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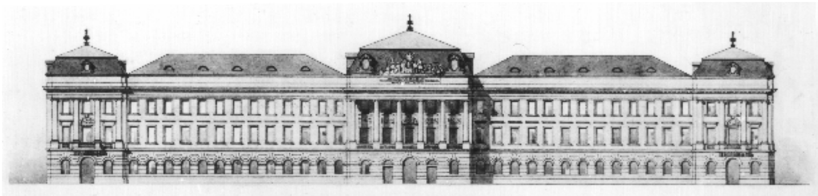

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ABTEILUNG DATENBANKEN UND ARTIFICIAL INTELLIGENCE

The Complexity Landscape of Claim-Augmented Argumentation Frameworks

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Wolfgang Dvořák **Alexander Greßler**
Anna Rapberger **Stefan Woltran**

Institut für Logic and Computation
Abteilung Datenbanken und
Artificial Intelligence
Technische Universität Wien
Favoritenstr. 9
A-1040 Vienna, Austria
Tel: +43-1-58801-18403
Fax: +43-1-58801-918403
sek@dbai.tuwien.ac.at
www.dbai.tuwien.ac.at

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Wolfgang Dvořák [1] **Alexander Greßler** [2]
Anna Rapberger [3] **Stefan Woltran** [4]

Abstract. Claim-augmented argumentation frameworks (CAFs) provide a formal basis to analyze conclusion-oriented problems in argumentation by adapting a claim-focused perspective; they extend Dung AFs by associating a claim to each argument representing its conclusion. This additional layer offers various possibilities to generalize abstract argumentation semantics, i.e. the re-interpretation of arguments in terms of their claims can be performed at different stages in the evaluation of the framework: One approach is to perform the evaluation entirely at argument-level before interpreting arguments by their claims (inherited semantics); alternatively, one can perform certain steps in the process (e.g., maximization) already in terms of the arguments' claims (claim-level semantics). The inherent difference of these approaches not only potentially results in different outcomes but, as we will show in this paper, is also mirrored in terms of computational complexity. To this end, we provide a comprehensive complexity analysis of the four main reasoning problems with respect to claim-level variants of preferred, naive, stable, semi-stable and stage semantics and complete the complexity results of inherited semantics by providing corresponding results for semi-stable and stage semantics. Moreover, we show that deciding, whether for a given framework the two approaches of a semantics coincide (concurrence) can be surprisingly hard, ranging up to the third level of the polynomial hierarchy.

[1]Institute of Logic and Computation, TU Wien, Austria. E-mail: dvorak@dbai.tuwien.ac.at
[2]Institute of Logic and Computation, TU Wien, Austria. E-mail: agressle@dbai.tuwien.ac.at
[3]Institute of Logic and Computation, TU Wien, Austria. E-mail: arapberg@dbai.tuwien.ac.at
[4]Institute of Logic and Computation, TU Wien, Austria. E-mail: woltran@dbai.tuwien.ac.at

The Complexity Landscape of Claim-Augmented Argumentation Frameworks

Wolfgang Dvořák, Alexander Greßler, Anna Rapberger, Stefan Woltran
TU Wien, Institute of Logic and Computation, Austria
{dvorak,agressle,arapberg,woltran}@dbai.tuwien.ac.at

Abstract

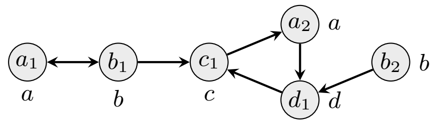
Claim-augmented argumentation frameworks (CAFs) provide a formal basis to analyze conclusion-oriented problems in argumentation by adapting a claim-focused perspective; they extend Dung AFs by associating a claim to each argument representing its conclusion. This additional layer offers various possibilities to generalize abstract argumentation semantics, i.e. the re-interpretation of arguments in terms of their claims can be performed at different stages in the evaluation of the framework: One approach is to perform the evaluation entirely at argument-level before interpreting arguments by their claims (inherited semantics); alternatively, one can perform certain steps in the process (e.g., maximization) already in terms of the arguments' claims (claim-level semantics). The inherent difference of these approaches not only potentially results in different outcomes but, as we will show in this paper, is also mirrored in terms of computational complexity. To this end, we provide a comprehensive complexity analysis of the four main reasoning problems with respect to claim-level variants of preferred, naive, stable, semi-stable and stage semantics and complete the complexity results of inherited semantics by providing corresponding results for semi-stable and stage semantics. Moreover, we show that deciding, whether for a given framework the two approaches of a semantics coincide (concurrence) can be surprisingly hard, ranging up to the third level of the polynomial hierarchy.

1 Introduction

Abstract argumentation (Dung, 1995) is nowadays acknowledged as the core reasoning mechanism for argumentation in the broad sense (Atkinson et al., 2017), in particular in instantiation-based approaches (see e.g. (Gorogiannis & Hunter, 2011)). This instantiation process starts from a (typically inconsistent) knowledge base, from which all possible arguments are constructed. An argument contains a claim and a support, the latter being a subset of the knowledge base. The relationship between arguments is then settled, for instance an argument α attacks argument β if the claim of α contradicts (parts of) the support of β. The resulting network is then interpreted as an abstract argumentation framework (AF) and semantics for AFs are used to obtain a collection of jointly acceptable sets of arguments, commonly referred to as extensions. In a final step these extensions are then reinterpreted in terms of the claims of the accepted arguments, thus restating the result in the domain of the initial setting.

Recent research (Baroni & Riveret, 2019; Dvořák, Rapberger, & Woltran, 2020) has addressed the fact that the re-interpretation part is not as obvious as it seems at first glance. For instance, consider preferred semantics, which is defined at the AF level as subset-maximal admissible sets (a set is admissible if it attacks all its attackers). When looking for preferred extensions in terms of claims, we can either (a) take the preferred extensions of the AF and replace each argument by its claim, or (b) take the admissible sets of the AF, replace each argument by its claim, and then select the subset-maximal ones from the resulting set of extensions.

Example 1. *Consider the following AF where each argument is labelled with its claim.*



The admissible sets are given by $\emptyset$,$\{a_1\}$,$\{b_1\}$,$\{b_2\}$,$\{a_1, b_2\}$, $\{b_1, b_2\}$, $\{a_2, b_1\}$, $\{a_1, b_2, c_1\}$, *and* $\{a_2, b_1, b_2\}$. *Selecting the subset-maximal admissible sets before replacing each argument by its claim (option (a)) thus yields the preferred claim-sets* $\{a, b, c\}, \{a, b\}$*; observe that swapping those steps (option (b)) results in the unique claim-set* $\{a, b, c\}$.

Option (a) which we shall call *inherited semantics* in what follows, is often used implicitly in instantiation-based argumentation and has been explicitly studied in (Dvořák & Woltran, 2020). Option (b) has recently been advocated in (Dvořák et al., 2020) as an alternative way to lift concepts behind argumentation semantics to claim-based semantics; we will refer to the latter as *claim-level semantics* since parts of the semantic selection process takes place on the claim- rather than on the argument-level. As discussed in (Rapberger, 2020), there are logic programming semantics that, in the standard instantiation model (Caminada, Sá, Alcântara, & Dvořák, 2015a, 2015b), correspond to claim-level semantics and cannot be captured with inherited semantics.

To be independent from a particular instantiation schema, Dvořák and Woltran (Dvořák & Woltran, 2020) introduced claim-augmented frameworks (CAFs), which are AFs where each argument is assigned a claim; hence, a CAF is given by a triple $(A, R, claim)$ where (A, R) constitutes an AF and function $claim$ maps arguments A to claims (indeed Example 1 provides an example for a CAF). They also introduced the important subclass of well-formed CAFs which restricts the assignment of claims in the sense that arguments with the same claim have to attack the same set of arguments (thus reflecting the instantiation model for attacks outlined above). AF semantics σ are then lifted to CAFs by setting $\sigma_c((A, R, claim)) = claim(\sigma(A, R))$ in order to obtain inherited CAF semantics. Claim-level semantics follow a different line of definition as sketched in Example 1 for the case of preferred semantics. We will introduce them in Section 2.

We have already seen that the two approaches differ in the above example; Dvořák et al. (Dvořák et al., 2020) have analyzed these differences in detail, also showing that there are some semantics where the two approaches coincide on the class of well-formed CAFs. What remains open is the question whether this difference is mirrored in terms of computational complexity (an analysis for CAF semantics has so far been only conducted for (most of) the inherited semantics (Dvořák & Woltran, 2020); the results show an occasional increase of complexity compared to the corresponding AF semantics). Another question is how hard it is to decide for a given CAF whether the two approaches of a semantics deliver the same result.

We tackle these two questions via a thorough complexity analysis. Our main contributions are as follows:

- We settle the computational complexity of all the claim-level semantics, i.e. stable, naive, preferred, semi-stable, and stage semantics, introduced in (Dvořák et al., 2020) for the main decision problems of credulous and skeptical acceptance, verification, and testing for non-empty extensions. Among our findings is that for naive semantics, the claim-level variant is harder than its inherited counterpart, while for preferred semantics, it is the inherited variant that shows higher complexity.
- We also provide complexity results for inherited semi-stable and stage semantics which have not been investigated in (Dvořák & Woltran, 2020). As it turns out, for these two semantics the complexity of the inherited and claim-level variants coincides.
- We determine the complexity of the concurrence problem, i.e. whether for a given CAF and a semantics, the inherited and claim-level variant of that semantics coincide. Note that showing this problem to be easy would suggest that there are relatively natural classes of CAFs which characterize whether or not the two variants collapse. However, as we will see, concurrence can be surprisingly hard, up to the third level of the polynomial hierarchy.

2 Preliminaries

We introduce (abstract) argumentation frameworks (Dung, 1995) and fix U as countable infinite domain of arguments.

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where $A \subseteq U$ is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. $E \subseteq A$* attacks *b if $(a, b) \in R$ for some $a \in E$; we denote by $E^+_F = \{b \in A \mid \exists a \in E : (a, b) \in R\}$ the set of arguments defeated by E. We call $E^\oplus_F = E \cup E^+_F$ the* range *of E in F. An argument $a \in A$ is* defended *(in F) by E if $b \in E^+_F$ for each b with $(b, a) \in R$.*

Semantics for AFs are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions cf, adm, $naive$, prf, stb, sem and stg which stand for conflict-free, admissible, naive, preferred, stable, semi-stable and stage, respectively.

Definition 2. *Let $F = (A, R)$ be an AF. A set $E \subseteq A$ is* conflict-free (in F)*, if there are no $a, b \in E$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of conflict-free sets in F. For $E \in cf(F)$ we have $E \in adm(F)$ if each $a \in E$ is defended by E in F. For $E \in cf(F)$, we define*

- *$E \in naive(F)$, if there is no $D \in cf(F)$ with $E \subset D$;*
- *$E \in prf(F)$, if $E \in adm(F)$ and $\nexists D \in adm(F)$: $E \subset D$;*
- *$E \in stb(F)$, if $E^\oplus_F = A$;*
- *$E \in sem(F)$, if $E \in adm(F)$ and $\nexists D \in adm(F)$: $E^\oplus_F \subset D^\oplus_F$;*
- *$E \in stg(F)$, if there is no $D \in cf(F)$ with $E^\oplus_F \subset D^\oplus_F$.*

Next we introduce CAFs (Dvořák & Woltran, 2020).

Definition 3. *A* claim-augmented argumentation framework (CAF) *is a triple $(A, R, claim)$ where (A, R) is an AF and $claim : A \to C$ assigns a claim to each argument in A; C is a set of possible claims. The claim-function is extended to sets in the natural way, i.e. $claim(E) = \{claim(a) \mid a \in E\}$. A CAF $(A, R, claim)$ is* well-formed *if $\{a\}^+_{(A,R)} = \{b\}^+_{(A,R)}$ for all $a, b \in A$ with $claim(a) = claim(b)$.*

Well-formed CAFs naturally appear as result of instantiation procedures where the construction of the attack relation depends on the claim of the attacking argument. However, formalisms which handle argument strengths or allow for preference relations over arguments (assumptions/defeasible rules) typically violate the property of well-formedness.

Semantics for CAFs. Here we give a short recap of *inherited semantics* and *claim-level semantics* for CAFs. We will first introduce inherited semantics (i-semantics).

Definition 4. *For a CAF $CF = (A, R, claim)$ and an AF semantics σ, we define* i-σ semantics *as $\sigma_c(CF) = \{claim(E) \mid E \in \sigma((A, R))\}$. We call $E \in \sigma((A, R))$ with $claim(E) = S$ a σ_c-realization of S in CF.*

Next we discuss claim-level semantics (cl-semantics) for CAFs. Central for cl-variants of stable, semi-stable and stage semantics is the following notion of claim-defeat.

Definition 5. *Let $CF = (A, R, claim)$, $E \subseteq A$ and $c \in claim(A)$. E* defeats *c (in CF) if E attacks every $a \in A$ with $claim(a) = c$. We define $\nu_{CF}(E) = \{c \in claim(A) \mid E \text{ defeats } c \text{ in } CF\}$.*

We will next introduce the notion of range for a claim-set S. As different realizations of S might yield different sets of defeated claims, the range of S is in general not unique and depends on the particular realization E of S. Observe that in well-formed CAFs, each claim-set possesses a unique range as each realization attacks the same arguments.

Definition 6. *For a CAF $CF = (A, R, claim)$, $S \subseteq claim(A)$ and a semantics σ, let $\mathcal{N}_\sigma^{CF}(S) = \{\nu_{CF}(E) \mid E \in \sigma((A,R)), claim(E) = S\}$. For each $S' \in \mathcal{N}_\sigma^{CF}(S)$, we call $S \cup S'$ a* range *of S in CF.*

We are now ready to introduce cl-semantics for CAFs.

