

Dynamic Argumentation Semantics (Work in Progress)

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Joint work with Ryuta Arisaka, Marcos Cramer, Jérémie Dauphin,
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Before we begin

Je n'ai fait celle-ci plus longue que parce que je n'ai pas eu le loisir de la faire plus courte.

Blaise Pascal, "Lettres Provinciales" (1657)

Before we begin

My to do list for formal argumentation

1. Formal argumentation as foundations for informal one?
2. Argumentation as inference vs argumentation as dialogue?
3. Pro & con vs attack graphs
4. Multi-valued argumentation: translations, bilattices
5. Aligning Dung AF and ADF research
6. Structured argumentation: prioritized rules: translations?
7. Representation results
8. Quantitative agenda
9. Dynamic agenda: AF can learn from ADF? (e.g. AFT)
10. Sequence semantics, attack semantics, defense semantics, update semantics, multi-sorted argumentation, Triple-A, ...

Before we begin

Why I don't use ADFs:

My favourite papers on argumentation semantics

- | | |
|--|-----|
| ▶ Dung 1995 and extensions | ADF |
| ▶ Baroni 2005 context (with translation) | ??? |
| ▶ Baroni 2007 principles | ??? |
| ▶ Baroni 2014 interface, IO (with translation) | ??? |
| ▶ Amgoud ranking | ??? |
| ▶ Dynamic semantics | ??? |

Outline

HOFA: Handbook Of Formal Argumentation

Dung's theory of abstract argumentation

Principle-based approach for abstract argumentation

Dynamic principles

Equivalence of argumentation frameworks

Dynamic argumentation semantics

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Dung's theory of abstract argumentation

Principle-based approach for abstract argumentation

Dynamic principles

Equivalence of argumentation frameworks

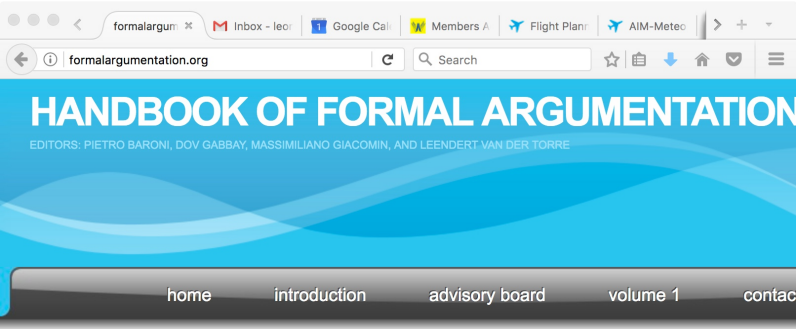
Dynamic argumentation semantics

COMMA 2014

The screenshot shows a web browser window with the following elements:

- Browser Tabs:** COMMA 2014, Inbox - leor, Google Cal, Members A, Flight Plan, AIM-Metec.
- Address Bar:** comma2014.arg.dundee.ac.uk
- Navigation Menu:** Overview, Call for Papers, Program, Organisation, Registration, Accepted Papers.
- Logo:** COMMA2014 THE SCOTTISH HIGHLANDS.
- Banner:** A scenic image of a lake and mountains. Text on the right reads: "Fifth International Conference on Computational Modelling" and "September 9th - 13th 2014". The University of Aberdeen logo is visible in the bottom right corner of the banner.

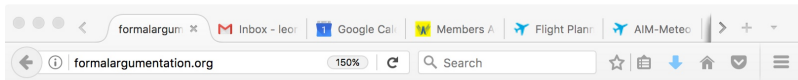
COMMA handbook



Handbook Of Formal Argumentatio

The HOFA initiative aims at producing a series of volumes providing a comprehensive state of the art and future research perspectives in the lively interdisciplinary field of formal argumentation. It is meant to be an open community effort and a service to current and future researchers in the field.

COMMA handbook, volume 1



Volume 1: FOUNDATIONS

PART A. INTRODUCTION

- Preface
- 1. Argumentation theory in formal and computational perspective (Van Eemeren & Verheij)
- 2. Historical overview of formal argumentation (Prakken)
- 3. Requirements analysis for formal argumentation (Gordon)

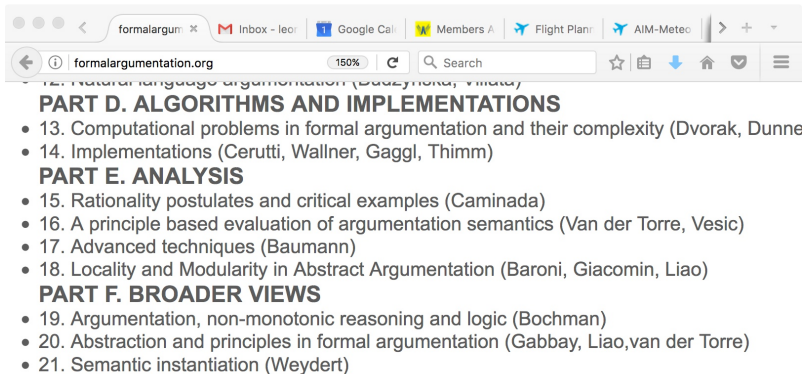
PART B. ARGUMENTATION FORMALISMS

- 4. Dung's abstract argumentation (Baroni, Caminada & Giacomin)
- 5. Abstract Dialectical Frameworks (Brewka, Ellmauthaler, Strass, Wallner, Woltran)
- 6. Abstract rule-based argumentation (Modgil & Prakken)
- 7. Assumption-based argumentation (Fan, Schulz & Toni)
- 8. Argumentation based on logic programming (Garcia & Simari)
- 9. Argumentation based on classical logic (Bernard & Hunter)

PART C. ARGUMENTATION AND DIALOGUES

- 10. Argument-Based Entailment as Discussion (Caminada)
 - 11. Argument schemes (Macagno, Reed & Walton)
 - 12. Natural language argumentation (Budzynska, Villata)
-

COMMA handbook, volume 1 (ctd)



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12. Natural language argumentation (Budzynski, Vintar)

PART D. ALGORITHMS AND IMPLEMENTATIONS

- 13. Computational problems in formal argumentation and their complexity (Dvorak, Dunne)
- 14. Implementations (Cerutti, Wallner, Gaggl, Thimm)

PART E. ANALYSIS

- 15. Rationality postulates and critical examples (Caminada)
- 16. A principle based evaluation of argumentation semantics (Van der Torre, Vesic)
- 17. Advanced techniques (Baumann)
- 18. Locality and Modularity in Abstract Argumentation (Baroni, Giacomin, Liao)

PART F. BROADER VIEWS

- 19. Argumentation, non-monotonic reasoning and logic (Bochman)
- 20. Abstraction and principles in formal argumentation (Gabbay, Liao, van der Torre)
- 21. Semantic instantiation (Weydert)

COMMA handbook, volume 2

First volume will be a stimulus for formal argumentation research

- ▶ First volume is baseline for the chapters in the second volume
- ▶ Also research in informal argumentation, e.g. ECA community
- ▶ Mathematical analysis
(unify structured theories, generalise, axiomatic analysis, ...)

