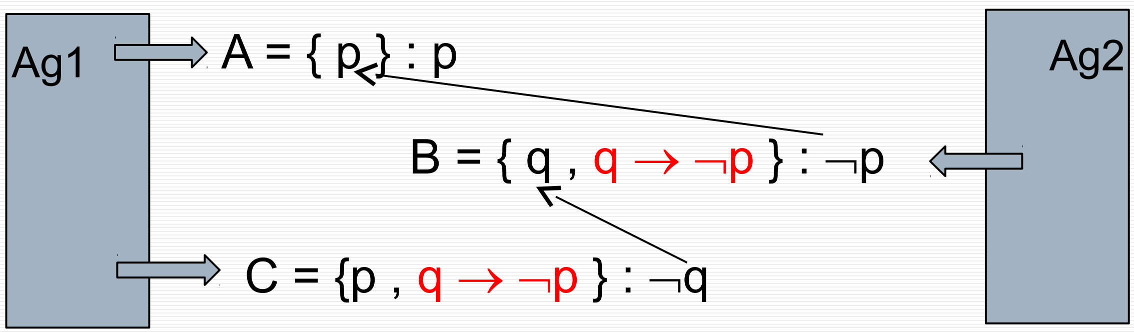
1) if $\square \in \Delta$ then $(\{\square\}, \emptyset, \square) \in \mathit{Args}$

2) if $(\Delta, \emptyset, \square)$, $(\Gamma, \emptyset, \neg \square) \in \mathit{Args}$ then $(\emptyset, \Delta \cup \Gamma, \bot) \in \mathit{Args}$

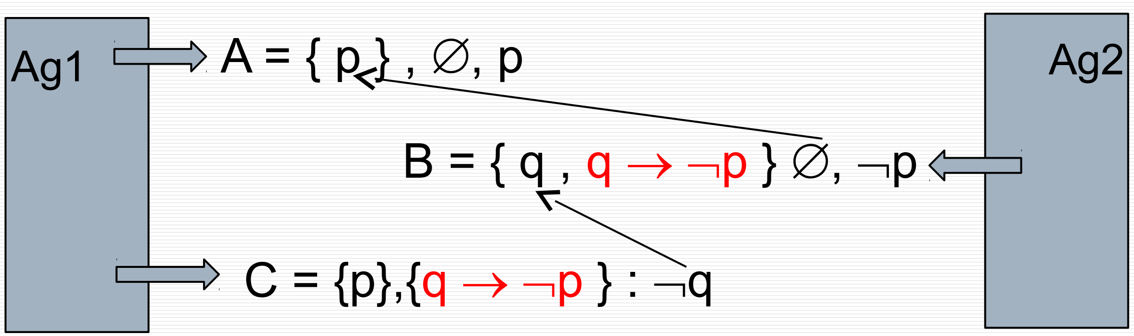
3) if $(\Delta \cup \Gamma, \emptyset, \square) \in \mathit{Args}$ and Γ syntactically disjoint from $\Delta \cup \{\square\}$ then
$(\Delta, \emptyset, \square) \in \mathit{Args}$ or $(\Gamma, \emptyset, \bot) \in \mathit{Args}$
(if proof theory licenses construction of irrelevant arguments)

4) if $(\Delta, \emptyset, \square) \in \mathit{Args}$ and Δ syntactically disjoint from $\{\square\}$ then
$(\Delta, \emptyset, \bot) \in \mathit{Args}$
(if proof theory excludes construction of irrelevant arguments)

Dialectical Argumentation Solves the Foreign Commitment Problem



Ag1 makes a foreign commitment to the premise $q \rightarrow \neg p$



Ag1 argues that given p, and supposing your premise $q \rightarrow \neg p$

Dialectical Classical Logic Argumentation: Results

- We have shown that all rationality postulates satisfied by **partially instantiated** AFs

  1) Non contamination (non-interference and crash resistance postulates)

  2) Consistency

  3) Sub-argument closure

  4) Closure under strict rules

  (where 2), 3) and 4) satisfied assuming **any preference relation**)

- We have shown satisfaction of key properties of Dung Argument Frameworks (non-trivial since dialectical attacks on an X vary depending on the set containing X)

  1) The fundamental lemma

  2) Monotonicity of the Defense (Characteristic) Function

A dialectical characterisation of Resource Bounded Preferred Subtheories

- $<_{KB}$ is a total ordering on the formulae in a KB of classical formulae and can be used to define Preferred Subtheories non-monotonic inference [1]
- One can define Preferred Subtheories inference assuming restricted inferential capabilities $\vdash_r \subseteq \vdash_{CL}$ s.t.

  1) if [illegible] $\in \Delta$ then Δ [illegible] $\vdash_r$

  2) if Δ [illegible] and $\vdash_r$ Δ $\neg$[illegible] then $\vdash_r$ Δ $\vdash_r \bot$

- We provide a dialectical account of Preferred Subtheories inference under limited resources

1. G. Brewka. Preferred subtheories: An extended logical framework for default reasoning. In *Proc. 11th International Joint Conference on Artificial Intelligence*, 1043–1048, 1989.

Resource Bounded Proof Theory for Construction of Classical Logic Arguments

- Arguments are *intelim* natural deduction proofs that use *intelim* introduction rules and elimination rules for connectives [1]

- No rules make use of virtual information (assumptions), e.g., standard $\to$I and $\vee$E rules are *not* intelim rules

- Instead just one rule of bivalence (RB), for example:

$$\dfrac{\dfrac{\dfrac{[\neg p] \quad p \vee q}{q} \quad q \to r}{r} \qquad \dfrac{[p] \quad p \to r}{r}}{r}$$

1. M D'Agostino, M Finger, D M Gabbay/. Semantics and Proof Theory of Depth-Bounded Boolean Logics. In: ***Theoretical Computer Science*** 480:43–68, 2013.

Resource Bounded Proof Theory for Construction of Classical Logic Arguments

- Degree k of nested use of RB equates with stepwise increments in computational complexity/cognitive effort
- $\Gamma \mid\!-_k \; \alpha$ can be decided in polynomial $O(n^{2k+2})$ time, where n = number of symbols occurring in $\Gamma \cup \{\alpha\}$ (hence quadratic for k = 0)
- $\mid\!-_\infty \; = \; \mid\!-_{CL}$
- Each ($Args_k$, Def_k) satisfies all rationality postulates
- Intelim natural deduction defined so as to exclude irrelevant proofs [1]

1. M. D'Agostino, D.Gabbay, and S. Modgil, 'M. D'Agostino, D.M. Gabbay, S. Modgil. *Normality and non-contamination in depth-bounded natural deduction for classical propositional logic.* Technical Report, http://www.filosofia.unimi.it/dagostino/wp-content/uploads/2017/07/TR_final.pdf

Conclusions

- Argumentative characterisations of non-monotonic inference enable individual agent reasoning and dialogue

- Our account of *dialectical* classical logic argumentation (*Cl-Arg*):

1. Drops consistency and subset minimality (replacing latter with notion of relevance that can be addressed proof theoretically);

2. Identifies minimal assumptions on arguments for inclusion in an AF;

3. Models Socratic dialectical move

and is provably rational

Future Work

- Argumentative formalisations of non-monotonic logics integrating classical and defeasible reasoning (e.g. Default Logic) inherit impracticality of *Cl-Arg* for resource bounded agents.

- General ASPIC+ framework in which arguments constructed by a mix of deductive and defeasible reasoning from premises to claim *cf Default Logics*

We are currently applying dialectical ontology and evaluation to the general ASPIC+ framework

Thank you for your attention

Questions ?