Definition 7. *For a CAF $CF = (A, R, claim)$ and $S \subseteq claim(A)$, we define*
- *$S \in cl\text{-}prf(CF)$ if $S \in adm_c(CF)$ and there is no $T \in adm_c(CF)$ with $S \subset T$;*
- *$S \in cl\text{-}naive(CF)$ if $S \in cf_c(CF)$ and there is no $T \in cf_c(CF)$ with $S \subset T$;*
- *$S \in cl\text{-}stb_\tau(CF)$, $\tau \in \{cf, adm\}$, if there is $S' \in \mathcal{N}_\tau^{CF}(S)$ with $S \cup S' = claim(A)$;*
- *$S \in cl\text{-}sem(CF)$ if there is $S' \in \mathcal{N}_{adm}^{CF}(S)$ s.t. there is no $T \in adm_c(CF)$, $T' \in \mathcal{N}_{adm}^{CF}(T)$ with $S \cup S' \subset T \cup T'$;*
- *$S \in cl\text{-}stg(CF)$ if there is $S' \in \mathcal{N}_{cf}^{CF}(S)$ s.t. there is no $T \in cf_c(CF)$, $T' \in \mathcal{N}_{cf}^{CF}(T)$ with $S \cup S' \subset T \cup T'$.*

We say that a set $E \subseteq A$ realizes a cl-σ claim-set S in CF if $claim(E) = S$, $E \in cf((A,R))$ ($E \in adm((A,R))$ respectively) and $S \cup \nu_{CF}(E)$ satisfies the respective requirements, e.g., $S \cup \nu_{CF}(E) = claim(A)$ for τ-cl-stable semantics. We call E also a cl-σ-realization of S in CF.

3 Computational Problems

We consider the following decision problems with respect to a CAF-semantics σ:
- *Credulous Acceptance* ($Cred_\sigma^{CAF}$): Given a CAF $CF = (A, R, claim)$ and claim $c \in claim(A)$, is c contained in some $S \in \sigma(CF)$?
- *Skeptical Acceptance* ($Skept_\sigma^{CAF}$): Given a CAF $CF = (A, R, claim)$ and claim $c \in claim(A)$, is c contained in each $S \in \sigma(CF)$?
- *Verification* (Ver_σ^{CAF}): Given a CAF $CF = (A, R, claim)$ and a set $S \subseteq claim(A)$, is $S \in \sigma(CF)$?
- *Non-emptiness* (NE_σ^{CAF}): Given a CAF $CF = (A, R, claim)$, is there a non-empty set $S \subseteq claim(A)$ such that $S \in \sigma(CF)$?

We furthermore consider these reasoning problems restricted to well-formed CAFs and denote them by $Cred_\sigma^{wf}$, $Skept_\sigma^{wf}$, Ver_σ^{wf}, and NE_σ^{wf}. Moreover, we denote the corresponding decision problems for AFs (which can be obtained by defining $claim$ as the identity function) by $Cred_\sigma^{AF}$, $Skept_\sigma^{AF}$, Ver_σ^{AF}, and NE_σ^{AF}. Finally, we introduce a new decision problem which asks whether the two variants of a semantics coincide on a given CAF.
- *Concurrence* (Con_σ^{CAF}): Given a CAF CF, does it hold that $\sigma_c(CF) = cl\text{-}\sigma(CF)$?

For stable semantics, we write $Con_{stb_\tau}^{CAF}$ to specify the considered cl-stable variant ($\tau \in \{adm, cf\}$). The concurrence problem restricted to well-formed CAFs is denoted Con_σ^{wf}.

Tables 1 & 2 depict known complexity results for AF semantics (Dimopoulos & Torres, 1996; Dunne & Bench-Capon, 2002; Dvořák & Woltran, 2010; Dvořák & Dunne, 2018); and for inherited CAF semantics (Dvořák & Woltran, 2020). Note that Table 2 lacks results for semi-stable and stage semantics which have not been studied yet in terms of complexity. We close this gap and complement these results by an analysis of the claim-level variants.

Table 1: Complexity of AFs.

σ	$Cred_\sigma^{AF}$	$Skept_\sigma^{AF}$	Ver_σ^{AF}	NE_σ^{AF}
cf	in P	trivial	in P	in P
adm	NP-c	trivial	in P	NP-c
stb	NP-c	coNP-c	in P	NP-c
prf	NP-c	Π_2^P-c	coNP-c	NP-c
$naive$	in P	in P	in P	in P
sem	Σ_2^P-c	Π_2^P-c	coNP-c	NP-c
stg	Σ_2^P-c	Π_2^P-c	coNP-c	in P

Table 2: Complexity for $\Delta \in \{CAF, wf\}$ of inherited semantics. Results that deviate from the corresponding results for AFs are bold-face.

σ	$Cred_\sigma^\Delta$	$Skept_\sigma^\Delta$	Ver_σ^{CAF}/Ver_σ^{wf}	NE_σ^Δ
cf_c	in P	trivial	$\mathbf{NP}$-c / in P	in P
adm_c	NP-c	trivial	$\mathbf{NP}$-c / in P	NP-c
stb_c	NP-c	coNP-c	$\mathbf{NP}$-c / in P	NP-c
prf_c	NP-c	Π_2^P-c	$\boldsymbol{\Sigma_2^\mathsf{P}}$-c / coNP-c	NP-c
$naive_c$	in P	$\mathbf{coNP}$-c	$\mathbf{NP}$-c / in P	in P

4 Complexity of Reasoning Problems

The forthcoming analysis yields the following high level picture: Credulous and skeptical reasoning as well as deciding existence of a non-empty extension is of the same complexity as in AFs except for the notable difference that skeptical reasoning with respect to cl-naive semantics goes up two levels in the polynomial hierarchy and is thus also more expensive than deciding skeptical acceptance for i-naive semantics which has been shown to be coNP-complete. For well-formed CAFs, skeptical reasoning admits the same complexity for both claim-level and inherited naive semantics but remains more expensive than in AFs.

For general CAFs, the verification problem is more expensive than for AFs for all of the considered semantics. Comparing claim-level and inherited semantics we observe that the complexity of the verification problem for cl-preferred semantics drops while the complexity for cl-naive semantics admits a higher complexity than their inherited counterparts; the claim-level and inherited variants of stable, semi-stable and stage semantics admit the same complexity. For well-formed CAFs, the complexity of the verification problem coincides with the known results for AFs.

Theorem 1. *The complexity results for CAFs depicted in Table 3 hold.*

In the following we provide proofs for the results in Table 3. We will first discuss the membership proofs of the considered decision problems. To begin with, we will

Table 3: Complexity of CAFs. Results that deviate from the corresponding AF results are in bold-face; results that deviate from those w.r.t. inherited semantics are underlined.

σ	$\mathit{Cred}_\sigma^{CAF}$	$\mathit{Skept}_\sigma^{CAF}$	$\mathit{Ver}_\sigma^{CAF}$	NE_σ^{CAF}
sem_c	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^\mathsf{P}}$**-c**	NP-c
stg_c	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^\mathsf{P}}$**-c**	in P
$\mathit{cl\text{-}stb}_{adm}$	NP-c	coNP-c	**NP-c**	NP-c
$\mathit{cl\text{-}stb}_{cf}$	NP-c	coNP-c	**NP-c**	NP-c
$\mathit{cl\text{-}prf}$	NP-c	Π_2^P-c	<u>**DP-c**</u>	NP-c
$\mathit{cl\text{-}naive}$	in P	<u>$\boldsymbol{\Pi_2^\mathsf{P}}$**-c**</u>	<u>**DP-c**</u>	in P
$\mathit{cl\text{-}sem}$	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^\mathsf{P}}$**-c**	NP-c
$\mathit{cl\text{-}stg}$	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^\mathsf{P}}$**-c**	in P

give poly-time respectively coNP procedures for deciding whether a given set of arguments E is a σ-realization for $\sigma \in \{\mathit{cl\text{-}stb}_{adm}, \mathit{cl\text{-}stb}_{cf}, \mathit{cl\text{-}sem}, \mathit{cl\text{-}stg}\}$. This lemma yields upper bounds for the respective reasoning problems; notice that the complexity goes up one level in the polynomial hierarchy since one requires an additional guess for E.

Lemma 1. *Given a CAF $CF = (A, R, \mathit{claim})$ and some $E \subseteq A$. Deciding whether E realizes (1) a τ-cl-stable claim-set in CF for $\tau \in \{adm, cf\}$ is in* P*; (2) a cl-semi-stable (cl-stage) claim set in CF is in* coNP.

Proof. Checking admissibility (conflict-freeness) of E is in P (cf. Table 1); moreover, $\nu_{CF}(E)$ can be computed in polynomial time by looping over all claims $c \in \mathit{claim}(A)$ and adding each c to $\nu_{CF}(E)$ if E attacks each occurrence of c in CF. For τ-cl-stable semantics, it remains to check whether $\mathit{claim}(E) \cup \nu_{CF}(E) = \mathit{claim}(A)$ to verify that E realizes a τ-cl-stable claim-set in CF. For cl-semi-stable (cl-stage) semantics, we have to check that each $E' \subseteq A$ with $\mathit{claim}(E') \cup \nu_{CF}(E') \supset \mathit{claim}(E) \cup \nu_{CF}(E)$ is not admissible (conflict-free). This can be solved in coNP by a standard guess & check algorithm, i.e. guess a set and verify that it is admissible (conflict-free), compute the claims and verify that they are a proper superset of the claims of the original set, yielding a coNP algorithm to verify that E realizes a cl-semi-stable (cl-stage) claim-set in CF. ☐

We use this lemma to show membership results for $\mathit{Ver}_\sigma^{CAF}$, $\sigma \in \{\mathit{cl\text{-}stb}_\tau, \mathit{cl\text{-}sem}, \mathit{cl\text{-}stg}\}$: For a CAF $CF = (A, R, \mathit{claim})$, $S \subseteq \mathit{claim}(A)$, one can verify $S \in \sigma(CF)$ by guessing a set of arguments $E \subseteq A$ with $\mathit{claim}(E) = S$ and checking whether E is a σ-realization of S. The latter is in P, respectively coNP by Lemma 1, yielding NP- and Σ_2^P-procedures for the respective semantics. DP-membership of $\mathit{Ver}_\sigma^{CAF}$ for $\sigma \in \{\mathit{cl\text{-}prf}, \mathit{cl\text{-}naive}\}$ is by (1) checking that a given claim-set S is admissible (conflict-free) and (2) verifying subset-maximality of S. The former has been shown to be NP-complete (cf. Table 2); the latter is in coNP: Guess a set of arguments E such that $S \subset \mathit{claim}(E)$ and check admissibility (conflict-freeness) of E. Σ_2^P-membership of $\mathit{Ver}_{\sigma_c}^{CAF}$ for $\sigma \in \{sem, stg\}$ is by guessing a set $E \subseteq A$ and checking $E \in \sigma((A, R))$ which is coNP-complete by known results for AFs (cf. Table 1).

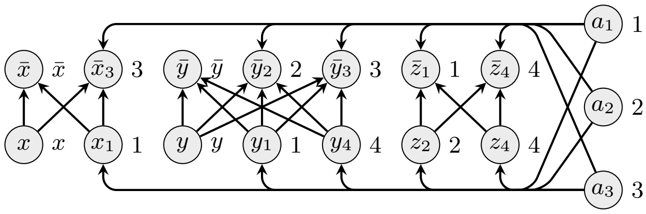


Figure 1: CAF from the proof of Proposition 1 for the formula $\forall xy \exists z \varphi$, where φ is given by the clauses $\{\{x, y, \neg z\}, \{\neg y, z\}, \{\neg x, \neg y\}, \{y, z, \neg z\}\}$.

Membership proofs for $\mathit{Skept}_\sigma^{CAF}$ are via the complementary problem: For a claim $c \in \mathit{claim}(A)$, guess a set $E \subseteq A$ such that $c \notin \mathit{claim}(E)$ and check $\mathit{claim}(E) \in \sigma(CF)$. For $\sigma \in \{\mathit{cl\text{-}stb}_\tau, \mathit{cl\text{-}sem}, \mathit{cl\text{-}stg}\}$, the latter can be verified in P respectively coNP by Lemma 1; for $\sigma \in \{\mathit{cl\text{-}prf}, \mathit{cl\text{-}naive}\}$, we use the result for $\mathit{Ver}_\sigma^{CAF}$, i.e., $\mathit{claim}(E) \in \sigma(CF)$ can be verified via two NP-oracle calls, which shows that $\mathit{Skept}_\sigma^{CAF}$ is in Π_2^P; for $\sigma \in \{\mathit{sem}_c, \mathit{stg}_c\}$, it suffices to check $E \in \mathit{sem}((A, R))$ or $E \in \mathit{stg}((A, R))$–both are in coNP (cf. Table 1)–to derive the desired upper bound.

Membership for $\mathit{Cred}_\sigma^{CAF}$ follows the same line of reasoning for $\sigma \in \{\mathit{cl\text{-}stb}_\tau, \mathit{cl\text{-}sem}, \mathit{cl\text{-}stg}, \mathit{sem}_c, \mathit{stg}_c\}$. For cl-preferred and cl-naive semantics, we exploit the fact a claim $c \in \mathit{claim}(A)$ is credulously accepted with respect to cl-preferred (cl-naive) semantics iff it is contained in some i-admissible (i-conflict-free) claim-set and thus the complexity of $\mathit{Cred}_\theta^{CAF}$ for $\theta \in \{\mathit{cf}_c, \mathit{adm}_c\}$ (cf. Table 2) applies.

Finally, NE_σ^{CAF} for $\sigma \in \{\mathit{sem}_c, \mathit{stg}_c, \mathit{cl\text{-}prf}, \mathit{cl\text{-}naive}, \mathit{cl\text{-}sem}, \mathit{cl\text{-}stg}\}$ coincides with either NE_{adm}^{AF} or NE_{cf}^{AF} and we get the complexity directly from Table 1. For $\sigma \in \{\mathit{cl\text{-}stb}_{adm}, \mathit{cl\text{-}stb}_{cf}\}$, NE_σ^{CAF} can be verified by guessing a non-empty set $E \subseteq A$ and utilizing Lemma 1 (1).

We now turn to the hardness results. First observe that one can reduce AF decision problems to the corresponding problems for CAFs by assigning each argument a unique claim. Thus CAF decision problems generalize the corresponding problems for AFs and are therefore at least as hard. It remains to provide hardness proofs for the decision problems with higher complexity. We will first present a reduction from $QSAT_2^\forall$ to show Π_2^P-hardness of $\mathit{Skept}_{\mathit{cl\text{-}naive}}^{CAF}$ before we address the verification problem: DP-hardness with respect to cl-preferred and cl-naive semantics is by reductions from SAT-UNSAT; Σ_2^P- hardness with respect to i-semi-stable and i-stage semantics are by reductions from credulous reasoning for AFs with the respective semantics; the remaining hardness results are shown via reductions from appropriate decision problems for inherited semantics.