Three research programs in volume 2:

1. Extensions of Dung's framework and abstract semantics
2. Numerical argumentation and strength of arguments & attacks
3. Dynamics of argumentation and dialogue

HOFA: Handbook Of Formal Argumentation

Dung's theory of abstract argumentation

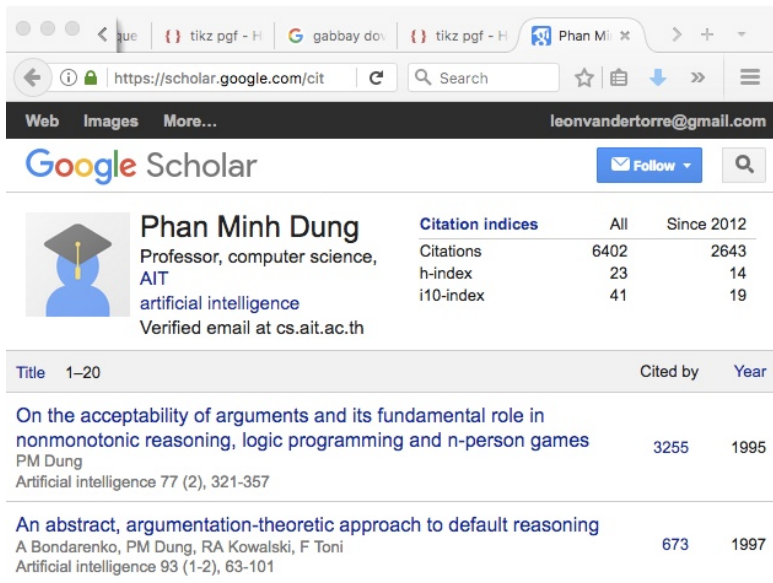
Principle-based approach for abstract argumentation

Dynamic principles

Equivalence of argumentation frameworks

Dynamic argumentation semantics


Dung



The image shows a screenshot of a web browser displaying a Google Scholar profile for Phan Minh Dung. The browser's address bar shows the URL <https://scholar.google.com/cit>. The profile includes a blue graduation cap icon, the name "Phan Minh Dung", and his affiliation as a Professor of computer science at AIT. His research interests in artificial intelligence and a verified email at cs.ait.ac.th are listed. A table of citation indices shows 6402 total citations, with 2643 since 2012, and h-index and i10-index values of 23 and 41, respectively. Two publications are listed with their titles, authors, journals, and citation counts.

Web Images More... leonvandertorre@gmail.com

Google Scholar Follow

 **Phan Minh Dung**
Professor, computer science,
AIT
artificial intelligence
Verified email at cs.ait.ac.th

Citation indices		All	Since 2012
Citations		6402	2643
h-index		23	14
i10-index		41	19

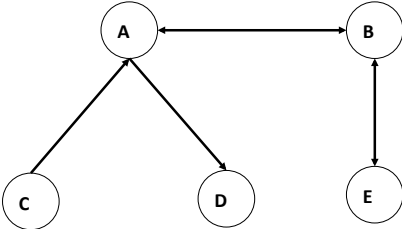
Title	1-20	Cited by	Year
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games	PM Dung Artificial intelligence 77 (2), 321-357	3255	1995
An abstract, argumentation-theoretic approach to default reasoning	A Bondarenko, PM Dung, RA Kowalski, F Toni Artificial intelligence 93 (1-2), 63-101	673	1997

Dung's theory in the handbook

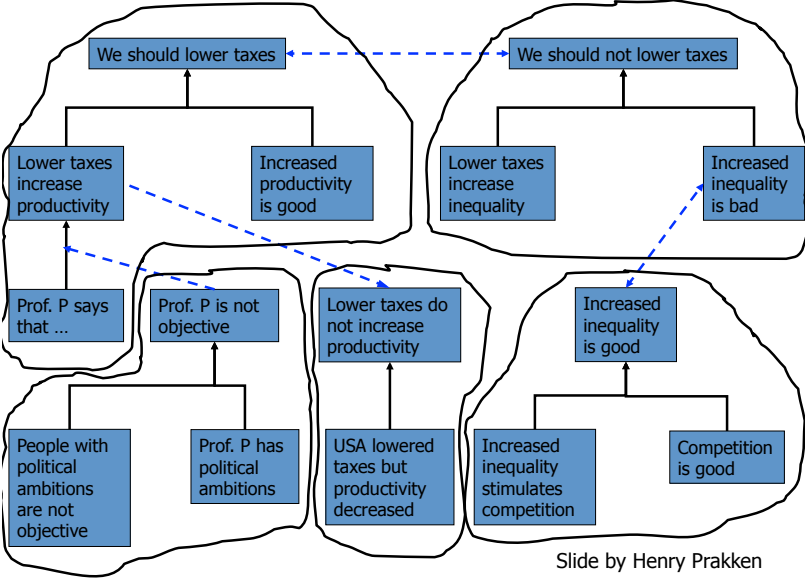
- ▶ Dung's theory (mainly, its framework and language) constitutes a turning point for the modern stage of formal argumentation theory.
 - ▶ Like possible worlds semantics for theory of modality.
- ▶ Nothing can remain the same as before [Dung, 1995].
 - ▶ It should be a focal point of reference for any study of argumentation, even if (especially if) it is critical about it.
 - ▶ In modal logics, the introduction of the possible worlds semantics has led to a paradigm shift, both in tools and new subjects of studies.
- ▶ This is still not fully accepted in formal argumentation theory.
 - ▶ The handbook will reflect the new stage of the development of formal argumentation theory.



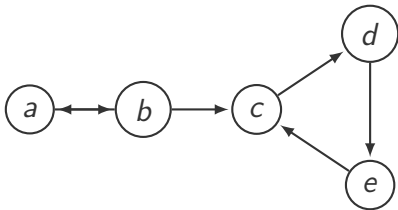
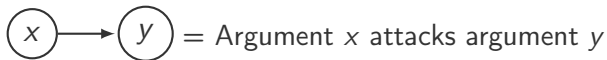
Structured Argumentation



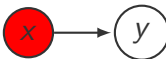

Dung's theory





Example: 2-3 cycle



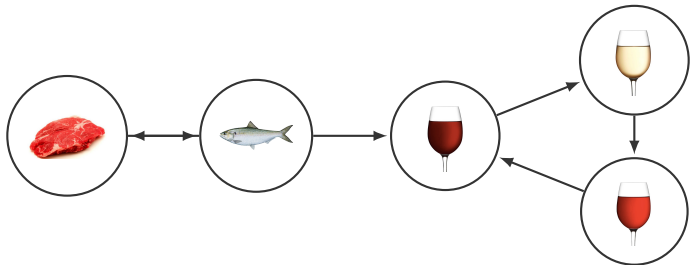
Gunfight rules:

▶ if all attackers are  then 

▶ if some attacker is  then 

Example: 2-3 cycle, food and wine

$\textcircled{x} \rightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

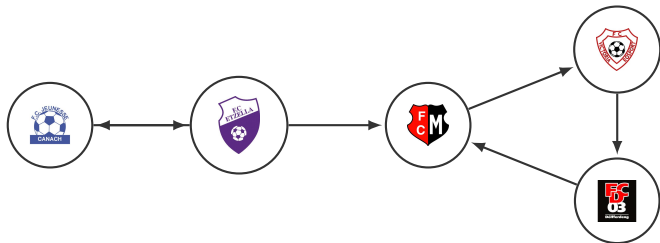
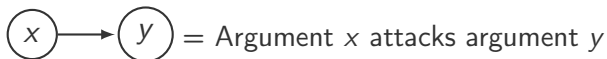


Gunfight rules:

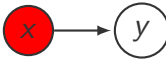



▶ if all attackers are $\textcircled{x} \rightarrow \textcircled{y}$ then $\textcircled{x} \rightarrow \textcircled{y}$

▶ if some attacker is $\textcircled{x} \rightarrow \textcircled{y}$ then $\textcircled{x} \rightarrow \textcircled{y}$