Proposition 1. $\mathit{Skept}_{\mathit{cl\text{-}naive}}^{CAF}$ *is* Π_2^P*-hard.*

Proof. We present a reduction from $QSAT_2^\forall$; see Figure 1 for an illustration. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^\forall$, where φ is a 3-CNF given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. We construct a CAF $CF = (A, R, \mathit{claim})$ as follows: For each clause cl_i, we introduce three arguments representing the literals contained in cl_i and assign them claim i; moreover, we add arguments representing literals over Y and assign them unique names; furthermore, we add arguments $a_1, \dots, a_{n-1}$ with claims $1, \dots, n-1$; formally, $A = \{x_i \mid x \in cl_i, i \leq n\} \cup \{\bar{x}_i \mid \neg x \in cl_i, i \leq n\} \cup Y \cup \bar{Y} \cup \{a_1, \dots, a_{n-1}\}$ where $\bar{Y} = \{\bar{y} \mid y \in Y\}$, and $claim(x_i) = claim(\bar{x}_i) = claim(a_i) = i$, $claim(y) = y$, $claim(\bar{y}) = \bar{y}$. We introduce conflicts between each argument representing a variable $x \in X$ and its negation; moreover, the additional $n-1$ arguments attack every argument $x_i, \bar{x}_i$ representing literals in clauses cl_i; i.e., $R = \{(x_i, \bar{x}_j), \mid i, j \leq n\} \cup \{(y, \bar{y}_i), (y_i, \bar{y}), (y, \bar{y}) \mid y \in Y\} \cup \{(a_i, x_j), (a_i, \bar{x}_j) \mid i < n, j \leq n\}$.

It can be shown that Ψ is valid iff the claim n is skeptically accepted with respect to cl-naive semantics in CF: For every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{a_1, \dots, a_{n-1}\}$ is conflict-free in (A, R) by construction, and therefore $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n-1\}$ is in $cf_c(CF)$. Consequently, n is skeptically accepted with respect to cl-naive semantics iff for every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n\}$ is cl-naive. It suffices to check that for every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n\}$ is cl-naive iff there is $Z' \subseteq Z$ such that $Y' \cup Z'$ is a model of φ. □

Hardness results for verification admits a higher complexity compared to AFs. We first recall the standard reduction that provides the basis for DP-hardness of verification with respect to cl-preferred semantics and reappears in Section 5.

Reduction 1. *Let φ be given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in X. We construct (A, R) with*
- *$A = X \cup \bar{X} \cup C \cup \{\varphi\}$, with $\bar{X} = \{\bar{x} \mid x \in X\}$;*
- *$R = \{(x, cl) \mid cl \in C, x \in cl\} \cup \{(\bar{x}, cl) \mid cl \in C, \neg x \in cl\} \cup \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(cl_i, \varphi) \mid i \leq n\}$.*

Proposition 2. *$Ver^{CAF}_{cl\text{-}prf}$ is DP-hard.*

Proof. We present a reduction from SAT-UNSAT. Let (φ_1, φ_2) be an instance of SAT-UNSAT, where φ_i, $i = 1, 2$, is given over a set of clauses C_i over atoms in X_i with $X_1 \cap X_2 = \emptyset$. We will construct a CAF CF which consists of two independent frameworks $CF_i = (A_i, R_i, claim_i)$, $i = 1, 2$, both representing one of the formulas φ_1, φ_2: For the formula φ_i, let (A_i, R_i) be defined as in Reduction 1. Let $CF_i = (A_i, R'_i, claim_i)$ with $R'_i = R_i \cup \{(cl, cl) \mid cl \in C_i\}$; moreover, we define $claim_i(x) = claim_i(\bar{x}) = x$ for all $x \in X_i$, $claim_i(cl) = d$ for all $cl \in C_i$ and $claim_i(\varphi_i) = \varphi_i$. We define $CF = CF_1 \cup CF_2$ as the component-wise union of CF_1 and CF_2.

It can be checked that φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of CF_i. Since X_i is i-admissible in CF_i (for an adm_c-realization, consider $X' \cup \{\bar{x} \mid x \notin X'\}$ for any $X' \subseteq X_i$), we furthermore obtain that φ_i is unsatisfiable iff X_i is a cl-preferred claim-set of CF_i. Since CF_1 and CF_2 are unconnected and have no common arguments, we have $cl\text{-}prf(CF) = \{S \cup T \mid S \in cl\text{-}prf(CF_1), T \in cl\text{-}prf(CF_2)\}$. Thus $X_1 \cup X_2 \cup \{\varphi_1\}$ is cl-preferred in CF iff φ_1 is satisfiable and φ_2 is unsatisfiable. □

DP-hardness of verification with respect to cl-naive semantics can be shown via a reduction from SAT-UNSAT by combining ideas from the previous propositions. As in Proposition 2, one constructs two independent frameworks CF_1, CF_2 representing the formulas (3-CNFs) φ_1, φ_2 with sets of clauses $C_1 = \{cl_1, \dots, cl_m\}$ respectively $C_2 = \{cl_{m+1}, \dots, cl_n\}$. The construction is similar to the one in Proposition 1, i.e., one introduces an argument with claim i for each literal in a clause $cl_i \in C_j$ and adds $|C_j| - 1$ arguments with claims $1, \dots, m-1$ respectively $m+1, \dots, n-1$. One can show that $\{1, \dots, n-1\}$ is cl-naive in $CF_1 \cup CF_2$ iff φ_1 is satisfiable and φ_2 is unsatisfiable.

Proposition 3. *$Ver^{CAF}_{cl\text{-}naive}$ is DP-hard.*

In the following, we show Σ^{P}_2-hardness of the verification problem for CAFs with respect to i-semi-stable and i-stage semantics, utilizing a reduction from the respective credulous acceptance problem for AFs.

Proposition 4. *$Ver^{CAF}_{sem_c}$ and $Ver^{CAF}_{stg_c}$ are Σ^{P}_2-hard.*

Proof. We present a proof for $Ver^{CAF}_{sem_c}$, the proof for $Ver^{CAF}_{stg_c}$ is analogous. For an instance (A, R), $b \in A$ of $Cred^{AF}_{sem}$, we construct a CAF $CF = (A', R, claim)$ with $A' = A \cup \{x\}$, $x \notin A$ and $claim(b) = c_1$, $claim(a) = c_2$ for all $a \in A' \setminus \{b\}$. Then, as the argument x is not involved in any attack, it is contained in every semi-stable extension of (A', R) and thus, as $claim(x) = c_2$, c_2 is contained in every i-semi-stable claim-set of CF. Furthermore, as CF contains only two claims, the only candidates for i-semi-stable claim-sets are $\{c_1, c_2\}$ and $\{c_2\}$. Moreover, as b is the only argument with claim c_1, $\{c_1, c_2\}$ is i-semi-stable iff b is contained in some semi-stable set of arguments in (A', R). Thus, b is credulously accepted in (A, R) w.r.t. semi-stable semantics iff $\{c_1, c_2\}$ is i-semi-stable in CF. Σ^{P}_2-hardness of $Ver^{CAF}_{sem_c}$ thus follows from known results for AFs. □

Finally, we provide hardness results for cl-semi-stable, τ-cl-stable and cl-stage semantics. We will present reductions from the verification problem of suitable inherited semantics. To that end, we consider the following translations.

Definition 8. *For a CAF $CF = (A, R, claim)$, we define $Tr_1(CF) = (A', R', claim')$ with*
- *$A' = A \cup \{a' \mid a \in A\}$;*
- *$R' = R \cup \{(a, a'), (a', a') \mid a \in A\}$; and*
- *$claim'(a) = claim(a)$ for $a \in A$, $claim(a') = c_a$ for $a' \in \{a' \mid a \in A\}$ and fresh claims $c_a \notin claim(A)$.*

Moreover, we define $Tr_2(CF) = (A', R'_2, claim')$ with $R'_2 = R' \cup \{(a, b') \mid (a, b) \in R\}$; and $Tr_3(CF) = (A', R'_3, claim')$ with $R'_3 = R'_2 \cup \{(b, a) \mid (a, b) \in R\} \cup \{(a, b) \mid a \in A, (b, b) \in R\}$.

It can be shown that Tr_1 maps i-preferred semantics to cl-semi-stable semantics, while Tr_2 (Tr_3) maps inherited to claim-level stable (respectively stage) semantics.

Lemma 2. *For a CAF $CF = (A, R, claim)$,*
1. *$prf_c(CF) = prf_c(Tr_1(CF)) = cl\text{-}sem(Tr_1(CF))$;*
2. *$stb_c(CF) = stb_c(Tr_2(CF)) = cl\text{-}stb_\tau(Tr_2(CF))$ for $\tau \in \{adm, cf\}$;*
3. *$stg_c(CF) = stg_c(Tr_3(CF)) = cl\text{-}stg(Tr_3(CF))$.*

Table 4: Complexity of well-formed CAFs. Results that deviate from general CAFs (cf. Table 3) are in bold-face.

σ	$Cred_\sigma^{wf}$	$Skept_\sigma^{wf}$	Ver_σ^{wf}	NE_σ^{wf}
sem_c	Σ_2^{P}-c	Π_2^{P}-c	**coNP**-c	NP-c
stg_c	Σ_2^{P}-c	Π_2^{P}-c	**coNP**-c	in P
$cl\text{-}stb_{cf}$	NP-c	coNP-c	in **P**	NP-c
$cl\text{-}stb_{adm}$	NP-c	coNP-c	in **P**	NP-c
$cl\text{-}naive$	in P	**coNP**-c	in **P**	in P
$cl\text{-}prf$	NP-c	Π_2^{P}-c	**coNP**-c	NP-c
$cl\text{-}sem$	Σ_2^{P}-c	Π_2^{P}-c	**coNP**-c	NP-c
$cl\text{-}stg$	Σ_2^{P}-c	Π_2^{P}-c	**coNP**-c	in P

Lower bounds for Ver_σ^{CAF}, $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}, cl\text{-}sem, cl\text{-}stg\}$, thus follow from the results of the respective inherited semantics: For a given CAF $CF = (A, R, claim)$ and a set of claims $S \subseteq claim(A)$, one can check $S \in \sigma'_c(CF)$, $\sigma' \in \{stb, prf, stg\}$, by applying the respective translation and checking whether S is a σ-realization in the resulting CAF. This concludes the proof of Theorem 1.

We next turn to the complexity of well-formed CAFs.

Theorem 2. *The complexity results for well-formed CAFs depicted in Table 4 hold.*

First observe that all upper bounds from Theorem 1 carry over since well-formed CAFs are a special case of CAFs. It remains to give improved upper bounds for verification with respect to all of the considered semantics as well as for $Skept_{cl\text{-}naive}^{wf}$. The latter also requires a genuine hardness proof as it remains harder than the corresponding problem for AFs even in the well-formed case. For the remaining semantics, we obtain hardness results from the corresponding problems for AFs since they constitute a special case of the respective problems for CAFs.

We first discuss improved upper bounds for verification. For preferred as well as for both variants of cl-stable semantics, membership is immediate by the corresponding results for inherited semantics as the respective semantics collapse in the well-formed case (Dvořák et al., 2020). For the remaining semantics, we exploit the following observation (Dvořák & Woltran, 2020).

Lemma 3. *Let $CF = (A, R, claim)$ be well-formed. For $S \subseteq claim(A)$, let $E_0(S) = \{a \in A \mid cl(a) \in S\}$, $E_1(S) = E_0(S) \setminus E_0(S)^+_{(A,R)}$, and $E_2 = \{a \in E_1(S) \mid b \in E_1(S)^+_{(A,R)}$ for all $(b, a) \in R\}$. Then $S \in cf_c(CF)$ iff $S = claim(E_1(S))$ and $S \in adm_c(CF)$ iff $S = claim(E_2(S))$.*

To check whether a set $S \subseteq claim(A)$ is cl-naive in a given well-formed CAF $CF = (A, R, claim)$, we utilize Lemma 3 to test (i) $S \in cf_c(CF)$ and (ii) $S \cup \{c\} \notin cf_c(CF)$ for all $c \in claim(A) \setminus S$, which yields a poly-time procedure for Ver_{naive}^{wf}. For inherited as well as claim-level semi-stable and stage semantics, we first compute $E_1(S)$, respectively $E_2(S)$ in P (cf. Lemma 3). For cl-semi-stable (cl-stage) semantics, utilize Lemma 1 to check in coNP whether $E_2(S)$ ($E_1(S)$) realizes a cl-semi-stable (cl-stage) claim set; similarly, for i-semi-stable (i-stage) semantics, we check that $E_2(S) \in sem((A, F))$ ($E_1(S) \in stg((A, F))$), which is known to be coNP-complete.

Finally, we will discuss coNP-completeness of skeptical reasoning in well-formed CAFs w.r.t. cl-naive semantics.

Proposition 5. *$Skept_{cl\text{-}naive}^{wf}$ is coNP-complete.*

Proof. As the verification problem is in P, the membership is by a standard guess and check algorithm. Hardness can be shown via a reduction from UNSAT: For a formula φ with clauses $C = \{cl_1, \ldots, cl_n\}$ over the atoms X, let (A, R) be defined as in Reduction 1. We define $CF = (A', R', claim)$ with $A' = A \setminus \{\varphi\}$ and $R' = R \setminus \{(cl_i, \varphi) \mid i \leq n\}$, moreover, we set $claim(x) = x$, $claim(\bar{x}) = \bar{x}$, $claim(cl_i) = \bar{\varphi}$. Observe that CF is indeed well-formed. It can be checked that φ is unsatisfiable iff $\bar{\varphi}$ is skeptically accepted with respect to cl-naive semantics. □

5 Deciding Concurrence

This section examines the complexity of deciding concurrence of the different variants of the considered semantics. Our results (cf. Table 5) reveal that deciding concurrence is in general computationally hard; observe that for semi-stable and stage semantics, the problem is complete for the third level of the polynomial hierarchy.

Table 5: Complexity of deciding Con_σ^{CAF} and Con_σ^{wf}.

	prf	$naive$	stb_τ	sem	stg
Con_σ^{CAF}	Π_2^{P}-c	coNP-c	Π_2^{P}-c	Π_3^{P}-c	Π_3^{P}-c
Con_σ^{wf}	trivial	in coNP	trivial	Π_2^{P}-c	Π_2^{P}-c

Theorem 3. *The complexity results depicted in Table 5 hold.*

In what follows, we will present upper bounds for the (non-trivial) problems and discuss Π_3^{P}-hardness of deciding concurrence for semi-stable and stage semantics.

Membership of deciding concurrence is by the following generic guess and check procedure for the complementary problem: To verify for a given (well-formed) CAF $CF = (A, R, claim)$ that $\sigma_c(CF) = cl\text{-}\sigma(CF)$ one first guesses a set of claims $S \subseteq claim(A)$ and checks whether $S \in \sigma_c(CF)$ and $S \notin cl\text{-}\sigma(CF)$ or vice versa. The complexity of the procedure thus follows from the corresponding results for verification with respect to the considered semantics.

For preferred and naive semantics, we get improved upper bounds by the following observation: If a CAF CF admits incomparable i-preferred (i-naive) claim-sets then both variants of the respective semantics coincide; that is, for $\sigma \in \{prf, naive\}$, $\sigma_c(CF) = cl\text{-}\sigma(CF)$ if and only if $\sigma_c(CF)$ is incomparable. Thus it suffices to verify incomparability of $\sigma_c(CF)$. We give a Σ_2^{P} (NP resp.) procedure for the complementary problem: Guess $E, G \subseteq A$ and check (i) $E, G \in \sigma((A, R))$ and (ii) $claim(E) \subset claim(G)$. The former is in coNP for prf (in P for $naive$) by Table 1.

We next extend Reduction 1 in order to show Π_3^{P}-hardness of concurrence with respect to semi-stable semantics.

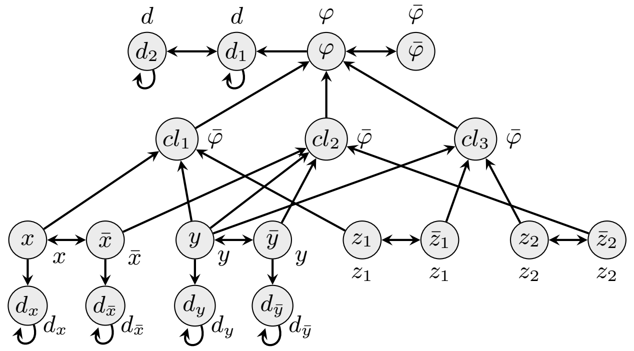


Figure 2: Reduction 2 for the formula $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ with clauses $\{\{z_1, x, y\}, \{\neg x, \neg y, \neg z_2, y\}, \{\neg z_1, z_2, y\}\}$.

Reduction 2. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^{\exists}$, where φ is given by a set of clauses $\mathcal{C} = \{cl_1, \ldots, cl_n\}$ over atoms in $V = X \cup Y \cup Z$. We can assume that there is a variable $y_0 \in Y$ with $y_0 \in cl_i$ for all $i \leq n$ (otherwise we can add such a y_0 without changing the validity of Ψ). Let (A, R) be the AF constructed from φ as in Reduction 1. We define $CF = (A', R', claim)$ with*

- *$A' = A \cup \{d_1, d_2, \bar{\varphi}\} \cup \{d_v, d_{\bar{v}} \mid v \in X \cup Y\}$;*
- *$R' = R \cup \{(a, d_a), (d_a, d_a), \mid a \in X \cup \bar{X} \cup Y \cup \bar{Y}\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi), (\varphi, d_1)\} \cup \{(d_i, d_j) \mid i, j \leq 2\}$;*
- *$claim(v) = claim(\bar{v}) = v$ for $v \in Y \cup Z$; $claim(cl_i) = \bar{\varphi}$ for $i \leq n$; $claim(d_i) = d$ for $i = 1, 2$; $claim(a) = a$ else.*

An illustrative example of the reduction is given in Figure 2. Next we provide some properties for the reduction making use of the observation that for any instance of $QSAT_3^{\exists}$, each i-semi-stable and each cl-semi-stable claim-set in the resulting CAF is of the form $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$; in fact, it can be shown that each such set is $cl\text{-}sem$-realizable. Note that this is not the case for i-semi-stable semantics (as a counter-example, consider $e = \bar{\varphi}$ and $X = \{x\}$ in Figure 2).

Lemma 4. *Let $CF = (A, R, claim)$ be as in Reduction 2 for an instance $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ of $QSAT_3^{\exists}$. Then, (1) $cl\text{-}sem(CF) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$; (2) $sem_c(CF) \subseteq cl\text{-}sem(CF)$; and (3) $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\} \in sem_c(CF)$ for all $X' \subseteq X$.*

Proposition 6. *Con_{sem}^{CAF} is Π_3^{P}-hard.*

Proof. Let $CF = (A, R, claim)$ be the CAF generated by Reduction 2 from $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$. We show Ψ is valid iff $sem_c(CF) \neq cl\text{-}sem(CF)$. Since $sem_c(CF) \subseteq cl\text{-}sem(CF)$ by Lemma 6 (2), the latter reduces to $sem_c(CF) \subset cl\text{-}sem(CF)$. By Lemma 6 (3), this is the case if there is some $X' \subseteq X$ such that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ is not sem_c-realizable.

Assume Ψ is valid, then there is $X' \subseteq X$ such that $\Psi' = \forall Y \exists Z \varphi(X', Y, Z)$ is valid ($\varphi(X', Y, Z)$ is the formula which arises after replacing each $x \in X$ with $\top$ in case $x \in X'$ and $\bot$ if $x \notin X'$). One can show that $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \notin sem_c(CF)$: Towards a contradiction, assume there is a sem_c-realization E of S (observe that $\bar{\varphi} \in E$ and $d_1, d_2 \notin E^{\oplus}_{(A,R)}$). Let $Y' = E \cap Y$ and consider the set $D = M \cup \{\bar{v} \mid v \notin M\} \cup \{\varphi\}$, where $M = X' \cup Y' \cup Z'$ is a model of φ (since Ψ' is valid, there is such a $Z' \subseteq Z$). It can be checked that D is admissible; moreover, D attacks d_1 since $\varphi \in D$. Thus $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to $E \in sem((A, R))$.

In case Ψ is not valid, one can show that for all $X' \subseteq X$, $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \in sem_c(CF)$. Let $X' \subseteq X$. Since Ψ is not valid, there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $X' \cup Y' \cup Z'$ is not a model of φ. It can be shown that $X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \mathcal{C}' \cup \{\bar{\varphi}\}$, where $Z' \subseteq Z$ and $\mathcal{C}' \subseteq \mathcal{C}$ being all clauses which are not satisfied, is semi-stable in (A, R). Thus $sem_c(CF) = cl\text{-}sem(CF)$. □

Π_3^{P}-hardness of Con_{stg}^{CAF} also uses Reduction 2; in fact, we have $stg_c(CF) = sem_c(CF)$ and $cl\text{-}stg(CF) = cl\text{-}sem(CF)$ for all CAFs CF generated via the reduction.

Well-formed CAFs. For well-formed CAFs, cl-preferred and i-preferred as well as all considered variants of stable semantics coincide (Dvořák et al., 2020) thus the respective problems become trivial. Since for semi-stable and stage semantics, the complexity for verification drops for both variants, we get the Π_2^{P}-membership results. Hardness is by a reduction from $QSAT_2^{\forall}$ by appropriate adaptions of Reduction 1. Concurrence for well-formed CAFs with respect to naive semantics is a special case of CAFs and is therefore in coNP; establishing a corresponding lower bound remains an open problem.

6 Discussion

In this work we complemented complexity results for inherited semantics and provided a full complexity analysis of claim-level semantics. We highlight three observations here: (a) for both approaches the verification problem is harder than in the AF setting, which is in particular relevant when it comes to the enumeration of extensions; (b) however, when restricted to well-formed CAFs the complexity of verification drops to the complexity of AFs; and (c) the complexity of inherited and claim-level semantics differs for naive and preferred semantics. Our complexity analysis paves the way for reduction-based implementation (Charwat, Dvořák, Gaggl, Wallner, & Woltran, 2015) of the considered semantics which is next on our agenda.

We also settled the complexity of the concurrence problem, i.e., deciding whether two variants of a semantics coincide on a CAF. The concurrence problem is in the tradition of the well-known coherence problem (Dunne & Bench-Capon, 2002), whose complexity for inherited semantics has been studied in (Dvořák & Woltran, 2020); for claim-based semantics this remains for future research. While we focused on two different claim-based argumentation semantics in this paper, exploring further concepts of claim-focused evaluation – as also recently addressed in (Baroni & Riveret, 2019) indicating alternative ways of lifting semantics to the claim-level – is a further point on our agenda.

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A Appendix

In this appendix we provide full proofs for the main results of the paper.

We occasionally make use of relations between semantics (Dvořák & Woltran, 2020; Dvořák et al., 2020). For any CAF CF:

$$\begin{aligned} stb_c(CF) \subseteq sem_c(CF) \subseteq prf_c(CF) \subseteq adm_c(CF) \\ stb_c(CF) \subseteq stg_c(CF) \subseteq naive_c(CF) \subseteq cf_c(CF) \end{aligned}$$

Moreover,

$$\begin{aligned} stb_c(CF) \subseteq \textit{cl-stb}_{adm}(CF) \subseteq \textit{cl-stb}_{cf}(CF) \\ \textit{cl-prf}(CF) \subseteq prf_c(CF) \\ \textit{cl-sem}(CF) \subseteq prf_c(CF) \\ \textit{cl-naive}(CF) \subseteq naive_c(CF) \\ \textit{cl-stg}(CF) \subseteq naive_c(CF) \end{aligned}$$

If the CAF CF is well-formed, we additionally have $stb_c(CF) = \textit{cl-stb}_{adm}(CF) = \textit{cl-stb}_{cf}(CF)$, furthermore $prf_c(CF) = \textit{cl-prf}(CF)$.

We will make use of the following notations: For a propositional atom x, we will denote its negation by $\bar{x}$. Moreover, we write $\bar{X} = \{\bar{x} \mid x \in X\}$ for any set of atoms X. Furthermore, let $[n] = \{1, \dots, n\}$ for any $n \in \mathbb{N}$.

Proofs of Section 4

Theorem 1 (restated). The complexity results for CAFs depicted in Table 3 hold.

Proposition 1 (restated). $\textit{Skept}^{CAF}_{\textit{cl-naive}}$ is Π^{P}_2-hard.

Proof. We present a reduction from $QSAT_2^{\forall}$. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is a 3-CNF given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. We construct a CAF $CF = (A, R, claim)$ as follows: For each clause cl_i, we introduce three arguments representing the literals contained in cl_i and assign them the claim cl_i; moreover, we add separate arguments for each literal in $Y \cup \bar{Y}$ and assign them their unique name; furthermore, we define $n-1$ additional arguments $a_1, \dots, a_{n-1}$ with claim $cl_1, \dots cl_{n-1}$; formally, $A = \{x_i \mid x \in cl_i, i \leq n\} \cup \{\bar{x}_i \mid \bar{x} \in cl_i, i \leq n\} \cup Y \cup \bar{Y} \cup \{a_1, \dots, a_{n-1}\}$ and $claim(x_i) = claim(\bar{x}_i) = claim(a_i) = i$ for all $i \leq n$, $claim(y) = y$, $claim(\bar{y}) = \bar{y}$. We introduce conflicts between each argument representing a variable $x \in X$ and its negation; moreover, the additional $n-1$ arguments attack every argument representing literals x_i, $\bar{x}_i$ in a clause; i.e., $R = \{(x_i, \bar{x}_j), \mid i, j \leq n\} \cup \{(y, \bar{y}_i), (y_i, \bar{y}), (y, \bar{y}) \mid y \in Y\} \cup \{(a_i, x_j), (a_i, \bar{x}_j) \mid i < n, j \leq n\}$.

We will first prove the following observation: (a) For every $Y' \subseteq Y$, $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup [n] \in \textit{cl-naive}(CF)$ iff there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. Let $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup [n]$.

First assume $S \in \textit{cl-naive}(CF)$. Consider a cf_c-realization E of S and let $Z' = \{z \in Z \mid z_i \in E\}$. Then $M = Y' \cup Z'$ is a model of φ: Consider an arbitrary clause cl_i. Since $[n] \subseteq S$, there is some argument with claim i in E, that is, either $a_i \in E$ or $x_i \in E$ or $\bar{x}_i \in E$ for some $x \in X$ (observe that $y_i \in E$ iff $y \in E$ and $\bar{y}_i \in E$ iff $\bar{y} \in E$, thus a further case distinction for $y \in Y$, $\bar{y} \in \bar{Y}$ is not required). We have that $a_i \notin E$ since $n \in S$ and for each argument b with $claim(b) = n$ we have $(a_i, b) \in R$. Thus there is $x \in X$ such that either $x_i \in E$ or $\bar{x}_i \in E$. In the former case, we have $x \in M$ and thus M satisfies cl_i, in the latter case $x \notin M$ and thus cl_i is satisfied. We obtain that M is a model of φ.

Now assume there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. Let $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{x_i \mid x \in M\} \cup \{\bar{x}_i \mid x \notin M\}$. E is conflict-free since $a_i \notin E$ for all $i < n$; other conflicts appear only between arguments $x_i, \bar{x}_j$ referring to the same atom x. Moreover, as M is a model of φ, we have that for each clause cl_i, there is either a positive literal $x \in cl_i$ with $x \in M$ or a negative literal $\bar{x} \in cl_i$ with $x \notin M$. Thus $[n] \subseteq claim(E)$; moreover, $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq claim(E)$ and therefore $claim(E) = S$. S is a maximal cl-conflict-free claim-set since $S \cup \{c\} \notin cf_c(CF)$ for any $c \in (Y \cup \bar{Y}) \setminus S$ as each realization of $S \cup \{c\}$ contains $y, \bar{y}$ for some $y \in Y$. Thus $S \in \textit{cl-naive}(CF)$.

We show that Ψ is valid iff the claim n is skeptically accepted with respect to cl-naive semantics in CF.

Assume Ψ is not valid. Then there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $M = Y' \cup Z'$ does not satisfy φ. Let $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup [n-1]$. Observe that S is i-conflict-free, witnessed by the cf_c-realization $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{a_1, \dots, a_{n-1}\}$. S is cl-naive since $S \cup \{n\} \notin cf_c(CF)$ by (a) and $S \cup \{c\} \notin cf_c(CF)$ for any $c \in (Y \cup \bar{Y}) \setminus S$ as each realization of $S \cup \{c\}$ contains $y, \bar{y}$ for some $y \in Y$. Thus n is not skeptically accepted with respect to cl-naive semantics in CF.