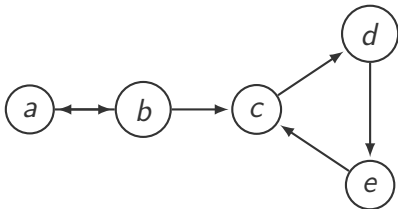
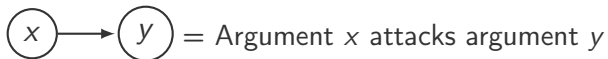
Example: 2-3 cycle, soccer tournament



Gunfight rules:

- ▶ if all attackers are  then 
- ▶ if some attacker is  then 

Example: 2-3 cycle, abstract

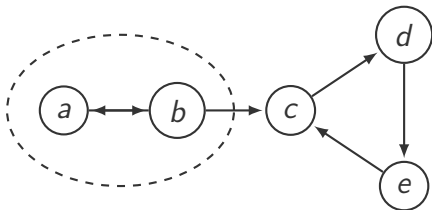
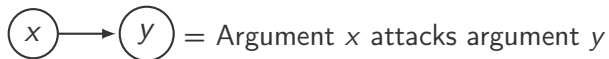


Gunfight rules:


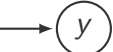


▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$


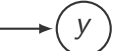


▶ if some attacker is $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$

Example: 2-3 cycle, directionality

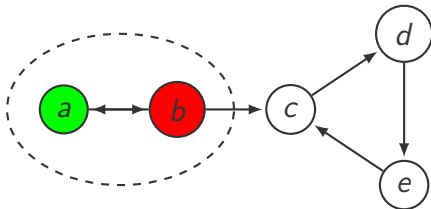
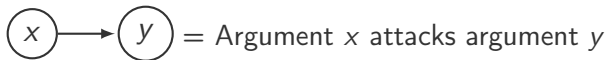


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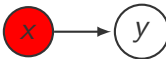
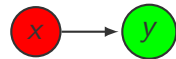
▶ if all attackers are   then  



▶ if some attacker is   then  

Example: 2-3 cycle, directionality

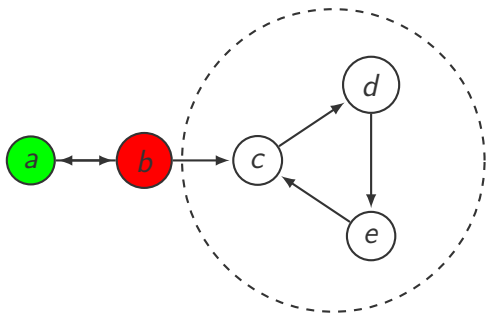
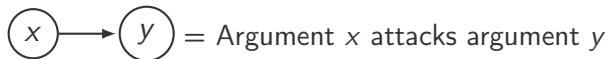


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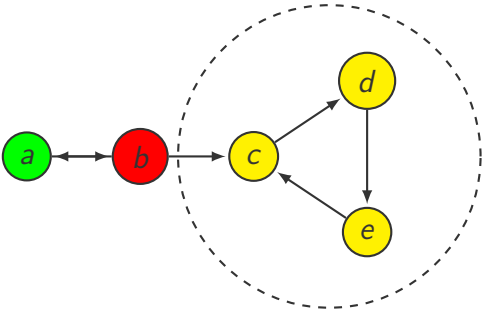
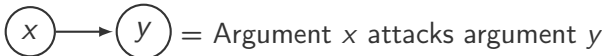


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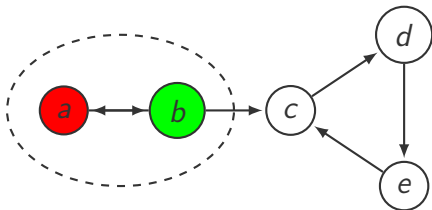
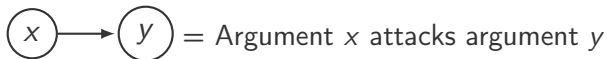


Gunfight rules:

▶ if all attackers are → then →

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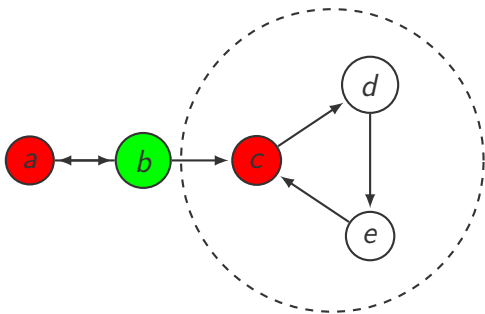
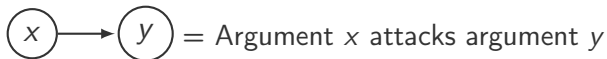


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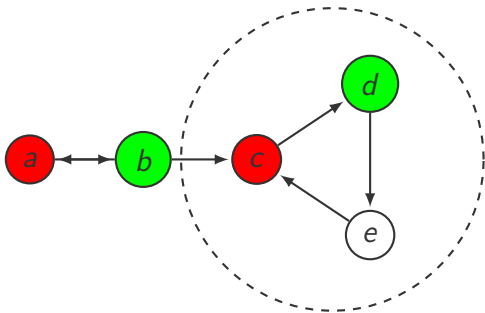
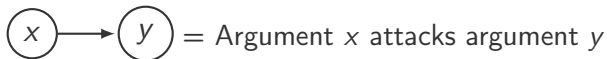


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Example: 2-3 cycle, reinstatement



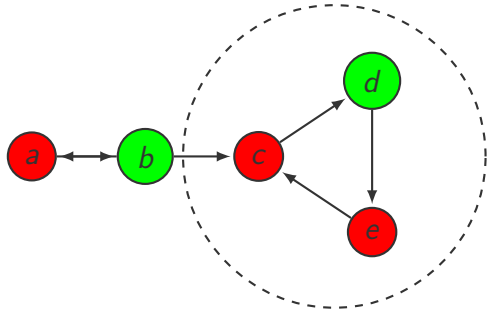
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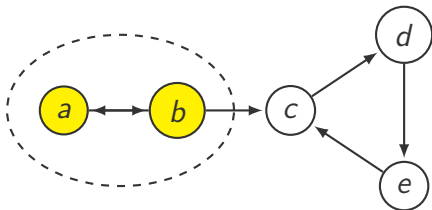
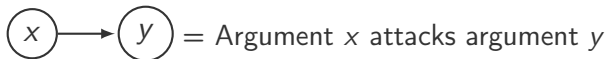
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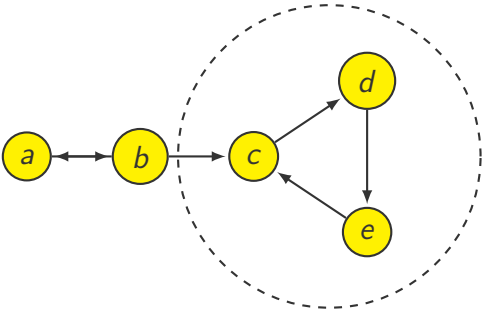
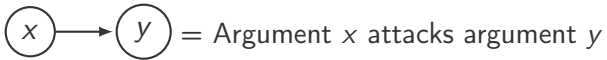


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




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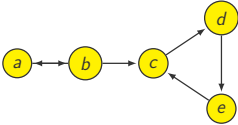
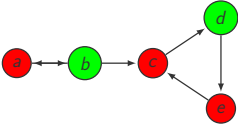
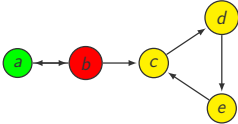


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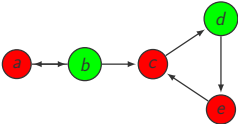
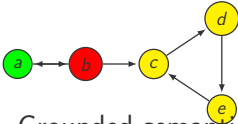
- ▶ if all attackers are  \rightarrow  then  \rightarrow 
- ▶ if some attacker is  \rightarrow  then  \rightarrow 

Example: agree to disagree

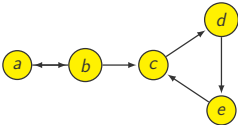
Complete semantics:



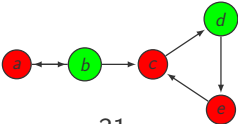
Preferred semantics:



Grounded semantics:



Stable semantics:



Summary: Dung and dynamics

- ▶ Dynamics implicit in directionality and reinstatement

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Principle-based approach, BG07, vdTorre & Vesic

Two problems:

- ▶ *Choice problem*: If there are many semantics, then how to choose one semantics from this set of alternatives in a particular application?
- ▶ *Search problem*: How to guide the search for new and hopefully better argumentation semantics?