Assume n is not skeptically accepted with respect to naive semantics in CF. Then there is a set $S \in \textit{cl-naive}(CF)$ such that $n \notin S$. Observe that S contains $Y' \cup \{\bar{y} \mid y \notin Y\}$ for some $Y' \subseteq Y$ by construction. Let $Y' = S \cup Y$. We show that for all $Z' \subseteq Z$, $Y' \cup Z'$ is not a model of φ: Towards a contradiction assume there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. By (a), $T = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup [n] \in \textit{cl-naive}(CF)$. Thus $T \supset S$ since $n \notin S$, contradiction to S being cl-naive in CF. It follows that Ψ is not valid. $\square$

Proposition 2 (restated). $\textit{Ver}^{CAF}_{\textit{cl-prf}}$ is DP-hard.

Proof. We present a reduction from SAT-UNSAT. Let (φ_1, φ_2) be an instance of SAT-UNSAT, where φ_i, $i = 1, 2$, is given over a set of clauses C_i over atoms in X_i with $X_1 \cap X_2 = \emptyset$. We will construct a CAF CF which consists of two independent frameworks $CF_i = (A_i, R_i, claim_i)$, $i = 1, 2$, both representing one of the formulas φ_1, φ_2: For the formula φ_i, let (A_i, R_i) be defined as in Reduction 1. Let $CF_i = (A_i, R_i', claim_i)$ with $R_i' = R_i \cup \{(cl, cl) \mid cl \in C_i\}$; moreover, we define $claim_i(x) = claim_i(\bar{x}) = x$ for all $x \in X_i$, $claim_i(cl) = d$ for all $cl \in C_i$ and $claim_i(\varphi_i) = \varphi_i$. We define $CF = CF_1 \cup CF_2$ as the component-wise union of CF_1, CF_2.

We will show that $X_1 \cup X_2 \cup \{\varphi_1\}$ is cl-preferred in CF iff φ_1 is satisfiable and φ_2 is unsatisfiable. Since CF_1 and CF_2 are unconnected and have no common arguments (and thus $\mathit{cl\text{-}prf}(CF) = \{S \cup T \mid S \in \mathit{cl\text{-}prf}(CF_1), T \in \mathit{cl\text{-}prf}(CF_2)\}$), it suffices to show that (a) φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of CF_i and (b) φ_i is unsatisfiable iff X_i is a cl-preferred claim-set of CF_i. The latter follows from (a) since X_i is i-admissible in CF_i independently of the satisfiability of φ_i (for an adm_c-realization, consider $X' \cup \{\bar{x} \mid x \notin X'\}$ for any $X' \subseteq X_i$).

We show φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of CF_i:

Assume φ_i is satisfiable and consider a model M of φ_i. Let $E = M \cup \{\bar{x} \mid x \notin M\}$. We show that $E' = E \cup \{\varphi_i\}$ is admissible in (A_i, R_i'): First observe that E is admissible since each $a \in X_i \cup \bar{X}_i$ defends itself. Since M satisfies φ_i, we have that for any clause $cl \in C_i$, there is either $x \in cl$ with $x \in M$ or $\bar{x} \in cl$ with $x \notin M$, thus E attacks each $cl \in C$. Consequently, E defends φ_i; we conclude that E' is admissible in (A_i, R_i'). Moreover, $claim(E')$ is a subset-maximal i-admissible claim-set since $claim(E') = A_i \setminus \{d\}$, that is, $claim(E')$ contains every claim $c \in claim(A_i)$ which is assigned to non-self-attacking arguments. Thus $claim(E') = X_i \cup \{\varphi_i\}$ is cl-preferred in CF_i.

Now assume $X_i \cup \{\varphi_i\}$ is cl-preferred in CF_i. Let E be a adm_c-realization of $X_i \cup \{\varphi_i\}$ and let $M = E \cap X_i$. Consider an arbitrary clause $cl \in C_i$. Since $\varphi_i \in E$ is defended by E we have that E attacks cl, thus there is either an argument $x \in E$ such that $(x, cl) \in R_i'$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl) \in R_i'$. In the former case, we have $x \in M$ and thus M satisfies cl, in the latter case $x \notin M$ and thus cl is satisfied. We obtain that M is a model of φ_i. □

Proposition 3 (restated). $Ver^{CAF}_{\mathit{cl\text{-}naive}}$ is DP-hard.

Proof. Let (φ_1, φ_2) be an instance of SAT-UNSAT, where φ_1 is a 3-CNF given over a set of clauses $C_1 = \{cl_1, \ldots, cl_m\}$ over atoms in X, φ_2 is a 3-CNF given over a set of clauses $C_2 = \{cl_{m+1}, \ldots, cl_n\}$ over atoms in Y. We construct a CAF $CF = CF_1 \cup CF_2$ which consists of two independent frameworks where CF_1 represents φ_1 and CF_2 represents φ_2. For both CF_1, CF_2, we introduce three arguments for each clause cl_i representing the literals in the clause and assign them claim i; moreover, we introduce conflicts between each variable and its negation; furthermore, we add arguments $A_1' = a_1, \ldots, a_{m-1}$ (respectively $A_2' = a_{m+1}, \ldots, a_{n-1}$) with claims $1, \ldots, n-1$ (respectively $m+1, \ldots, n-1$) where each a_i attacks every argument representing a literal in φ_1 (respectively φ_2). Formally, $CF_j = (A_j, R_j, claim_j)$, $j = 1, 2$ is given by $A_j = A_j' \cup \{x_i \mid x \in cl_i, i \leq m\} \cup \{\bar{x}_i \mid \bar{x} \in cl_i, cl_i \in C_j\}$; $R_j = \{(x_i, \bar{x}_k) \mid i \neq k\} \cup \{(a_i, b) \mid a_i \in A_j', b \in A \setminus A_j'\}$; and $claim_j(x_i) = claim(\bar{x}_i) = i$.

We show φ_1 is satisfiable and φ_2 is unsatisfiable iff $[n-1] \in \mathit{cl\text{-}naive}(CF)$ by proving (a) φ_1 is satisfiable iff $[m] \in \mathit{cl\text{-}naive}(CF_1)$. (b) φ_2 is unsatisfiable iff $\{m+1, \ldots, n-1\} \in \mathit{cl\text{-}naive}(CF_2)$. Since CF_1, CF_2 are unconnected and $claim(A_1) \cap claim(A_2) = \emptyset$, we have $naive_c(CF) = \{S \cup T \mid S \in naive_c(CF_1), T \in naive_c(CF_2)\}$. Thus φ_1 is satisfiable and φ_2 is unsatisfiable iff $[n-1] \in \mathit{cl\text{-}naive}(CF)$.

(a) First assume φ_1 is satisfiable and consider a model M of φ_1. Let $E = \{x_i \mid x \in M, i \leq m\} \cup \{\bar{x}_i \mid x \notin M, i \leq m\}$. E is conflict-free by construction. We show that $claim(E) = [m]$. Let $cl_i \in C_1$. Since φ_1 is satisfiable, there is either $x \in M \cap cl_i$ or $\bar{x} \in cl_i$ such that $x \notin M$. Thus either $x_i \in E$ with $claim(x_i) = i$ or $\bar{x}_i \in E$ with $claim(\bar{x}_i) = i$ and therefore $claim(E) = [m]$.

Now assume $[m] \in \mathit{cl\text{-}naive}(CF)$ and let E be a cf_c-realization of $[m]$. Let $M = \{x \mid \exists i \leq n : x_i \in E\}$. Now, consider an arbitrary clause $cl_i \in C_1$. Then E contains an argument with claim i, that is, either $x_i \in E$ or $\bar{x}_i \in E$. In the former case, $x \in M$ and thus cl_i is satisfied. In the latter case, $x \notin M$ as $\bar{x}_i$ is in conflict with all arguments x_j and thus cl_i is satisfied. We obtain that M is a model of φ and thus φ_1 is satisfiable.

(b) First notice that $claim(A_2') = \{m+1, \ldots, n-1\} \in cf_c(CF_2)$ by construction. By (a), φ_2 is unsatisfiable iff $\{m+1, \ldots, n\} \notin \mathit{cl\text{-}naive}(CF_2')$. As $claim(A_2') \subset \{m+1, \ldots, n\}$, we have φ_2 is unsatisfiable iff $\{m+1, \ldots, n\} \notin \mathit{cl\text{-}naive}(CF_2)$ iff $\{m+1, \ldots, n-1\} \in \mathit{cl\text{-}naive}(CF_2)$. □

Lemma 2 (restated). For a CAF $CF = (A, R, claim)$,

1. $prf_c(CF) = prf_c(Tr_1(CF)) = \mathit{cl\text{-}sem}(Tr_1(CF))$;
2. $stb_c(CF) = stb_c(Tr_2(CF)) = \mathit{cl\text{-}stb}_\tau(Tr_2(CF))$ for $\tau \in \{adm, cf\}$;
3. $stg_c(CF) = stg_c(Tr_3(CF)) = \mathit{cl\text{-}stg}(Tr_3(CF))$.

We prove the three statements separately. First, we show $prf_c(CF) = prf_c(Tr_1(CF)) = \mathit{cl\text{-}sem}(Tr_1(CF))$.

Proof. Let $Tr_1(CF) = CF' = (A', R', claim')$. The proof proceeds in three steps:

(i) We first show that $C \in cf_c(CF)$ if and only if $C \in cf_c(CF')$ and further that $prf_c(CF) = prf_c(CF')$.
$\Rightarrow$: Let E be a cf_c-realization of C in (A, R). As $E \subseteq A$, it cannot contain any a'. Thus, $E \in cf((A', R'))$, as all additional attacks contain at least one argument a', which are not contained in E and therefore $C \in cf_c(CF')$.
$\Leftarrow$: Let E be a cf_c-realization of C in (A', R'). As all arguments a' are self-attacking, $E \cap A' = \emptyset$. Therefore, as $R \subseteq R'$, $E \in cf((A, R))$ and thus $C \in cf_c(CF)$.

Moreover, also $E \in adm((A, R))$ if and only if $E \in adm((A', R'))$, as $E \cap A' = \emptyset$. Now, as preferred extensions are subset maximal admissible sets, we further obtain that $E \in prf((A, R))$ if and only if $E \in prf((A', R'))$ and thus, $prf_c(CF) = prf_c(CF')$.

(ii) Next, to show that $prf_c(CF') \subseteq \mathit{cl\text{-}sem}(CF')$, let $C \in prf_c(CF')$ and E be a prf_c-realization of C in (A', R'). Furthermore, towards a contradiction, let $F \in adm((A', R'))$ and $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$. As $E \in prf((A', R'))$, there must be some $a \in E \setminus F$. Furthermore, as all arguments $b' \in A' \setminus A$ are self-attacking, it must hold that $a \in A$ and thus, by the construction of Tr_1, there must be some argument a' such that a is the only argument attacking a' and a' is the only argument with claim $claim'(a')$. Therefore, $claim'(a') \in \nu_{CF'}(E)$

but $claim(a') \notin claim'(F) \cup \nu_{CF'}(F)$, contradicting that $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$. Thus, such a set F cannot exist and therefore, $prf_c(CF') \subseteq cl\text{-}sem(CF')$.

(iii) Finally, to show that $cl\text{-}sem(CF') \subseteq prf_c(CF')$, let $C \in cl\text{-}sem(CF')$ and $E \subseteq A'$ be a admissible set witnessing C. Towards a contradiction, let $F \subseteq prf((A', R'))$ such that $E \subset F$. Then, $C \cup \nu_{CF'}(E) \subseteq claim'(F) \cup \nu_{CF'}(F)$. Furthermore, as $E \subset F$, there must be some $a \in F \setminus E$ and thus some $a' \in A'$ attacked by a. As, by the construction of Tr_1, a' is the only argument with claim $claim'(a')$ and is only attacked by a (except for itself), $claim'(a') \in claim'(F) \cup \nu_{CF'}(F)$ and $claim'(a') \notin C \cup \nu_{CF'}(E)$ and thus $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$, contradicting that $C \in cl\text{-}sem(CF')$. Thus, such a set F cannot exist and therefore, $cl\text{-}sem(CF') \subseteq prf_c(CF')$. □

Next, we show the second statement that $stb_c(CF) = stb_c(Tr_2(CF)) = cl\text{-}stb_\tau(Tr_2(CF))$ for $\tau \in \{adm, cf\}$.

Proof. Let $Tr_2(CF) = CF' = (A', R', claim')$. Since $stb_c(CF) \subseteq cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$ holds for any CAF CF, it suffices to show that (i) $stb_c(CF) \subseteq stb_c(CF')$ and (ii) $cl\text{-}stb_{cf}(CF') \subseteq stb_c(CF)$.

First observe that (a) for every set of arguments $E \subseteq A$, E attacks the argument a' in CF' iff $a \in E \cup E^+_{(A,R)}$. Indeed, E attacks an argument a' iff either $a \in E$ or if there is $b \in E$ such that $(b, a) \in R$.

(i) Let $S \in stb_c(CF)$ and consider a stb_c-realization $E \subseteq A$. We show that E is stable in CF': First notice that E is conflict-free since we introduced no attacks between existing arguments in CF'. Moreover, E attacks every argument $a \in A' \setminus E$: Clearly, E attacks every argument $a \in A \setminus E$; moreover, E attacks every $a' \in \{a' \mid a \in A\}$ by (a) since $E \cup E^+_{(A,R)} = A$.

(ii) Let $S \in cl\text{-}stb_{cf}(CF')$, then there is a set $E \in A'$ such that $E \in cf((A', R'))$ and $claim(E) \cup \nu_{CF'}(E) = claim(A')$. We show that $E \in stb((A, R))$. First observe that $E \subseteq A$ since each argument $a' \in \{a' \mid a \in A\}$ is self-attacking; moreover, E is conflict-free in (A, R). We show that E attacks every argument $a \in A \setminus E$: We have $\{c_a \mid a \in A\} \subseteq \nu_{CF'}(E)$ since $claim(E) \cup \nu_{CF'}(E) = claim(A')$. Thus E attacks each argument a' in CF'. We conclude by (a) that $a \in E \cup E^+_{(A,R)}$ for every argument $a \in A$. We have shown that $E \in stb((A, R))$ and, consequently, $S \in stb_c(CF)$. □

Finally we show $stg_c(CF) = stg_c(Tr_3(CF)) = cl\text{-}stg(Tr_3(CF))$.

Proof. Let $Tr_3(CF) = CF' = (A', R', claim')$. The proof proceeds in three steps:

(i) First, observe that $cf((A, R)) = cf((A', R'))$ as all added arguments are self-attacking and we only add attacks between arguments $\{a, b\} \subseteq A$ if there was already one in at least one direction or the attacked argument was self-attacking. Moreover, $\{\emptyset\} \in stg_c(CF)$ if and only if all arguments are self-attacking which is the case if and only if $\{\emptyset\} \in cl\text{-}stg(CF)$.