Classification argumentation semantics based on principles.

- ▶ AKA axiomatic (e.g. voting), or postulate based (e.g. AGM).

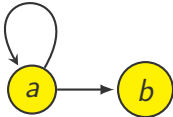
E.g., the principle of resolution was defined, well before resolution based semantics were defined

Principle-based approach, BG07, vdTorre & Vesic

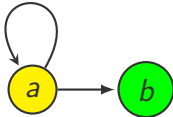
	Defence	Admiss.	Strong adm.	Naivety	Ind. CF	Reinst.	Weak reinst.	CF-reinst.
complete	✓	✓	×	×	×	✓	✓	✓
grounded	✓	✓	✓	×	×	✓	✓	✓
preferred	✓	✓	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	×	✓	✓	✓
semi-stable	✓	✓	×	×	×	✓	✓	✓
ideal	✓	✓	×	×	×	✓	✓	✓
eager	✓	✓	×	×	×	✓	✓	✓
p-complete	✓	✓	×	×	✓	×	×	×
p-grounded	✓	✓	✓	×	✓	×	×	×
p-preferred	✓	✓	×	×	✓	×	×	×
p-stable	✓	✓	×	✓	✓	✓	✓	✓
naive	×	×	×	✓	×	×	×	✓
CF2	×	×	×	✓	×	×	✓	✓
stage	×	×	×	✓	×	×	×	✓
stage2	×	×	×	✓	×	×	✓	✓

Table: Admissibility and reinstatement

Admissibility vs naive based semantics







ambiguity propagation



ambiguity blocking

A derived gunfight rule:

- ▶ if some attacker is  →  then not  → 

Principle-based approach, BG07, vdTorre & Vesic

	I-max.	Allowing abstention	Crash resistance	Non-interference	Direct.	Weak-direct.	Semi-direct.
complete	×	✓	✓	✓	✓	✓	✓
grounded	✓	✓	✓	✓	✓	✓	✓
preferred	✓	×	✓	✓	✓	✓	✓
stable	✓	×	×	×	×	✓	×
semi-stable	✓	×	✓	✓	×	×	×
ideal	✓	✓	✓	✓	✓	✓	✓
eager	✓	✓	✓	✓	✓	✓	✓
p-complete	×	✓	✓	✓	×	×	✓
p-grounded	✓	✓	✓	✓	✓	✓	✓
p-preferred	✓	×	✓	✓	×	×	✓
p-stable	✓	×	×	×	×	✓	×
naive	✓	×	✓	✓	×	×	✓
CF2	✓	×	✓	✓	✓	✓	✓
stage	✓	×	✓	✓	×	×	×
stage2	✓	×	✓	✓	✓	✓	✓

Table: Directionality

Principle-based approach, BG07, vdTorre & Vesic

	\Downarrow_{\cap}^E -sk. ad.	\Downarrow_W^E -sk. ad.	\Downarrow_S^E -sk. ad.	\Downarrow_{\cap} -res. ad.	\Downarrow_W -res. ad.	\Downarrow_S -res. ad.
complete	✓	✓	×	×	×	×
grounded	✓	✓	✓	×	×	×
preferred	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	✓	✓
semi-stable	×	×	×	✓	✓	×
ideal	×	×	×	×	×	×
eager	×	×	×	×	×	×
p-complete	×	×	×	×	×	×
p-grounded	×	×	×	✓	×	×
p-preferred	×	×	×	×	×	×
p-stable	×	×	×	✓	✓	×
naive	✓	✓	✓	✓	✓	✓
CF2	✓	✓	×	×	×	×
stage	×	×	×	✓	✓	×
stage2	×	×	×	×	×	×

Table: Skepticism and resolution adequacy

Principle-based approach, BG07, vdTorre & Vesic

	Succinctness	Tightness	Conflict- -sensitivity	Com- -closure	SCC- -recursiveness	Cardinality
complete	×	×	×	✓	✓	1+
grounded	×	✓	✓	✓	✓	1
preferred	×	×	✓	✓	✓	1+
stable	×	✓	✓	✓	✓	0+
semi-stable	×	×	✓	✓	×	1+
ideal	×	✓	✓	✓	×	1
eager	×	✓	✓	✓	×	1+
p-complete	×	×	×	×	×	1+
p-grounded	×	✓	✓	✓	×	1
p-preferred	×	✓	✓	✓	×	1+
p-stable	×	✓	✓	✓	×	0+
naive	×	✓	✓	✓	×	1+
CF2	✓	✓	✓	✓	✓	1+
stage	×	✓	✓	✓	×	1+
stage2	✓	✓	✓	✓	✓	1+

Table: SCC recursiveness

Definition (Semantics)

An extension-based semantics is a function σ such that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, we have $\sigma(\mathcal{F}) \in 2^{2^{\mathcal{A}}}$. The elements of $\sigma(\mathcal{F})$ are called *extensions*.

The principle-based approach consists of three steps.

1. Define a general function with domain and codomain.
 - ▶ This function will be the object of study.
2. Define the principles.
 - ▶ Existing functions can be checked against the principles,
 - ▶ New functions can be defined satisfying given sets of principles.
3. Classify and study sets of principles.
 - ▶ Representation theorems for sets of principles can be defined.

Definition (Semantics)

An extension-based semantics is a function σ such that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, we have $\sigma(\mathcal{F}) \in 2^{2^{\mathcal{A}}}$. The elements of $\sigma(\mathcal{F})$ are called *extensions*.

Note:

- ▶ Not every function can be used as an argumentation semantics.
- ▶ A semantics is fundamentally different from a principle.
- ▶ Many more semantics can be defined.
- ▶ Principles are not necessarily requirements or postulates.
- ▶ Lack of AGM style representation theorems and similar results.

Labeling-based semantics

Definition (Semantics)

A labeling-based semantics is a function σ such that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, we have $\sigma(\mathcal{F}) \subseteq Lab$, where Lab is the set of functions from \mathcal{A} to $\{in, out, undec\}$. The elements of $\sigma(\mathcal{F})$ are called *labelings*.

Standard reduction:

- ▶ Argument is accepted iff in, rejected iff out or undec
- ▶ Argument is in iff accepted, out iff rejected and one of its attackers is accepted, undec otherwise

Label depends only on labels of its attackers (gunfight rules).