(ii) Regarding $stg_c(CF) = stg_c(CF')$: For every maximal (with regard to $\subseteq$) $E \in cf(A', R')$, $A \subseteq E \cup E^+_{(A',R')}$, as all arguments in A are either contained or, due to the fact that E is maximal, are attacked by E. Thus, such sets E, due to the fact that all arguments a' are self-attacking, are the only witnessing candidates for the extensions in $stg_c(CF)$ and $stg_c(CF')$. Furthermore, by construction of Tr_3, $E \cup E^+_{(A',R')} = A \cup \{a' \in A' \mid a \in E \cup E^+_{(A,R)}\}$ and thus $E \cup E^+_{(A,R)}$ will be maximal if and only if $E \cup E^+_{(A',R')}$ is maximal.

(iii) Finally, $stg_c(CF') = cl\text{-}stg(CF')$ follows by observing that the claims of all arguments in A' are unique. □

Theorem 2 (restated). The complexity results for CAFs depicted in Table 4 hold.

To show that $\mathit{Skept}^{wf}_{naive}$ is coNP-complete we make use of the following reduction which will also reappear in several proofs from Section 5.

Reduction 3. *Let φ be given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in X and let (A, R) be as in Reduction 1. We define (A', R') with $A' = A \setminus \{\varphi\}$ and $R' = R \setminus \{(cl_i, \varphi) \mid i \leq n\}$.*

Proposition 5 (restated). $\mathit{Skept}^{wf}_{naive}$ is coNP-complete.

Proof. For a well-formed CAF $CF = (A, R, claim)$, one can verify skeptical acceptance of a claim $c \in claim(A)$ by (1) guessing a set $E \subseteq A$ such that $c \notin claim(E)$; (2) checking if $claim(E)$ is a cl-naive claim-set of CF. The latter can be verified in polynomial time, yielding a NP-procedure for the complementary problem.

Hardness can be shown via a reduction from UNSAT: For a formula φ with clauses $C = \{cl_1, \ldots, cl_n\}$ over the atoms X, we define $CF = (A, R, claim)$ where (A, R) is defined as in Reduction 3 and $claim(x) = x$, $claim(\bar{x}) = \bar{x}$, $claim(cl_i) = \bar{\varphi}$. Observe that CF is well-formed. We show φ is satisfiable iff $\bar{\varphi}$ is not skeptically accepted in CF.

In case φ is satisfiable, then there is a model $M \subseteq X$ of φ. Consider $E = M \cup \{\bar{x} \mid x \notin M\}$, which is conflict-free and cannot be extended by any argument cl_i assigned with claim $\bar{\varphi}$: Indeed, since each clause cl_i is satisfied by M, there is either a positive literal $x \in cl_i$ with $x \in M$ or a negative literal $\bar{x} \in cl_i$ with $x \notin M$, thus cl_i is attacked by E in (A, R). Moreover, we have that for each $x \in X$, either $x \in E$ (and thus $x \in claim(E)$) or $\bar{x} \in E$ (and thus $\bar{x} \in claim(E)$) and $(x, \bar{x}) \in R$. Consequently, $claim(E)$ is maximal among i-conflict-free claim-sets and thus $claim(E) \in cl\text{-}naive(CF)$. It follows that $\bar{\varphi}$ is not skeptically accepted in CF.

Now assume $\bar{\varphi}$ is not skeptically accepted in CF, then there is a set $S \in cl\text{-}naive(CF)$ such that $\bar{\varphi} \notin S$. For a cf_c-realization E of S, we have $M = E \cap X$ is a model of φ: Consider an arbitrary clause cl_i. As $\bar{\varphi} \notin S$ we have that E attacks cl_i, thus there is either an argument $x \in E$ such that $(x, cl_i) \in R$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl_i) \in R$. In the former case, we have $x \in M$ and thus M satisfies cl_i, in the latter case $\bar{x} \notin M$ and thus cl_i is satisfied. We obtain that M is a model of φ. □

Proofs of Section 5

Theorem 2 (restated). The complexity results depicted in Table 5 hold.

Proposition 7. *For a CAF* $CF = (A, R, claim)$, *for* $\sigma \in \{prf, naive\}$, $\sigma_c(CF) = cl\text{-}\sigma(CF)$ *if and only if* $\sigma_c(CF)$ *is incomparable.*

Proof. Let $\sigma = prf$ (the proof for $\sigma = naive$ is analogous). Assume $prf_c(CF)$ is incomparable and let $S \in prf_c(CF)$. Then $S \in adm_c(CF)$. Now assume there is $T \in adm_c(CF)$ with $T \supset S$. Consider a adm_c-realization E of T in CF and let $E' \in prf((A, R))$ with $E \subseteq E'$. But then $claim(E') \in prf_c(CF)$ and $claim(E') \supseteq T \supset S$, contradiction to $prf_c(CF)$ being incomparable. □

Proposition 8. Con^{CAF}_{prf} *is* Π^{P}_2*-hard.*

Proof. We present a reduction from $Skept^{AF}_{prf}$: Given an instance (A, R), $a \in A$ from $Skept^{AF}_{prf}$. W.l.o.g. we can assume that the preferred extensions of (A, R) are non-empty (otherwise add an isolated argument). We construct $CF = (A', R', claim)$ with $A' = A \cup \{i, j\}$, $R' = R \cup \{(j, b), (b, j) \mid b \in A\}$, and $claim(a) = claim(j) = c_1$, $claim(b) = c_2$ for $b \in (A\setminus\{a\})\cup\{i\}$. Then $prf((A', R')) = \{E \cup \{i\} \mid E \in prf((A, R))\} \cup \{\{i, j\}\}$ since the argument i is isolated and thus appears in each extension; moreover, j mutually attacks each argument $b \in A$. For all extensions $D \in prf((A', R'))$ with $a \in D$ we have $claim(D) = \{c_1, c_2\}$; for all extensions $D \in prf((A', R'))$, $D \neq \{i, j\}$, with $a \notin D$, we have $claim(D) = \{c_2\}$; moreover, $claim(\{i, j\}) = \{c_1, c_2\}$ and thus we have $\{c_1, c_2\} \in prf_c(CF)$ independently of the considered instance. Thus a is not skeptically accepted in (A, R) with respect to preferred semantics iff $\{c_2\} \in prf_c(CF)$ iff $prf_c(CF)$ is not incomparable. □

Proposition 9. Con^{CAF}_{naive} *is* coNP*-hard.*

Proof. For hardness, we present a reduction from UNSAT: Let φ be given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over literals in X. W.l.o.g. we can assume that φ does not contain tautological clauses, i.e., there is no cl_i, $i \leq n$ with $x, \bar{x} \in cl_i$ for any $x \in X$. Let $A_X = \{a_x \mid x \in X\}$ and $A_{\bar{X}} = \{a_{\bar{x}} \mid x \in X\}$ and let (A, R) be defined as in Reduction 3. We construct a CAF $CF = (A', R', claim)$ with $A' = A \cup A_X \cup A_{\bar{X}}$; $R' = R \cup \{(a, b) \mid a \in A_X \cup A_{\bar{X}}, b \in C \cup X \cup \bar{X}\} \cup \{(cl_i, cl_j) \mid i \neq j \leq m\}$; and $claim(x) = claim(a_x) = x$, $claim(\bar{x}) = claim(a_{\bar{x}}) = \bar{x}$ and $claim(cl_i) = c$. We will show φ is unsatisfiable iff $naive_c(CF)$ is incomparable.

First asssume φ is satisfiable and consider a model M of φ. Let $E = M \cup \{\bar{x} \mid x \notin M\}$. Clearly, E is conflict-free; moreover, as M satisfies each clause cl_i there is either $x \in cl_i$ with $x \in M$ or $\bar{x} \in cl_i$ with $x \notin M$, thus E attacks each cl_i. As E is attacked by each $a \in A_X \cup A_{\bar{X}}$ and since E contains either x or $\bar{x}$ for each $x \in X$ and each pair $x, \bar{x}$ is conflicting, we can conclude that E is a subset-maximal conflict-free set. Moreover, $X \cup \bar{X} \in naive_c(CF)$ witnessed by the set $A_X \cup A_{\bar{X}}$ which is conflict-free and attacks every remaining claim $b \in A' \setminus (A_X \cup A_{\bar{X}})$. It follows that $naive_c(CF)$ is not incomparable since $claim(E) = E \in naive_c(CF)$ and $E \subset X \cup \bar{X}$.

Now assume φ is not satisfiable. We will show that $naive_c(CF) = \{X \cup \bar{X}\} \cup \{X' \cup \{\bar{x} \mid x \notin X'\} \cup \{c\} \mid X' \subseteq X\}$ (implying that $naive_c(CF)$ is incomparable).

(a) $\{X \cup \bar{X}\} \cup \{X' \cup \{\bar{x} \mid x \notin X'\} \cup \{c\} \mid X' \subseteq X\} \subseteq naive_c(CF)$: Consider $X' \subseteq X$ and let $E = X' \cup \{\bar{x} \mid x \notin X'\}$. E is conflict-free. As φ is not satisfiable, there is a clause cl_i such that $x \notin cl_i$ for all $x \in X'$ and $\bar{x} \notin cl_i$ for all $x \notin X'$, otherwise, X' would be a model of φ. Thus $E' = E \cup \{cl_i\} \in cf((A', R'))$. E' is a maximal conflict-free set since $E' \cup \{a\}$ is conflicting for each argument $a \notin E'$; thus $X' \cup \{\bar{x} \mid x \notin X'\} \cup \{c\} \in naive_c(CF)$. The statement follows as $X \cup \bar{X} \in naive_c(CF)$ by construction.

(b) $naive_c(CF) \subseteq \{X \cup \bar{X}\} \cup \{X' \cup \{\bar{x} \mid x \notin X'\} \cup \{c\} \mid X' \subseteq X\}$: Let $S \in naive_c(CF)$. First assume $c \notin S$ and consider a $naive_c$-realization E of S. As $c \notin S$ we have that E is in conflict with each $cl_i \in C$. As φ is unsatisfiable, we conclude that the only possible $naive_c$-realization of S is $A_X \cup A_{\bar{X}}$; indeed, in case $E \cap (X \cup \bar{X}) \neq \emptyset$, consider $M = E \cap X$, then M is a model of φ since for each clause cl_i, there is either $x \in cl_i$ with $x \in M$ or $\bar{x} \in cl_i$ with $x \notin M$. Thus $S = X \cup \bar{X}$ in case $x \notin S$. Now, assume $c \in S$ and consider a $naive_c$-realization E of S. Let cl_i be the argument realizing c in E. Notice that for every $x \in X$, either $\{x, cl_i\}$ or $\{\bar{x}, cl_i\}$ is conflict-free since there is no clause cl_i with $x, \bar{x} \in cl_i$. Thus there is $X' \subseteq X$ such that $X' \cup \{\bar{x} \mid x \notin X'\} \subseteq E$, that is, $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup \{c\}$ in case $c \in S$. □

Proposition 10. $Con^{CAF}_{stb_\tau}$, $\tau \in \{cf, adm\}$ *is* Π^{P}_2*-hard.*

Proof. We present a reduction from $QSAT^\forall_2$. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT^\forall_2$, where φ is given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in $X = Y \cup Z$. Let (A, R) be as in Reduction 3. We construct a CAF $CF = (A, R', claim)$ with $R' = R \cup \{(cl_i, cl_i) \mid i \leq n\}$ and $claim(y) = y$, $claim(\bar{y}) = \bar{y}$, $claim(v) = claim(cl_i) = c$ for $i \leq n$, $v \in Z \cup \bar{Z}$.

We will first show that (a) $cl\text{-}stb_\tau = \{Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \mid Y' \subseteq Y\}$: Each τ-cl-stable claim-set S contains either y or $\bar{y}$ by construction; moreover, $c \in S$ since c is not defeated by any conflict-free set of arguments $E \subseteq A$, thus each τ-cl-stable claim-set is of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\}$ for some $Y' \subseteq Y$. Moreover, each such set is stb_τ-realizable, since for any $Y' \subseteq Y$, $z \in Z$, the set $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{z\}$ is admissible (conflict-free) in (A, R') and attacks every $a \in A$ such that $claim(a) \notin claim(E)$.

We show Ψ is valid iff $stb_c(CF) = cl\text{-}stb_\tau(CF)$.

Assume Ψ is valid. Let $Y' \subseteq Y$. Then there is $Z' \subseteq Z$ such that φ is satisfied by $M = Y' \cup Z'$. Let $E = M \cup \{\bar{x} \mid x \notin M\}$. Since M satisfies each clause cl_i, there is either $x \in cl_i$ with $x \in M$ or there is $\bar{x} \in cl_i$ with $x \notin M$. It follows that each cl_i, $i \leq n$, is attacked by E. Since E is also conflict-free we have shown that E is a stable extension of (A, R) and therefore $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in stb_c(CF)$. As Y' was arbitrary, we have that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in$

$stb_c(CF)$ for all $Y' \subseteq Y$. We conclude that $stb_c(CF) = cl\text{-}stb_\tau(CF)$ by (a).

Assume $stb_c(CF) = cl\text{-}stb_\tau(CF)$. Let $Y' \subseteq Y$. By (a) we have that $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in cl\text{-}stb_\tau(CF) = stb_c(CF)$. Consider a stb_c-realization E of S and let $Z' = E \cap Z$. Then $M = Y' \cup Z'$ satisfies φ: Consider an arbitrary clause cl_i. As E attacks cl_i there is either an argument $x \in E$ with $(x, cl_i) \in R$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl_i) \in R$. In the former case, $x \in cl_i$ and $x \in M$ and thus cl_i is satisfied; in the latter case, $\bar{x} \in cl_i$ and $x \notin M$ and thus cl_i is satisfied. Thus M is a model of φ. We have shown that for every $Y' \subseteq Y$, there is $Z' \subseteq Z$ such that $Y' \cup Z'$ satisfies φ. It follows that Ψ is valid. □

We will now consider Π_3^P-hardness of semi-stable and stage semantics. We will make use of the following observations.

Lemma 5. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^\exists$ and let $CF = (A, R, claim)$ be as in Reduction 2. Then for all $E \in sem((A, R))$,*

1. *$\varphi \in E \Leftrightarrow \bar{\varphi} \notin E$;*
2. *$\varphi \in E \Leftrightarrow E^\oplus_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$;*
3. *$\bar{\varphi} \in E \Leftrightarrow E^\oplus_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$.*

Proof. (1) is immediate by construction.