Alternative reductions

Principles in COMMA handbook, volume 2

Three research programs:

1. Extensions of Dung's framework
Rarely lifted to principle-based approach
(exceptions: bipolar argumentation, ranking based semantics)
2. Numerical argumentation: strength of arguments and attacks
Bochum16 workshop: no principle-based approach yet
3. Dynamics of argumentation and dialogue
The Madeira workshops: mainly inspired by belief revision

HOFA: Handbook Of Formal Argumentation

Dung's theory of abstract argumentation

Principle-based approach for abstract argumentation

Dynamic principles

Equivalence of argumentation frameworks

Dynamic argumentation semantics

Dynamic principles

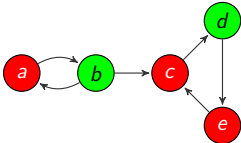
When is a point of view on argument acceptance robust w.r.t. addition/removal of attacks?

Examples:

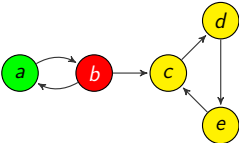
- ▶ Guido Boella, Souhila Kaci, Leon van der Torre: Dynamics in Argumentation with Single Extensions: Attack Refinement and the Grounded Extension (Extended Version). ArgMAS 2009: 150-159
- ▶ Tjitze Rienstra, Chiaki Sakama, Leon van der Torre: Persistence and Monotony Properties of Argumentation Semantics. TFAA 2015: 211-225

Our running example

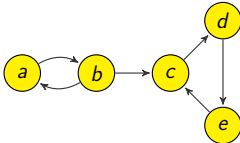
We use green, red and yellow to depict in, out and undecided arguments.



: L_1



: L_2



: L_3

	Complete	Grounded	Preferred	Semi-Stable	Stable
L_1	✓		✓	✓	✓
L_2	✓		✓		
L_3	✓	✓			

Addition Persistence Properties

Definition (*XY* Addition Persistence)

A semantics σ satisfies *XY* addition persistence iff every σ labelling of an AF F in which x is labelled X and y is labelled Y is still a σ labelling of F after adding an attack from x to y .

Addition Persistence Properties

Introduce a conflict.

Failure of II-addition persistence



Failure of IU-addition persistence



Failure of UI-addition persistence



Addition Persistence Properties

As we just saw, some properties fail:

- ▶ II-addition persistence
 - ▶ IU-addition persistence
 - ▶ UI-addition persistence
- } These cases fail because they introduce a conflict.

Other properties reflect reasonable principles:

- ▶ OO-addition persistence
 - ▶ OU-addition persistence
 - ▶ OI-addition persistence
 - ▶ IO-addition persistence
 - ▶ UO-addition persistence
 - ▶ OO-addition persistence
- } In these cases the added attack doesn't introduce a conflict, and doesn't invalidate the justification of the attacked argument's label.

Are these properties satisfied by the semantics we consider?

Addition Persistence Properties

Grounded:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	-	-	-

Complete:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	✓	-	-

Preferred:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	-	-
	I	✓	-	-

Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

Addition Persistence Properties

Grounded:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	-	-	-

Complete:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	✓	-	-

Preferred:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	-	-
	I	✓	-	-

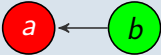
Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

Addition Persistence Properties

Failure of OI addition persistence under grounded semantics.

After adding an attack from a to b there is a new grounded labelling:



Addition Persistence Properties

Grounded:


		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	-	-	-

Complete:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	✓	-	-

Preferred:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	-	-
	I	✓	-	-



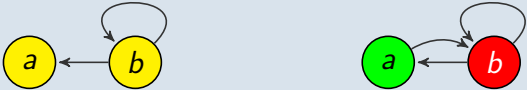
Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

Addition Persistence Properties

Failure of UU addition persistence under the preferred semantics.

After adding an attack from a to b there is a new preferred labelling:



Removal Persistence Properties

Grounded:

		X		
		O	U	I
Y	O	✓	✓	-
	U	✓	-	✓
	I	✓	✓	✓

Complete:

		X		
		O	U	I
Y	O	✓	✓	-
	U	✓	-	✓
	I	✓	✓	✓

Preferred:

		X		
		O	U	I
Y	O	✓	✓	-
	U	✓	-	✓
	I	✓	✓	✓

Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	✓
	I	-	✓	✓

Skeptical Monotony Properties

Grounded:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	-	-	-

Complete:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	-	-
	I	-	-	-

Preferred:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

Skeptical Monotony Properties

Grounded:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	✓	-
	I	-	-	-

Complete:

		X		
		O	U	I
Y	O	✓	✓	✓
	U	✓	-	-
	I	-	-	-

Preferred:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

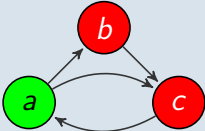
Semi-Stable:

		X		
		O	U	I
Y	O	-	-	-
	U	-	-	-
	I	-	-	-

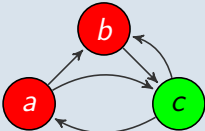
Skeptical Monotony Properties

Failure of OO skeptical monotony under the preferred semantics

The following AF has one preferred labelling:



The arguments *b* and *c* are both labelled **O**. If we add an attack $c \rightsquigarrow b$ then we obtain a new preferred labelling:



Summary: Dung, principles and dynamics

- ▶ Dynamics implicit in directionality and reinstatement
- ▶ Many principles have a dynamic flavor
- ▶ New dynamic principles based on expansion and contraction

HOFA: Handbook Of Formal Argumentation

Dung's theory of abstract argumentation

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Semantic equivalence

Two frameworks are equivalent if they have the same extensions.

The puzzle of Dung's theory

Do these two argumentation frameworks *mean* the same?

- ▶ Reinstatement (a is the reason for accepting c)



- ▶ Independent arguments (a is not a reason for accepting c)



Does argumentation semantics cover all aspects of the meaning?

Equivalence

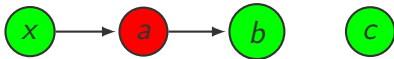
Two frameworks are strongly equivalent if each expansion has the same extensions.

The two frameworks are not strongly equivalent:

- ▶ Reinstatement (a was the reason for accepting c)



- ▶ Independent arguments (a was not a reason for accepting c)



Baumann's chapter in HOFA

82

Ringo Baumann

	<i>stg</i>	<i>stb</i>	<i>ss</i>	<i>eg</i>	<i>ad</i>	<i>pr</i>	<i>il</i>	<i>gr</i>	<i>co</i>	<i>na</i>	<i>cf2</i>	<i>stg2</i>
W	?	[1,3]	?	?	[2,1]	[3,1]	?	?	[2,1]	?	?	?
L	$k^*(stg)$	[4,9]	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	[4,10]	[4,11]	$k(na)$?	?
E	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$k(na)$	<i>id</i>	<i>id</i>
N	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$k(na)$	<i>id</i>	<i>id</i>
S	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(gr)$	$k^*(co)$	$k(na)$?	?
ND	?	$k(stb)$?	?	$k^*(ad)$ L, Att^{ad}	?	?	$k^*(gr)$ L, Att^{gr}	$k^*(co)$ L, Att^{co}	?	?	?
D	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>
LD	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>
U	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>	<i>id</i>

Figure 12: Extension-based Characterizations for Finite AFs

Summary: Dung, principles, equivalence and dynamics

- ▶ Dynamics implicit in directionality and reinstatement
- ▶ Many principles have a dynamic flavor
- ▶ New dynamic principles based on expansion and contraction
- ▶ Strong equivalence based on framework expansion

HOFA: Handbook Of Formal Argumentation

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Equivalence of argumentation frameworks

Dynamic argumentation semantics

Dynamic semantics (New!)

Inspired by attack semantics (Villata et al, 2011) and AFRA:

Definition (Semantics)

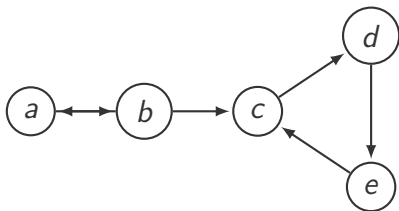
A dynamic semantics is a function d from argumentation frameworks to sets of its sub-frameworks, i.e. if $(\mathcal{A}', \mathcal{R}') \in d(\mathcal{A}, \mathcal{R})$, then we have $\mathcal{A}' \subseteq \mathcal{A}$ and $\mathcal{R}' \subseteq \mathcal{R}$.

Note:

- ▶ Extension is also a (trivial) graph
 - ▶ thus extension based semantics is special case
- ▶ Since domain = codomain, we can iterate!
- ▶ New idea: static semantics is fixpoint of this dynamic relation
- ▶ E.g. dynamic relation is breaking cycles

Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

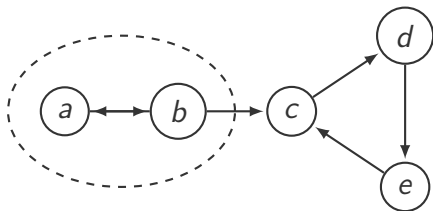


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if some attacker is $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if $\textcircled{x} \longrightarrow \textcircled{y}$ and no $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$

Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

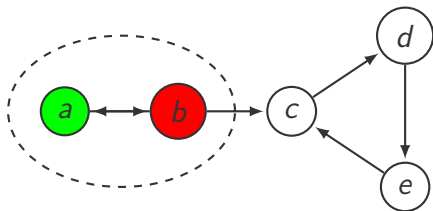


Gunfight rules:

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Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

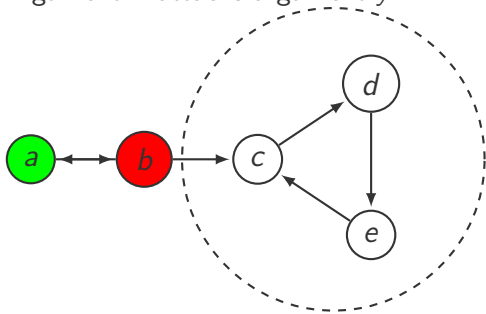


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if some attacker is $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if $\textcircled{x} \longrightarrow \textcircled{y}$ and no $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$

Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

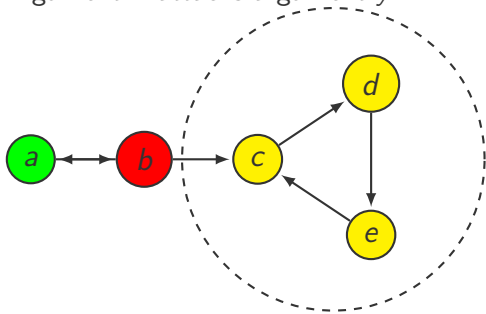


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
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Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

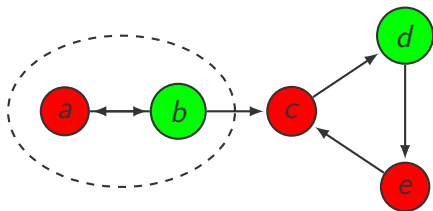


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
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Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

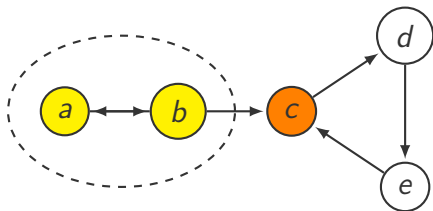


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if some attacker is $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if $\textcircled{x} \longrightarrow \textcircled{y}$ and no $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$

Five valued semantics

$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

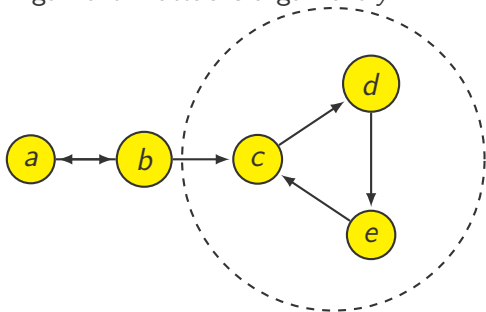


Gunfight rules:

- ▶ if all attackers are $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if some attacker is $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$
- ▶ if $\textcircled{x} \longrightarrow \textcircled{y}$ and no $\textcircled{x} \longrightarrow \textcircled{y}$ then $\textcircled{x} \longrightarrow \textcircled{y}$

Five valued semantics

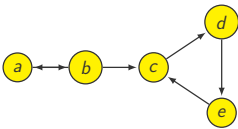
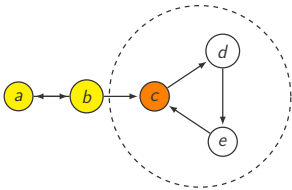
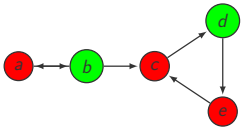
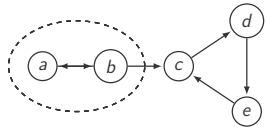
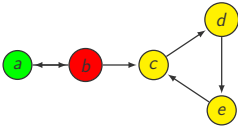
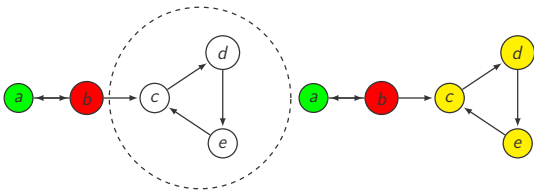
$\textcircled{x} \longrightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$



Gunfight rules:

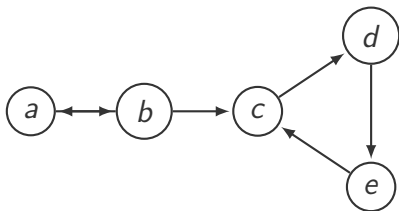
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Five valued semantics



Two valued semantics

$\textcircled{x} \rightarrow \textcircled{y} = \text{Argument } x \text{ attacks argument } y$

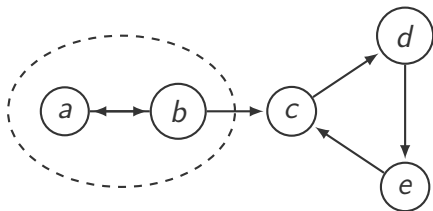


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Two valued semantics

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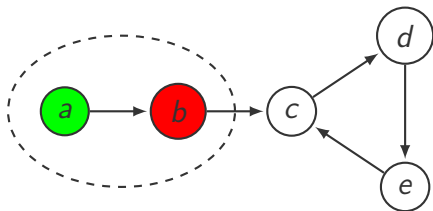


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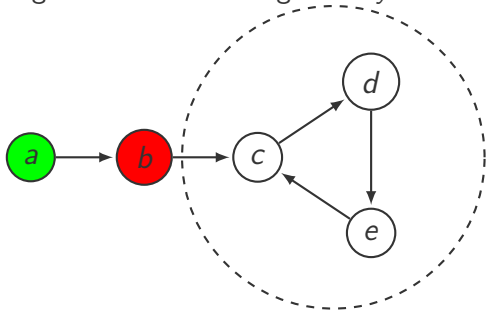