For (2), first assume $\varphi \in E$. Then $\bar{\varphi}, d_1 \in E^\oplus_{(A,R)}$ since $\varphi \in E$; also, $\varphi \in E$ only if E defends φ against each cl_i, $i \leq n$, thus each cl_i is attacked by E; moreover, each $a \in V \cup \bar{V}$ is either contained or attacked by E, otherwise, there is $D = E \cup \{a\}$ with $D^\oplus_{(A,R)} \supset E^\oplus_{(A,R)}$, contradiction to $E \in sem((A, R))$. Thus $V \cup \bar{V} \in E^\oplus_{(A,R)}$ and $d_a \in E^\oplus_{(A,R)}$ for $a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})$. In case $E^\oplus_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$, we have $\varphi \in E$ since φ is the only argument attacking d_1.

To show (3) first assume $\bar{\varphi} \in E$. Then $\varphi \in E^+_{(A,R)}$; moreover, each $a \in V \cup \bar{V}$ is either contained or attacked by E, otherwise, there is $D = E \cup \{a\}$ with $D^\oplus_{(A,R)} \supset E^\oplus_{(A,R)}$, contradiction to $E \in sem((A, R))$. Thus we have $V \cup \bar{V} \in E^\oplus_{(A,R)}$ and $d_a \in E^\oplus_{(A,R)}$ for $a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})$. Also, each cl_i is either attacked by E or defended by E (observe that there is at least one $i \leq n$ such that $cl_i \in E$, otherwise $D = (E \cup \{\varphi\}) \setminus \{\bar{\varphi}\}$ is admissible and $D^\oplus_{(A,R)} \supset E^\oplus_{(A,R)}$). The other direction follows since $d_1 \notin E^\oplus_{(A,R)}$ and thus $\varphi \notin E$. □

Lemma 6. *Let $CF = (A, R, claim)$ be as in Reduction 2 for an instance $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ of $QSAT_3^\exists$. Then, (1) $cl\text{-}sem(CF) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$; (2) $sem_c(CF) \subseteq cl\text{-}sem(CF)$; and (3) $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\} \in sem_c(CF)$ for all $X' \subseteq X$.*

Lemma 6 (restated). Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^\exists$ and let $CF = (A, R, claim)$ be as in Reduction 2. Then

1. $cl\text{-}sem(CF) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$;
2. $sem_c(CF) \subseteq cl\text{-}sem(CF)$; and
3. $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\} \in sem_c(CF)$ for all $X' \subseteq X$.

Proof. To prove the statements we will first show that (i) each cl-semi-stable and each i-semi-stable claim-set is of the form $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$: As $sem_c(CF) \subseteq prf_c(CF)$ and $cl\text{-}sem(CF) \subseteq prf_c(CF)$, it suffices to prove the statement for each i-preferred claim-set S. First observe that S cannot contain both $a, \bar{a}$ for $a \in X \cup \{\varphi\}$ since there is no cf_c-realization containing both $a, \bar{a}$. As each other claim in $claim(A) \setminus (V \cup \bar{V} \cup \{\varphi, \bar{\varphi}\})$ is self-attacking, it remains to show that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \subseteq S$ for some $X' \subseteq X$, for $e \in \{\varphi, \bar{\varphi}\}$. S contains $X' \cup \{\bar{x} \mid x \notin X'\}$ for some for some $X' \subseteq X$: Assume there is $x \in X$ such that $x, \bar{x} \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{x\}$. D is conflict-free since $\bar{x}, d_x \notin E$, moreover, $cl_i \notin E$ for each clause cl_i with $(x, cl_i) \in R$, since cl_i is not defended against the attack (x, cl_i). Also, D is admissible since E does not contain the only attacker $\bar{x}$ of x and $D \supset E$, contradiction to E being preferred in (A, R). S contains $Y \cup Z$: Assume there is $v \in Y \cup Z$ such that $v \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{v\}$. D is admissible since $\bar{v} \notin E$ by assumption $v \notin S$ and $D \supset E$, contradiction to E being preferred in (A, R). S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{\bar{\varphi}\}$. D is admissible since $\varphi \notin E$ and $D \supset E$, contradiction to E being preferred in (A, R).

(1) By (i), it suffices to show that (a) for all $X' \subseteq X$, $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ is cl-semi-stable; and (b) for all $X' \subseteq X$, $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ is cl-semi-stable.

(a) Let $X' \subseteq X$ and fix $Y' \subseteq Y$, $Z' \subseteq Z$. Let $\mathcal{C}' \subseteq \mathcal{C}$ denote the set of clauses cl_i which are not attacked by $X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}$. Let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \mathcal{C}' \cup \{\bar{\varphi}\}$. Then E is admissible and $claim(E) = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ and $\nu_{CF}(E) = \{d_a \mid a \in X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}\} \cup \{\varphi\}$. $claim(E) \cup \nu_{CF}(E)$ is subset-maximal among admissible sets since it contains every claim $c \in claim(A)$ which is assigned to non-self-attacking arguments; moreover, it contains a maximal set of claims among $\{d_v \mid v \in V \cup \bar{V}\}$ since it contains precisely one of $d_v, d_{\bar{v}}$ for each $v \in V$; furthermore observe that $d \notin \nu_{CF}(E)$ for all conflict-free sets $E \subseteq A$ since $d_2 \notin E^+_{(A,R)}$ for every $E \in cf((A, R))$.

(b) Let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \{\varphi\}$ for some $X' \subseteq X$, $Z' \subseteq Z$ and $Y' \subseteq Y$ with $y_0 \in Y'$. E defends φ as $y_0 \in cl_i$ for all $i \leq n$, thus E is admissible. Moreover, $claim(E) = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ and $\nu_{CF}(E) = \{d_a \mid a \in X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}\} \cup \{\bar{\varphi}\}$. Similar as above, we conclude that $claim(E) \cup \nu_{CF}(E)$ is subset-maximal.

(2) $sem_c(CF) \subseteq cl\text{-}sem(CF)$ by (i) and (1).

(3) Consider $X' \subseteq X$ and let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \{\varphi\}$ for some $Z' \subseteq Z$ and $Y' \subseteq Y$

with $y_0 \in Y'$. E defends φ as $y_0 \in cl_i$ for all $i \leq n$, thus E is admissible. Moreover, E is semi-stable since $E^{\oplus}_{(A,R)} = V \cup \bar{V} \cup \{d_a \mid a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})\} \cup \mathcal{C} \cup \{\varphi, \bar{\varphi}, d_1\}$ is subset-maximal: Assume there is $D \in adm((A,R))$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, that is, there is $e \in \{d_2\} \cup \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\}$ such that $e \in D^{\oplus}_{(A,R)}$; in particular, $e \in D^{+}_{(A,R)}$ because all considered arguments are self-attacking. Observe that $d_2 \notin D^{+}_{(A,R)}$ since its only attacker is self-attacking. In case $e = d_a$ for some $a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E$ we have $a \in D$ and $\bar{a} \in D$ and thus D is conflicting, contradiction to D being conflict-free. Thus we have shown that $claim(E) = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ is i-semi-stable. $\square$

Proposition 6 (restated). *Con^{CAF}_{sem}* is Π^{P}_3-hard.

Proof. Let $CF = (A, R, claim)$ be the CAF generated by Reduction 2 from $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$. We show Ψ is valid iff $sem_c(CF) \neq cl\text{-}sem(CF)$. Since $sem_c(CF) \subseteq cl\text{-}sem(CF)$ by Lemma 6 (2), the latter reduces to $sem_c(CF) \subset cl\text{-}sem(CF)$. By Lemma 6 (3), this is the case if there is some $X' \subseteq X$ such that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$ is not sem_c-realizable.

Assume Ψ is valid, that is, there is $X' \subseteq X$ such that for all $Y' \subseteq Y$, there is $Z' \subseteq Z$ such that $X' \cup Y' \cup Z'$ is a model of φ. We show $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \notin sem_c(CF)$. Towards a contradiction, assume there is $E \in sem((A,R))$ such that $claim(E) = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\}$. Then $\bar{\varphi} \in E$. By Lemma 5, $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$. Let $Y' = E \cap Y$. By assumption Ψ is valid, there is $Z' \subseteq Z$ such that $M = X' \cup Y' \cup Z'$ is a model of φ. Let $D = M \cup \{\bar{v} \mid v \notin M\} \cup \{\varphi\}$. D is conflict-free; moreover, D attacks every cl_i, $i \leq n$: Since M is a model of φ, we have that for all clauses cl_i, $i \leq n$, there is $v \in V$ such that either $v \in cl_i \cap M$ (in that case, $v \in D$ and $(v, cl_i) \in R$) or $\bar{v} \in cl_i$ and $v \notin M$ (in that case, $\bar{v} \in D$ and $(\bar{v}, cl_i) \in R$). It follows that D is admissible. We show that $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$: Clearly, $V \cup \bar{V} \subseteq D^{\oplus}_{(A,R)}$; also, $\mathcal{C} \subseteq D^{\oplus}_{(A,R)}$ as shown above; moreover, $\bar{\varphi}, d_1 \in D^{\oplus}_{(A,R)}$ since $\varphi \in D$. As D and E contain the same arguments $a \in X \cup \bar{X} \cup Y \cup \bar{Y}$ by construction, we furthermore have $\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} = \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus D\}$. It follows that $D^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$. Thus D is admissible and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to our assumption E is semi-stable in (A,R).

Assume Ψ is not valid. We show that for all $X' \subseteq X$, $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \in sem_c(CF)$. Fix $X' \subseteq X$. Since Ψ is not valid, there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $X' \cup Y' \cup Z'$ is not a model of φ. Fix $Z' \subseteq Z$ and let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \mathcal{C}' \cup \{\bar{\varphi}\}$, where $\mathcal{C}' \subseteq \mathcal{C}$ contains all clauses cl_i which are not attacked by $X' \cup Y' \cup Z' \cup \{\bar{a} \mid a \notin X' \cup Y' \cup Z'\}$. Then E is admissible and $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$. We show that E is semi-stable in (A,R). Assume there is $D \subseteq A$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$. First observe that D attacks the same arguments d_a, $a \in X \cup \bar{X} \cup Y \cup \bar{Y}$, as E and thus $X' \cup Y' \subseteq D$. By Lemma 5 and since $D^{\oplus}_{(A,R)}$ is strictly bigger than $E^{\oplus}_{(A,R)}$, we have that $D^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus D\} \cup \{d_2\})$. It follows that $\varphi \in D$. Let $Z'' = D \cap Z$. Then $M = X' \cup Y' \cup Z''$ is a model of φ: As each cl_i, $i \leq n$, is attacked by D, there is a literal $l \in D$ with $l \in cl_i$; now, if l is a positive literal, we have $l \in M$, in case l is a negative literal, we have $l \notin M$. Therefore φ is satisfied by M, contradiction to our initial assumption Ψ is not valid. It follows that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\bar{\varphi}\} \in sem_c(CF)$ for all $X' \subseteq X$. Thus $sem_c(CF) = cl\text{-}sem(CF)$ by Lemma 6. $\square$

We now turn to stage semantics.

Lemma 7. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT^{\exists}_3$ and let $CF = (A, R, claim)$ be as in Reduction 2. Then*

1. *$cl\text{-}sem(CF) = cl\text{-}stg(CF)$; and*
2. *$sem_c(CF) = stg_c(CF)$.*

Proof. To prove the statements we will first show that (i) each cl-stage and each i-stage claim-set is of the form $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$: Let $S \in stg_c(CF) \cup cl\text{-}stg(CF)$, $V = X \cup Y \cup Z$. First notice that $S \subseteq X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$, for $e \in \{\varphi, \bar{\varphi}\}$: S cannot contain both $a, \bar{a}$ for $a \in X \cup \{\varphi\}$ since there is no cf_c-realization E containing both $b, \bar{b}$, for $b \in X$, nor φ, b for $b \in \{\bar{\varphi}\} \cup \mathcal{C}$. It remains to show that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \subseteq S$ for some $X' \subseteq X$, for $e \in \{\varphi, \bar{\varphi}\}$.

Let $S \in stg_c(CF)$ and consider a stg_c-realization E of S. E contains $V' \cup \{\bar{v} \mid v \notin V'\}$ for some $V' \subseteq V$: Assume there is $v \in V$ such that $v, \bar{v} \notin E$ and let $D = (E \setminus \{cl_i \mid (v, cl_i) \in R\}) \cup \{v\}$. D is conflict-free since $\bar{v}, d_v \notin E$ and since $cl_i \notin E$ for each clause cl_i with $(v, cl_i) \in R$. Moreover, each such cl_i is attacked by D and thus $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A,R). Moreover, E contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin E$ and let $D = E \cup \{\bar{\varphi}\}$. D is conflict-free since $\varphi \notin E$ and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A,R).

Let $S \in cl\text{-}stg(CF)$. We will first show that S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $claim(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets. Let $D = E \cup \{\bar{\varphi}\}$. D is conflict-free since $\varphi \notin E$ and thus $claim(D) \cup \nu_{CF}(D) = claim(E) \cup \nu_{CF}(E) \cup \{\varphi, \bar{\varphi}\} \supset claim(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. S contains $X' \cup \{\bar{x} \mid x \notin X'\}$ and $Y \cup Z \subseteq S$: Assume there is $x \in X$ such that $x, \bar{x} \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $claim(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets. In case $\varphi \in S$, then $\varphi \in E$ and $\bar{\varphi} \notin E$, $cl_i \notin E$, $i \leq n$, since they are in conflict with φ. Then $D = E \cup \{x\}$ is conflict-free and properly extends E, thus $claim(D) \cup \nu_{CF}(D) \supset claim(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. In case $\bar{\varphi} \in E$, let $D = (E \setminus \{cl_i \mid (x, cl_i) \in R\}) \cup \{x, \bar{\varphi}\}$. D is conflict-free since $\bar{x}, d_x \notin E$, $cl_i \notin E$ for each clause

cl_i with $(v, cl_i) \in R$ and $\varphi \notin E$ by assumption $\bar{\varphi} \in S$. $claim(D) = claim(E) \cup \{x\}$ since the only arguments which have been removed from D are labelled with claim $\bar{\varphi}$ and D contains $\bar{\varphi}$; moreover, $\nu_{CF}(E) \subseteq \nu_{CF}(D)$ since φ is the only attacked argument of each cl_i and $(\bar{\varphi}, \varphi) \in R$. Consequently, $claim(D) \cup \nu_{CF}(D) \supset claim(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. $Y \cup Z \subseteq S$: Assume there is $v \in Y \cup Z$ such that $v \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $claim(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets and E does not contain $v, \bar{v}$ by assumption. Analogous to above, one can extend E appropriately to derive a contradiction to S being cl-stage.