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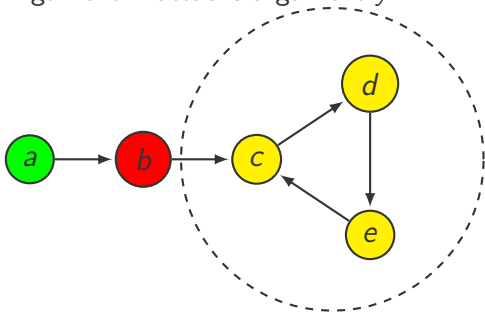


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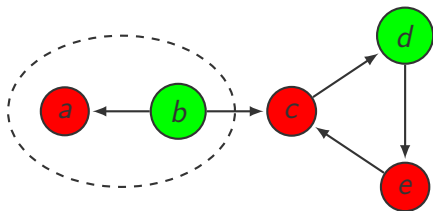


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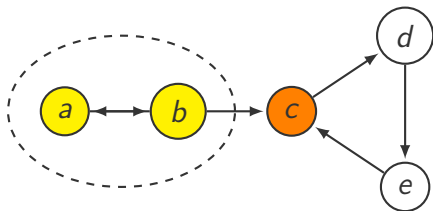


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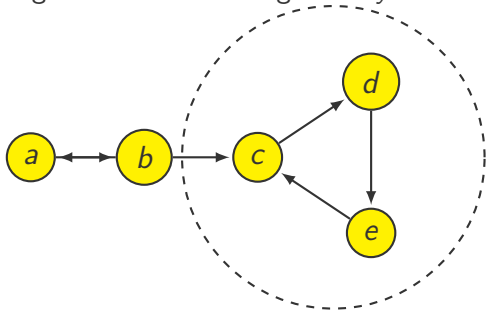


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Two valued semantics

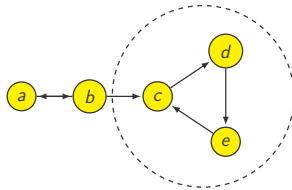
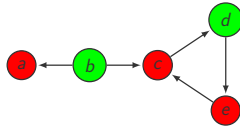
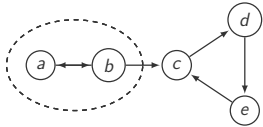
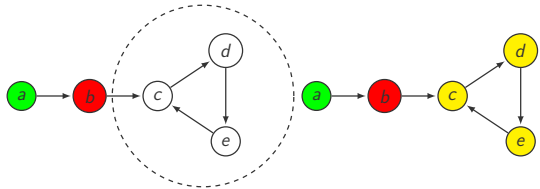
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Two valued semantics



Analysis

Principles for dynamic semantics, e.g.:

- ▶ Update relation is a tree, not a graph

Comparing dynamic semantics, e.g.:

- ▶ One update relation is a refinement of another one

Equivalence

Summary

- ▶ Formal foundations of abstract argumentation have just begun
- ▶ Two key challenges are strengths of arguments and dynamics
- ▶ Dynamics in Dung's theory: principles and strong equivalence
- ▶ Making dynamics explicit: dynamic argumentation semantics
 - ▶ Static semantics is fixpoint of dynamic semantics
 - ▶ Dynamic updates as breaking cycles
 - ▶ Five valued labelings (and more)
 - ▶ Two valued updates: remove attacks on accepted arguments
 - ▶ Update principles and comparing update functions
- ▶ Next: relate with algorithms, dialogue games, ...

To do

My to do list for formal argumentation

1. Formal argumentation as foundations for informal one?
2. Argumentation as inference vs argumentation as dialogue?
3. Pro & con vs attack graphs
4. Multi-valued argumentation: translations, bilattices
5. Aligning Dung AF and ADF research (next slide)
6. Structured argumentation: prioritized rules: translations?
7. Representation results
8. Quantitative agenda
9. Dynamic agenda: AF can learn from ADF? (e.g. AFT)
10. Sequence semantics, attack semantics, defense semantics, update semantics, multi-sorted argumentation, Triple-A, ...

To do

Is there a separation between Dung AF and ADF?

My favourite papers on argumentation semantics

- ▶ Dung 1995 and extensions ADF
- ▶ Baroni 2005 context (with translation) ???
- ▶ Baroni 2007 principles ???
- ▶ Baroni 2014 interface, IO (with translation) ???
- ▶ Amgoud ranking ???
- ▶ Dynamic semantics ???

**Temporal Dynamics of Support and Attack
Networks: From Argumentation to Zoology
Initial Results**

Howard Barringer¹, Dov Gabbay², and John Woods³

¹ School of Computer Science,
The University of Manchester,
Oxford Rd, Manchester M13 9PL, UK

² Department of Computer Science,
King's College London,
Strand, London WC2R 2LS, UK

³ Department of Philosophy,
University of British Columbia,
1866 Main Mall E370, Vancouver BC Canada V6T 1Z1

Further research: defence semantics

Beishui Liao and Leon van der Torre. Defense semantics of argumentation: encoding reasons for accepting arguments. Workshop on mining and reasoning with legal texts (MIREL@ICAIL2017).

Further research: multi-agent argumentation

Ryuta Arisaka, Ken Satoh and Leon van der Torre. Abstract Agent Argumentation (Triple-A). Workshop on mining and reasoning with legal texts (MIREL@ICAIL2017).

Does this theory have applications?

» Computer Science » Database Management & Information Retrieval

The Enterprise Engineering Series



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Enterprise Architecture at Work

Modelling, Communication and Analysis

Authors: **Lankhorst**, Marc

ArchiMate book (2005/2009/2013/2017, GS 1405 citations)

» Computer Science » Database Management & Information Retrieval

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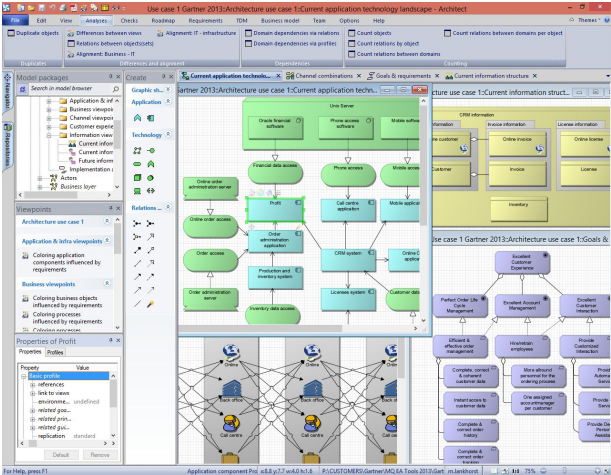
Enterprise Architecture at Work

Modelling, Communication and Analysis

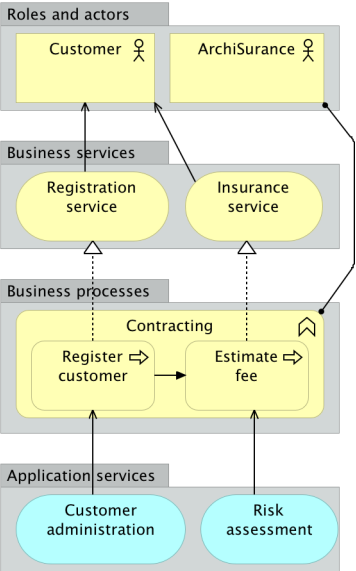
Authors: **Lankhorst**, Marc

ArchiMate tools

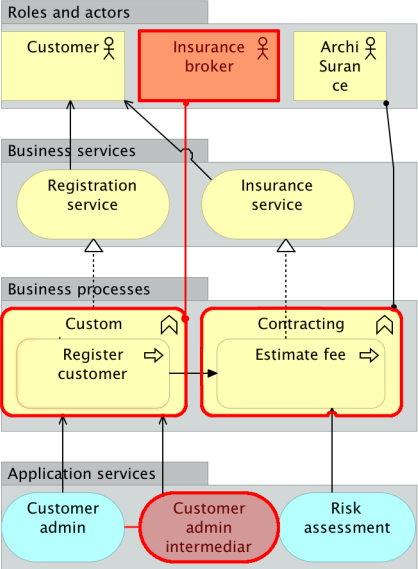
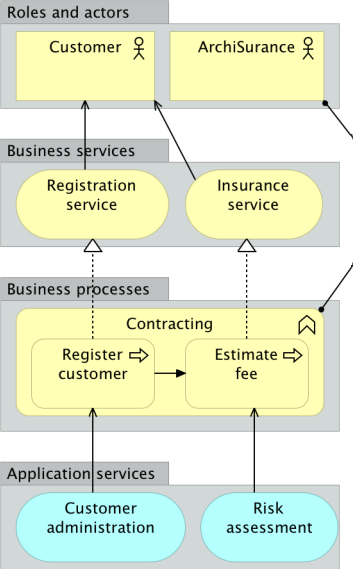
Example ArchiMate model in Bizzdesign Architect.



ArchiSurance: ArchiMate model



ArchiSurance: ArchiMate model



Decision Support System

Empirical study:

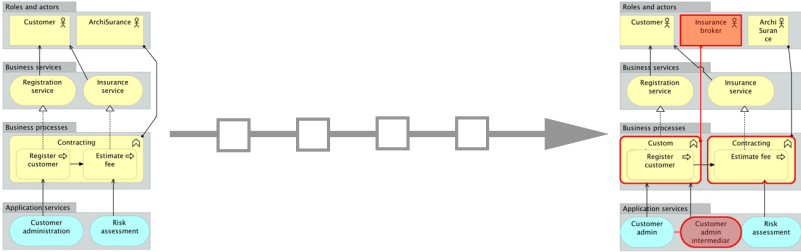
- ▶ Qualitative/quantitative questionnaire (35 architects).¹
- ▶ List of eight characteristics.

EA Planning is difficult:

- ▶ Highly uncertain plans (long-term) with many **assumptions**.
- ▶ Rapidly changing environment.
- ▶ Many *stakeholders*.

¹Insights from a Study on Decision Making in Enterprise Architecture (Dirk van der Linden, Marc van Zee), In *Proceedings of the 8th IFIP WG 8.1 Working Conference on the Practice of Enterprise Modeling (PoEM)*, 2015.

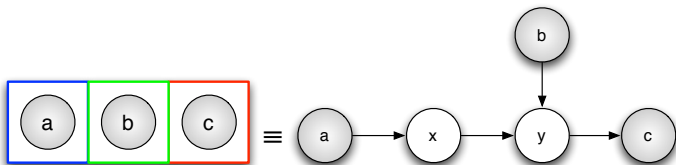
ArchiMate Change Management



Change management process

Why is Dung's theory so popular?

- ▶ Many people have criticized Dung
- ▶ Many people have extended Dung
 - ▶ Preferences, higher order, collective attack, numbers, structured, ...
- ▶ But Dung's theory has shown to be very robust
 - ▶ E.g. flattening based reductions



Villata S., Boella G., van der Torre L. *Argumentation Patterns*. 8th International Workshop on Argumentation in Multi-Agent Systems (ArgMAS 2011), p. 133-150, 2011.

- ▶ Compare Turing machine, possible worlds, and so on

Explaining the popularity of Dung's theory (reminder)

“major approaches to nonmonotonic reasoning in AI and logic programming are special forms of our theory of argumentation”

“logic programming as well as many major formalisms to nonmonotonic and defeasible reasoning in AI are argumentation systems. That means that all these systems are based on the same principle. They differ only by the structure of their arguments.”

For example, [Dung 1995]'s result on default logic is as follows:

“Let $T = (D, W)$ be a default theory. Let E be an R -extension of T and E' be a stable extension of $AF(T)$. Then

1. $arg(E)$ is a stable extension of $AF(T)$,
2. $at(E')$ is an R -extension of T .” [Dung 1995, Theorem 43.]

Explaining the popularity of Dung's theory (reminder)

Definition (Representation)

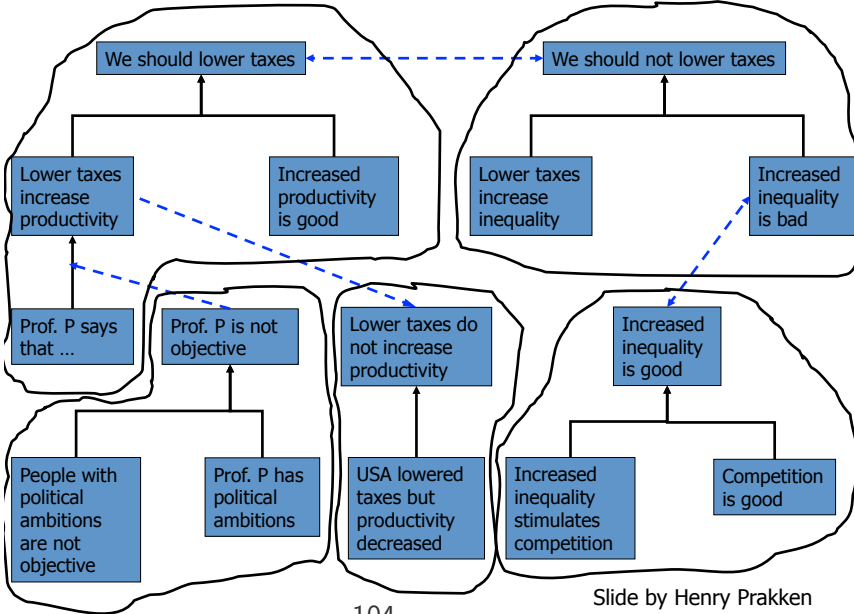
Given $m(T) = \{E_1, \dots, E_n\}$, a representation (AF, arg, at, sem) consists of a function from theories to argumentation frameworks $AF(T)$, a function from extensions to argument extensions $arg(E)$, a function from argument extensions to extensions $at(E')$, and an argumentation semantics sem .

Moreover, a representation has to satisfy the condition that $arg(E)$ is a sem extension of $AF(T)$, and $at(E')$ is an extension of T .

Definition (Compositional representation)

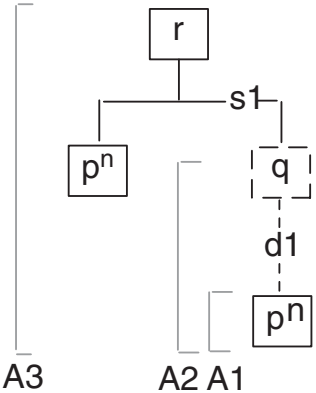
A representation (AF, arg, at, sem) is compositional if the function $AF(T)$ satisfies the following condition: If $T = T_1 \cup T_2$, then if $A_1, A_2 \in AF(T_1)$, then A_1 attacks A_2 in $AF(T)$ iff A_1 attacks A_2 in $AF(T_1)$.

Explaining the popularity of Dung's theory (reminder)



Explaining the popularity of Dung's theory (reminder)

Sanjay Modgil and Henry Prakken, ASPIC+ tutorial in Special Issue of Argumentation and Computation, 2014



$$A_1: p$$

$$A_2: A_1 \Rightarrow q$$

$$A_3: A_1, A_2 \rightarrow r$$