(1) Analogous to Lemma 6, one can show that $cl\text{-}stg(CF) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$.

(2) We will show (a) $stg_c(CF) \subseteq sem_c(CF)$; and (b) $sem_c(CF) \subseteq stg_c(CF)$.

To show (a), let $S \in stg_c(CF)$. By (i), either $\varphi \in S$ or $\bar{\varphi} \in S$. In case $\varphi \in S$, we have $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ for some $X' \subseteq X$, thus $S \in sem_c(CF)$ by Lemma 6. In case $\bar{\varphi} \in S$, we consider a stg_c-realization E of S. E is admissible: Each $a \in V \cup \bar{V} \cup \{\bar{\varphi}\}$ defends itself; also, $\varphi \notin E$ by (i); moreover, each $cl_i \in E$ is defended by E, otherwise there is $cl_i \in E$ which is not defended by E against some argument $a \in V \cup \bar{V}$, thus $\bar{a} \notin E$, that is, there is $v \in V$ such that $v, \bar{v} \notin E$, contradiction to (i). Thus E is semi-stable, otherwise there is some set $D \in adm((A, R)) \subseteq cf((A, R))$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A, R).

To show (b), let $S \in sem_c(CF)$ and consider a sem_c-realization E of S. Clearly, E is conflict-free. We show that $E \in stg((A, R))$. Towards a contradiction, assume that there is $D \in cf((A, R))$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$. Let $a \in D^{\oplus}_{(A,R)} \setminus E^{\oplus}_{(A,R)}$. By Lemma 5, either $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$ (in case $\varphi \in E$) or $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$ (in case $\bar{\varphi} \in E$); that is, $a \in \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\}$. Also, for all $v \in V$, either $d_v \in E^{\oplus}_{(A,R)}$ or $d_{\bar{v}} \in E^{\oplus}_{(A,R)}$, otherwise $v, \bar{v} \notin E$; let $E' = E \cup \{v\}$, then $(E')^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being semi-stable. In case $a = d_b$ for some $b \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\}$, we have $d_b, d_{\bar{b}} \in D^{\oplus}_{(A,R)}$ and thus $b, \bar{b} \in D$, contradiction to D being conflict-free. Moreover, $a \neq d_2$ since the only attacker d_1 of d_2 is self-attacking. Consider the case $a = d_1$, then $\varphi \in D$ since φ is the only attacker of d_1. Thus $cl_i \notin D$ for all $i \leq n$ by conflict-freeness of D; we conclude that D attacks each cl_i, $i \leq n$ since $cl_i \in E^{\oplus}_{(A,R)}$ for all $i \leq n$ and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$. Therefore D is admissible and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being semi-stable. □

Π^{P}_3-hardness of Con^{CAF}_{stg} thus follows from Lemma 7 and Proposition 6.

Proposition 11. *Con^{CAF}_{stg} is Π^{P}_3-hard.*

Next we consider concurrence of semi-stable and stage semantics in well-formed CAFs. We will adapt Reduction 1 as follows.

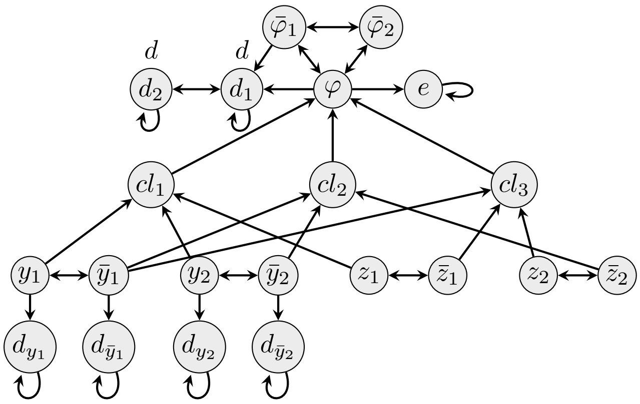

Figure 3: Reduction 4 for the formula $\forall Y \exists Z \varphi(Y, Z)$ where $\varphi(Y, Z)$ is given by the clauses $\{\{z_1, y_1, y_2\}, \{\bar{y}_1, \bar{y}_2, \bar{z}_2)\}, \{\bar{z}_1, \bar{y}_1, z_2\}\}$.

Reduction 4. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT^{\forall}_2$, where φ is given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. Let (A, R) be the AF constructed from φ as in Reduction 1. We define $CF = (A', R', claim)$ with*

- *$A' = A \cup \{e, d_1, d_2, \bar{\varphi}_1, \bar{\varphi}_2\}$;*
- *$R' = R \cup \{(a, d_a)(d_a, d_a) \mid a \in Y \cup \bar{Y}\} \cup \{(d_i, d_j) \mid i, j = 1, 2\} \cup \{(a, b) \mid a, b \in \{\varphi, \bar{\varphi}_1, \bar{\varphi}_2\}, a \neq b\} \cup \{(\varphi, e), (e, e), (\varphi, d_1), (\bar{\varphi}_1, d_1)\}$;*
- *$claim(d_1) = claim(d_2) = d$ and $claim(v) = v$ else.*

An example to illustrate the reduction is given in Figure 3.

Lemma 8. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT^{\forall}_2$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 4. Then*

1. *for all $E \in cf((A, R))$, $(claim(E))^+_{CF} = E^+_{(A,R)} \setminus \{d_1\}$;*
2. *every $S \in cf_c(CF)$ admits a unique realization in (A, R);*
3. *for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$, either $\varphi \in S$ or $\varphi_1 \in S$ or $\varphi_2 \in S$.*

Proof. (1) and (2) follow since $claim(a) = a$ for every non-self-attacking argument a; moreover, $d_2 \notin E^+_{(A,R)}$ for every conflict-free set E since d_1 is the only attacker of d_2; (3) is immediate by construction. □

Lemma 9. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT^{\forall}_2$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 4. Then $\sigma_c(CF) \subseteq cl\text{-}\sigma(CF)$.*

Proof. Consider $S \in \sigma_c(CF)$ and let E denote the unique σ_c-realization of S in (A, R). As $E \in \sigma((A, R))$, we have that $E \cup E^+_{(A,R)}$ is subset-maximal among admissible (conflict-free) extensions. We will show that $S \cup S^+_{CF}$ is subset-maximal among i-admissible (i-conflict-free) claim-sets. Towards a contradiction, assume $S \cup S^+_{CF}$ is not subset-maximal among i-admissible (i-conflict-free) claim-sets, that is, there is $T \in adm_c(CF)$ ($T \in cf_c(CF)$) with

$T \cup T^+_{CF} \supset S \cup S^+_{CF}$. Consider the unique cf_c-realization D of T in (A, R), then $D \cup D^+_{(A,R)} \setminus \{d_1\} = T \cup T^+_{CF} \supset S \cup S^+_{CF} = E \cup E^+_{(A,R)} \setminus \{d_1\}$. If either $d_1 \in D^+_{(A,R)}$ or $d_1 \notin E^+_{(A,R)}$ we are done since in this case, we have $D \cup D^+_{(A,R)} \supset E \cup E^+_{(A,R)}$, contradiction to E being semi-stable (stage) in (A, R). Thus we assume $d_1 \in E^+_{(A,R)}$ but $d_1 \notin D^+_{(A,R)}$. By Lemma 8, we have $\varphi_2 \in D$ since φ_2 does not attack d_1; also, $\varphi_1 \in E$ or $\varphi \in E$. In case $\varphi \in E$, we have $e \in E^+_{(A,R)}$, $e \notin D^+_{(A,R)}$ thus $e \in S \cup S^+_{CF}$ but $e \notin T \cup T^+_{CF}$, contradiction to the assumption $T \cup T^+_{CF} \supset S \cup S^+_{CF}$. In case $\varphi_2 \in D$ and $\varphi_1 \in E$, consider $D' = (D \cup \{\varphi_1\}) \setminus \{\varphi_2\}$. D' is admissible (conflict-free) as D is admissible (conflict-free) and exchanging φ_2 with φ_1 does neither add conflicts nor undefended arguments. Moreover, $d_1 \in (D')^+_{(A,R)}$ and $D \cup D^+_{(A,R)} = D' \cup (D')^+_{(A,R)} \setminus \{d_1\}$. Therefore $D' \cup (D')^+_{(A,R)} \supset E \cup E^+_{(A,R)}$, contradiction to E being semi-stable (stage) in (A, R). $\square$

Lemma 10. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^\forall$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 4. Then for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$, $\varphi \in S$ implies $S \in \sigma_c(CF) \cap cl\text{-}\sigma(CF)$.*

Proof. By Lemma 9, $\sigma_c(CF) \subseteq cl\text{-}\sigma(CF)$ thus it suffices to prove the statement for $S \in cl\text{-}\sigma(CF)$. Let E denote the unique cf_c-realization of S in (A, R). We will show $E \in \sigma((A, R))$. Towards a contradiction, assume there is $D \in adm((A, R))$ ($D \in cf((A, R))$) with $D \cup D^+_{(A,R)} \supset E \cup E^+_{(A,R)}$. As $\varphi \in E$ we have $d_1 \in E^+_{(A,R)}$ and thus $D \cup D^+_{(A,R)} \setminus \{d_1\} \supset E \cup E^+_{(A,R)} \setminus \{d_1\}$. By Lemma 8, $claim(D) \cup claim(D)^+_{(A,R)} = D \cup D^+_{(A,R)} \setminus \{d_1\} \supset E \cup E^+_{(A,R)} \setminus \{d_1\} = S \cup S^+_{CF}$, contradiction to S being cl-semi-stable (cl-stage) in CF. $\square$

Proposition 12. *Con^{wf}_σ, $\sigma \in \{sem, stg\}$, is Π_2^{P}-hard.*

Proof. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^\forall$ and let $CF = (A, R, claim)$ be as in Reduction 4.

We will show Ψ is valid iff $\sigma_c(CF) = cl\text{-}\sigma(CF)$.

First assume Ψ is valid. We show that in this case, $\varphi \in S$ for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$. By Lemma 10, this implies $S \in \sigma_c(CF) \cap cl\text{-}\sigma(CF)$ and thus $\sigma_c(CF) = cl\text{-}\sigma(CF)$.

By Lemma 9, it suffices to prove the statement for every $S \in cl\text{-}\sigma(CF)$. Towards a contradiction, assume there is $S \in cl\text{-}\sigma(CF)$ such that $\varphi \notin S$. Then $e \notin S \cup S^+_{CF}$. Let $Y' = S \cap Y$. Since Ψ is valid, there is $Z' \subseteq Z$ such that $Y' \cup Z'$ is a model of φ. Let $E = Y' \cup Z' \cup \{\bar{x} \mid x \notin Y' \cup Z'\} \cup \{\varphi\}$. Then $S' = claim(E)$ is i-admissible (i-conflict-free) and $S' \cup (S')^+_{CF} = claim(A) \setminus (\{d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\})$. We can conclude that $S' \cup (S')^+_{CF} \supset S \cup S^+_{CF}$ since $e \notin S \cup S^+_{CF}$ and $\{d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\} \not\subseteq S \cup S^+_{CF}$, contradiction to our initial assumption S is cl-semi-stable (cl-stage). It follows that $\varphi \in S$ for every $S \in cl\text{-}\sigma(CF)$.

Now assume Ψ is not valid, i.e., there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $Y' \cup Z'$ is not a model of φ. We will show that $\sigma_c(CF) \subset cl\text{-}\sigma(CF)$. Fix $Z' \subseteq Z$ and let $E = Y' \cup Z' \cup \{\bar{x} \mid x \notin Y' \cup Z'\}$. Moreover, let $E_1 = E \cup \mathcal{C}' \cup \{\varphi_1\}$ and $E_2 = E \cup \mathcal{C}' \cup \{\varphi_2\}$ where $\mathcal{C}' \subseteq \mathcal{C}$ contains all clauses cl_i such that $E \cap cl_i = \emptyset$. Clearly, $E_1, E_2 \in adm((A, R))$ ($E_1, E_2 \in cf((A, R))$) and thus $E_1 = claim(E_1), E_2 = claim(E_2) \in adm_c(CF)$ ($E_1 = claim(E_1), E_2 = claim(E_2) \in cf_c(CF)$). Observe that $(E_2)^\oplus_{(A,R)} \subset (E_1)^\oplus_{(A,R)}$ since d_1 is attacked by $\varphi_1 \in E_1$ but there is no $a \in E_2$ such that $(a, d_1) \in R$. It follows that $E_2 = claim(E_2) \notin \sigma_c(CF)$. We show that $E_2 \in cl\text{-}\sigma(CF)$ for $\sigma \in \{sem, stg\}$, that is, we show that $claim(E_2) \cup (E_2)^+_{CF} = claim(A) \setminus (\{e, d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\})$ is maximal among admissible (conflict-free) claim-sets: Towards a contradiction, assume there is $T \in adm_c(CF)$ ($T \in cf_c(CF)$) such that $T \cup T^+_{CF} \supset claim(E_2) \cup (E_2)^+_{CF}$. As $\{d_y \mid y \in Y'\} \cup \{d_{\bar{y}} \mid y \notin Y'\} \subseteq T^+_{CF}$ we have $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq T$ and T^+_{CF} does not contain any claim in $\{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\}$ since for every $y \in Y$, there is no conflict-free set attacking both d_y and $d_{\bar{y}}$. Moreover, $d \notin T^+_{CF}$ for every $T \in cf_c(CF)$ since d_1 and d_2 are the only attackers of d_2 and d_1 is self-attacking. It follows that $e \in T^+_{CF}$ and thus $\varphi \in T$. Consider the unique cf_c-realization D of T. Since $\varphi \in D$ we have we have $cl_i \notin D$ for every $i \leq n$ and thus each cl_i is attacked by D. Let $M = D \cap X$ and consider an arbitrary clause cl_i. As each cl_i is attacked by D, there is either $x \in D$ with $x \in cl_i$ or $\bar{x} \in D$ with $\bar{x} \in cl_i$. In the former case, we have $x \in M$ and thus cl_i is satisfied, in the latter case, $x \notin M$ and thus cl_i is satisfied. Thus M is a model of φ and $Y' \subseteq M$, contradiction to our initial assumption $Y' \cup Z''$ is not a model of φ for every $Z'' \subseteq Z$. $\square$